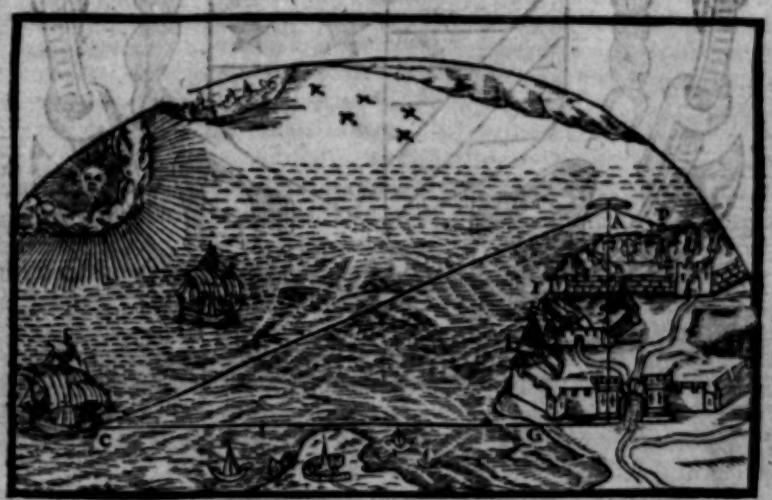
A GEOMETRICAL PRACTICAL TREATIZE NAMED PANTOMETRIA,

divided into three Bookes, LONGIMETRA, PLANIMETRA, and STEREOMETRIA, Containing rules manifolde for menſuration of all *Lines*, *Superficies* and *Solides*: with ſundrie ſtrange concluſions both by Inſtrument and without, and alſo by *Glaſſes* to ſet forth the true Deſcription or exact Platte of an whole *Region*. Firſt publiſhed by *Thomas Digges* Eſquire, and Dedicated to the Graue, Wiſe, and Honourable, Sir *Nicholas Bacon* Knight, Lord Keeper of the great Seale of England. With a Mathematicall diſcourſe of the fiue regular *Platonicall Solides*, and their *Metamorphoſis* into other fiue compound rare *Geometricall Bodyes*, conteyning an hundred newe *Theoremes* at leaſt of his owne *Inuention*, neuer mentioned before by anye other *Geometrician*.

LATELY REVIEWED BY THE AVTHOR himſelfe, and augmented with ſundrie *Additions*, *Diffinitions*, *Problemes* and rare *Theoremes*, to open the paſſage, and prepare away to the vnderſtanding of his Treatize of *Martiall Pyrotechnie* and great *Artillerie*, hereafter to be publiſhed.



AT LONDON
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ANNO. 1591.

MEDIOCRIA · FIRMA ·

To the right honorable my singular good Lorde Sir *Nicholas Bacon* Knight, Lorde Keeper *of the Great Seale of England.*

CAlling to memorie right Honourable, and my singular good Lorde, the great fauour your Lordſhip bare my Father in his life time, and the conference it pleaſed your honour to vſe with him touchyng the Sciences Mathematicall, eſpecially in Geometricall menſurations, peruſing alſo of late certaine volumes that he in his youthe time long ſithens had compiled in the Engliſh tongue, among other I found this *Geometricall Practiſe*, which my Father (if God had ſpared him life) minded to haue preſented your Honoure withall, but vntimely Death preuenting his determination, I thought it my part to accompliſh the ſame, aſwell for the ſatiſfaction of his deſire, as alſo to ſhew my ſelfe not vnmindefull of ſo many good turnes as your honor from time to time moſt aboundantly hath beſtowed on me, hauing therfore ſupplied ſuch partes of this Treatiſe as were leaſt obſcure or vnperfect, adioyning therevnto a Diſcourſe Geometricall of the fiue regulare or Platonicall bodyes, contayning ſundry Theoricall and practicall propoſitions of the manifolde proportions ariſing by mutuall conference of theſe Solides *Inſcripton*, *Circumſcripton* or *Tranſformation*, and now at the laſt fully finiſhed the ſame, I am bolde to exhibite and dedicate it to your Honor, as an eternall memoriall of your Lordſhips great fauoure towardes the furtherance of learning, and a publike teſtimonie of my bounden duetie: hoping your Honor will rather reſpect the good will wherewith it is preſented, than the worthineſſe of the preſent, not agreeable I confeſſe to the excellent knowe-ledge wherewith your Lordſhippe is indued, euen in the verye Fountaines themſelues whence theſe concluſions as ſprings or branches are deriued. And yet ſuch as I nothing doubt your Honor will both accept in good parte, and alſo at vacant leyſure from affaires of more importance, delite your ſelfe withall, the rather for that it containeth ſundry ſuch new inuented *Theoremes*, and other ſtrange concluſions, as no Geometers haue hitherto, in any language publiſhed. Whereby your Lordſhip ſhall not only incourage me hereafter to attempt greater matters, but alſo as it were with a ſoueraigne medicine preuent the poiſoned infection of enuious backbiting toongs: for as the veritie of theſe experiments and rules ſhall neuer be impugned, being ſo firmely grounded, garded, and defended with Geometricall Demonſtrations againſt whoſe puiſſance no ſubtile Sophiſtrie or craftie coloured arguments can preuaile, ſo thinke I there is none ſo impudently malitious, as wil or dare reproue them for vaine or improfitable, when they ſhall perceiue your Lordſhip (whoſe learned iudgement, grauitie and wiſedome, is ſufficiently knowne to the world) doth allow and accepte them as fragrant floures, ſelect and gathered out of the pleaſant gardines Mathematicall, meete to delite any noble, free, or well diſpoſed minde, and profitable frutes ſeruing moſt commodiouſly to ſundry neceſſary vſes in a publike weale, and ſuch as ſhall thereby receiue pleaſure or commoditie, muſt of dutie yeeld condigne thankes vnto your Lorſhip, vnder whoſe protection and patronage I haue not feared to ſend abroade (as a wandring pilgrime) this Orphane and fatherleſſe childe, the which as I perceiue of your Honor fauorably accepted, ſo meane I, God ſparing life, to imploy no ſmall portion of this my ſhorte and tranſitorie time in ſtoring our natiue tongue with Mathematicall demonſtrations, and ſome ſuch other rare experiments and practicall concluſions, as no forraine Realme hath hitherto beene, I ſuppoſe, partaker of. In the meane time I leaue longer to detaine your honour with my rude and homely tale, from more ſeriouſe and waightie affaires, committing your Lordſhip to the tuition of the Almightie, who graunt you a long healthfull honourable life, accompanied with perfect felicitie.

Your Honours moſt bounden
Thomas Digges.

The Preface to the Reader.

ALthough Geometrie for the certaintie thereof haue such a priuilidge as ſerue other Sciences, being ſo fortified with Demonſtration, that no Precept or Rule thereof for the vertue can be reproued, yet leaſt ſome, either of ignorance or malice ſhould affirme it vnprofitable, not ſeruing to any neceſſarie vſe in a publike Weale, I thought good ſomewhat to ſaye thereof, and firſt the ſentence of Plato written at the entrie into his Schole commeth to my remembraunce, [illegible] thereby excluding all ſuch as were ignorant of Geometrie as vnable or vnmeete to attaine higher ſecrets or miſteries of Philoſophie. Ariſtotle alſo intreating of Morall Philoſophie, in the fifth Booke of his Ethikes, with Geometricall Figures moſt beautifully painteth out Iuſtice, diſcerning and ſeuerally comparing ſundrie partes thereof with Geometricall and Arithmeticall proportions. But his Interpreters, whoſe workes are yet extant, for want of skill in theſe matters, haue ſo blemiſhed, darkned and defaced his meaning, that ſcarcely any reſemblance thereof ſhall to the Reader appeare. In ſundrie other his workes alſo of naturall Philoſophie, as the Phyſikes, Meteores, de Cœlo & Mundo, &c. yee ſhall finde ſundry Demonſtrations, that without Geometrie may not poſſibly be vnderſtanded. And to leaue Philoſophie, how neceſſarie it is to attaine exact knowledge in Aſtronomie, Muſike, Perſpectiue, Coſmographie and Nauigation, with many other Sciences and faculties, whoſo meanely trauaileth therein ſhall ſoone finde. But to omitte talking of Geometrie in generall, and to ſpeake priuatly of this Treatiſe, I thinke good firſt to open the order and effect thereof, and then ſomewhat to ſaye of the applyances. Like as euery body Geometricall is inuironed with Superficies, and euery Superficies incloſed with lines, ſo that it conſiſteth of theſe three, Longitude, Latitude, and Profunditie, and without conſideration of theſe three, no Solide may bee meaſured: ſo is this Treatiſe alſo diuided into three Bookes. The firſt entreating of lines ſheweth ſundry meanes to meaſure all manner Lengths, Heigthes, Diſtances, and Profundities. The ſecond termed Planimetra, ſets forth diuerſe meanes and rules to meaſure many folde Superficies, Plaine Comex and Concaue, whether they be compounde with ſtraight or circulare lines, or mixte of both. In the third named Stereometria, is ſet out the exact menſuration of ſundry ſolides, repleniſhed with a number of rules and precepts, gathered out of Euclide, Archimedes and Appolonius Pergeus his Conykes, wherein the Reader ſhall not a little delight himſelfe with the fineneſſe and ſubtiltie of their inuentions, eſpecially if he indeuor himſelfe to ſearch out the reaſon, cauſe, and demonſtration of them. And now ſomewhat to ſpeake of the commoditie of theſe concluſions, as the skilfull in Architecture can apply the Stereometria to ſerue his turne in preordinance and forecaſting both of the charges, quantities and proportion of all pareels neceſſarilie appertayning to any kind of buildings: ſo Planimetra may ſeruе for diſpoſing all manner grounde, Plattes of Cities, Townes, Fortes, Caſtles, Pallaces or other Edifices. The Marſhall of the fielde ſhall alſo moſt ſpeedely thereby appoint place conuenient for his Campe, diſtributing euery parte thereof according to the nvmber of his Men, Horſe, cariage, &c. Alſo in ſurueying, parting and diuiding of Lands and Woods, it is moſt requiſite a ſkill for exact as ſpeedye diſpatch therein, and thereof we haue notable recorde in Hiſtories how much this ſcience auailed the Egyptians, when as by the inundation of Nylus their whole countrey was ſo drowned that with ſlime of the water all their Bounds and markes were defaced: Yet certaine wiſe men ayded with knowledge in this ſcience, found out and diſtributed to euery man his owne. The other part named Longimetra, the ingenious practizioner will applie to Topographie, fortification, conducting of Mines vnder the earth, and ſhooting of great Ordinance. So that as there is no kinde of man, of what vocation or degree ſo euer he bee, but ſhall finde matter bothe to exerciſe his witte and diuerſly to pleaſure himſelfe, ſo ſurely for a Gentleman eſpecially that profeſſeth the warres, aſwell for diſcoueries made by Sea, as Fortification, placing of Campes, and conducting of Armies on the lande, howe neceſſaree it is to bee able exactlye to deſcribe the true

Plattes,

Plattes, Symetrie and proportion of Fortes, Campes, Townes and Countries, Coastes and Harboroughes, I thinke there are none so vnskilfull, but will confesse these Geometricall mensurations most requisite. But if any there be so rude, ignoraunt and vnlearned, or so much blinded with selfe liking, that cannot be content to acknowledge any thing requisite in a perfect Souldier that is wanting in themselues (for such surelye they are that moste arrogantlye will maintaine this fonde opinion) let them but regarde and marke the renowmed Captaine Alexander *the great, who had this knowledge in such high estimation, that hee seldome or neuer in his manifold Conquests would attempt any thing, whether it were Forte, Towne or countrey, but first he would haue the exact* Topographie *thereof, and thereupon inuent, deuise, and after execute his warlike Policies. On the contrarie side, for negligence of this knowledge wee maye reade howe* Aniball *the moste worthie and famous Souldier that euer hath beene (all circumstances duely wayed) was by that painefull, Graue and right valiant Captaine* Quintus Fabius Maximus *ledde and trayned into a Fielde inclosed with steepe hilles and deepe riuers, where this noble Consull had so inuironed his Hoaste by fortifying of two Mountaines, that hee with all his Armie were in marualeous ieopardie to haue perished for famine had not the craftie witte of* Aniball *inuented a present pollicie to escape such imminent calamitie.* Cyrus *also that great King and mightie Monarche, was he not through ignorance of* Topographie, *euen in pursuing of victorie intrapped and discomfited with all his power by the* Scithian *Queene* Thomiris *at the riuer of* Oaxis? *I passe ouer innumerable Examples of all ages, manifestly declaring the great aduantage or disaduantage a Captaine may receiue, aswell inuading as defending by the strong or weake situation of places, and by the foreknowledge or ignorance of them: neither is there any liberall or free minde, whether he be of profession Warlike or Ciuile, that will not take great delighte and pleasure to see howe by Arte a man may measure the Distances of places remote and farre a sunder, approching nighe none of them, and that aswell, yea and farre more exactly than if with Corde or Pole he should painfully passe them ouer. Hereupon did the Poetes faine* Atlas *of such huge and mightie personage, sustaining and vpholding the earth and mightie Masse of heauenly Spheres, for that this man notwithstanding he were imprisoned in a mortall carkasse, and thereby detayned in this moste inferior and vilest portion of the vniuersall world, fardest distant from that passing pleasant and beautifull frame of celestial Orbes, yet his diuine minde ayded with this science of* Geometricall *mensurations, founde out the Quantities, Distances, courses, and straunge intricate miraculous motions of these resplendent heauenly Globes of Sunne, Moone, Planets and Starres fixed, leauing the rules and precepts thereof to his posteritie.* Archimedes *also (as some suppose) with a Glasse framed by reuolution of a section Parabolicall, fixed the Romane nauie in the Sea comming to the siege of* Syracusa. *But to leaue these celestiall causes and things doone of antiquitie long agoe, my Father by his continuall painfull practises, assisted with Demonstrations* Mathematicall, *was able, and sundrie times hath by proportionall Glasses duely situate in conuenient Angles, not onely discouered things farre off, read letters, numbred peeces of money with the verye coyne and superscription thereof, cast by some of his freends of purpose vpon Downes in open Fields, but also seuen Myles off declared what hath beene doone at that instant in priuate places: Hee hath also sundrie times by the Sunne beames fixed Powder, and discharged Ordinance halfe a Mile and more distante, which things I am the boulder to report, for that there are yet liuing diuerse (of these his dooings)* Oculati testes, *and many other matters farre more strange and rare which I omit as impertinent to this place. But for inuention of these conclusions, I haue heard him say, nothing euer helped him so much as the exquisite knowledge he had by continuall practise attayned in Geometricall mensurations. And for Science in great Ordinance especially to shoote exactly at Randons (a qualitie not vnmeete for a Gentleman) without rules Geometricall, and perfect skill in these mensurations, hee shall neuer knowe any thing: thus haue I partely declared the pleasure and commoditie that anye well disposed minde may reape by these three Bookes of my Fathers. But somewhat to saye concerning the last Treatize of the fiue Regulare Bodyes*

and

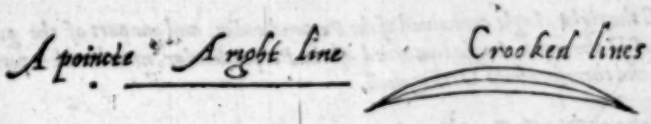
and their Metamorphosis adioyned by my selfe, for that I knowe some yea many shall there be that will not spare to say, as they haue done by others, it is a fonde toye, a meare curiouse trifle, serning to no vse or commoditie: surely I meane not greatly to labour to attaine the good opinion of such, hauing learned how the dispraise of the ignorant and vitious is no lesse commendation than the praise of the wise and vertuous, being a precedent infallible from the beginning and alway continuing that ignoraunce enuieth knowledge, and vice vertue. Euery man dislikeih or extolleth things accordingly as they are agreeable or repugnant to his appetite, and rather would I enioyning the company of Euclide, Archimedes, Appollonius Pergeus, and other Geometers writing of the like toyes and curiositues be condemned, than in publishing matter agreeable to suche mens phantasie and concepte, be of them highly commended, and of the learned laught at or condemned, but howsoeuer Epicurus, Midas, or other like giuen onely to lucre and worldlye pleasure lift to thinke there of, I nothing mistrust of such as couet the vnderstanding of matters harde and difficile, desiring the knowledge of things somewhat passing the reache and capacitie of the common sorte, wherein onely the nature of man surmounteth beastlye kinde, or by proofe and assaie in cases of like difficultie able to iudge themselues, this Treatize shall not bee misliked but thankefully receiued, and for the rest perswasions are but vaine, for as no wordes can adde stomacke or make the coward valiant, so surely such two footed Moules and Todes, whom destinie and nature hath ordayned to craule within the earth, and suck vpon the muck, may not possibly by any vehement exhortation be reduced or moued to taste or sauour any whit of vertue, science, or any such celestiall influence, my hope is if any faulte escape, as in such long and intricate tedious calculations of Irrationall numbers may happen to be some, the discrete and modest Reader will rather of curteise amendit, than with enuious cauillation vngratefullye requite my painefull trauayles, whereby I shall bee prouoked not onelye to publishe the Demonstrations of these and many moe strange and rare Mathematicall Theoremes, hitherto hidden and not knowne to the worlde, but also to Imprinte sundry other volumes of my Fathers, which hee long sithence compiled in the English toong, desiring rather with plaine and profitable conclusions to store his natiue language and benefit his Countreymen, than by publishing in the Latin rare and curious demonstrations, to purchase fame among straungers.

ELEMENTS OF GEOMETRIE, or Diffinitions.

A Poynt I call which cannot be deuided, whose parte is nothing.

A Line is a length without breadth or thickneſſe, whose extremities are two poynts.

The shortest drawne betweene two Poyntes is a straight line, the contrary are crooked lines.

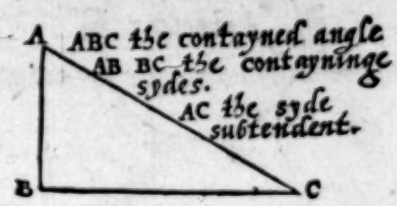


A Superficies is that hath length and breadth onely, being bounded or determined with lines.

A Plaine Superficies is that which lyeth equally and euenly betweene his lines or boundes.

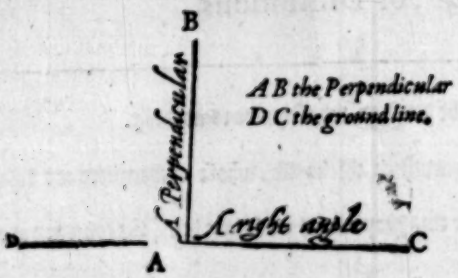
A Plaine Angle is the inclination of two lines lying in one plaine Superficies, concurring or meeting in a point:

If those lines that containe the Angle be straight, it is called a right lined Angle, and those two lines his containing sides, but if a third straight line bee drawne crosse the former from one to the other, that shall be called the subtending side.



OF ſtraight lined Angles there are three kindes, the Orthogonall, the Obtuſe, and the Acute Angle.

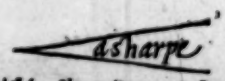
WHen any right line falleth Perpendicularly vpon an other, that is to ſay, making the Angles on either ſide equall, each of thoſe Angles is an Orthogonall or right Angle, and that falling line a Perpendicular.



B A C the right Angle contained of the Perpendicular, and one part of the ground line equall to B A D the right Angle contained of the Perpendicular, and the other portion of the ground line, and therefore both Orthogonall.

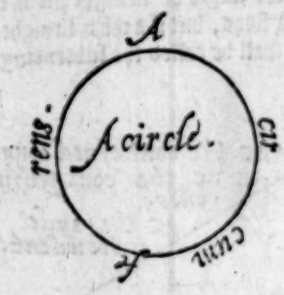
The Brode or Obtuſe Angle is greater than the Orthogonall.

The Acute or ſharpe, is leſſer than the right Angle.



A Figure is comprehended within limmites and boundes, whether it bee one or many:

A Circle is a plaine figure, determined with one line, which is called a Circumference, in whoſe mids there is a point named his Centre. From the which all right lines drawne to the circumference are equall.



A Semi-

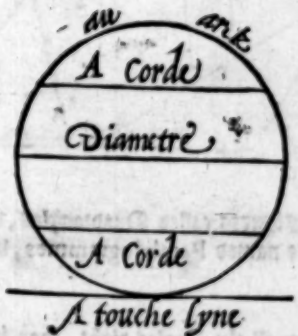
A Semicircle or halfe Circle, dooth conteine both the Dimetient and Centre of his circle, with the precise halfe of his circumference.

A Right line drawne through the Centre vnto the Circumference of both sides, is named his Diameter or Dimetient, the halfe of it is called his Semidiameter.

ALl Straight lines besides the Diameter in any Circle pulled from one part of the Circumference to the other, be called cordes.

THe portion of the Circumference from that Corde comprehended, is named an Arcke.

A Touch line is that toucheth a Circle in a Pointe.



AMong Right lined figures, such as haue onely three sides are Triangles, whereof there be sundrie sortes bearing seuerall names, according to the diuersitie of their sides and Angles.

IF the Triangles three sides bee euery of them of like length, it is called an Equilater Triangle.

Iſoſcheles is ſuch a Triangle as hath onelye two ſides like, the third being vnequall, and that is the Baſe.

Schalenum hath three vnequall ſides.

A Rightangled Triangle is ſuch a one as hath one right Angle.

An Obtuſiangle Triangle hath one obtuſe angle, and is called Ambligonium.

Oxigonium hath all acute or ſharpe Angles.

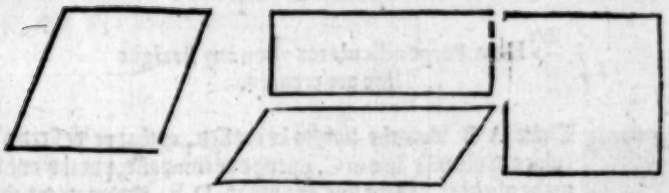
There bee alſo foure ſided Figures called Quadrangles, whoſe Oppoſite ſides and Angles are equall, ſuch are named Paralelogrammes, wherevnto there are but foure ſortes.

If all the ſides bee equall, and all the Angles right, than is that Paralelogramme called a Square.

If one ſide containing the right Angle, be longer than the other containing ſide, then is that figure called a Rectangle.

If all the ſides be equall, and no angle aright, then is it called Rombus.

But if it haue onely the Oppoſite ſides equall, and the other that containe an Angle vnequall, it ſhall be named Romboides.



All other Quadrangles are Trapezia.

POlygna, are such figures as haue moe than foure sides, whose Angles if they bee all like and equall, they are termed Equiangle Polygona.

ALl other plaine Superficies, whether they bee enuironed with straight or crooked lines, shall be named irregulare figures.

WHen two right lines drawne in one plaine Superficies, are so equedistantly placed, that though they were infinitelye extended on either side, yet would neuer meete nor concurre, they shall be called Parallels.

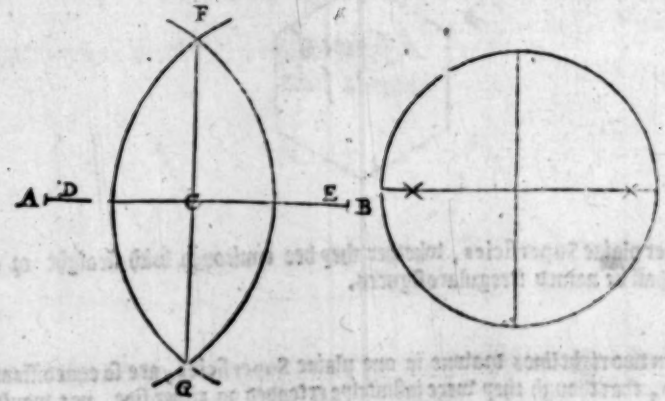
Parallels.

A Quadrant is the fourth parte of a Circle, included with two Semidiameters commonly diuided in 90. portions, which partes are named grades or degrees.

The fyrſt Chapter.

How Perpendiculares vpon any ſtraight line are creared.

Omit A B were the line to be croſſed, or that ye deſired a Perpendiculare or plumbe line in C, open your compaſſe, put the one foote in C, make of either ſide the line one pricke, D E. Nowe extend the compaſſe to the wideneſſe of bothe, or shorter, putting the one foote in D and the other immoueable, making an Arke ouer and vnder C. This doone diſcreetely, remooue the compaſſe from that Centre to E (remaining ſo opened) there fixe one foote, with the other croſſe the Arke afore made aboue and beneath C. where make two points, or theſe letters F G. Then take a Ruler, and lay him vpon bothe the points croſſing the centre C. Thus drawe your plumbe or ſquire line F C G. In like manner any line may be deuided in halfe, or circle in foure equall parts. See the Example heere following.

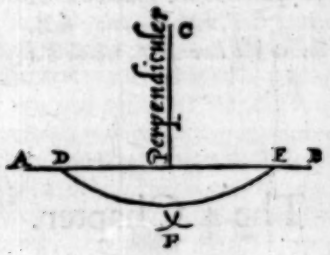


The ſecond Chapter

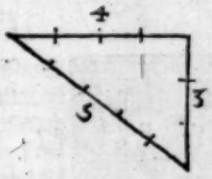
How perpendiculare or hanging lines are drawne from a poynt aſſigned to any right line lying in the ſame plaine Superficies.

Vppoſe C the point from whence I would let fall a Perpendiculare to the ſtreight line A. B. open therefore your compaſſe of ſuch wideneſſe, that placing the one foote in C, ye may reache with the other beyond the line A. B. and drawing an Arke, note where it maketh interſections with the ſame line, which in this Figure are marked with the letters D and E, the placing

cing one foote of your compaſſe in E, drawe an Arke directlye vnder C, and then fixing one foote of your compaſſe in D, croſſe the former arke in F, finally drawe the ſtreight line C F, for that is a perpendiculare to the line A B.



PIthagoras inuention might heere take place, who did finde theſe numbers 3.4.and 5. or like meaſures to make a right Angle:



The third Chapter.

From any point aſſigned to extend a Parallele to any other right line lying in the Superficies.

FRom the pointe aſſigned let fall a perpendiculare to that line, and from ſome other pointe in that line, ereare a perpendiculare, as yee were in the laſt Chapters taught, then opening your compaſſe to the lengthe of the perpendiculare let fall from the pointe aſſigned, meaſure out the like length in the perpendiculare ereared, beginning from the ground line: then laying a Ruler to the pointe aſſigned, and the ende of that length drawe a ſtreight line, for that ſhall be a Paralelle to the other.

Example.

Example.

ADmit A the point aſſigned, B C the line, A D. the perpendicular let fall from A, C F. the perpendiculare ereared from C. my compaſſe opened to the wydneſſe of A D, I ſet one foote in C, cutting C F, with the other in E, then applying my Ruler, I drawe the line A E, which is a Parallelo to B.

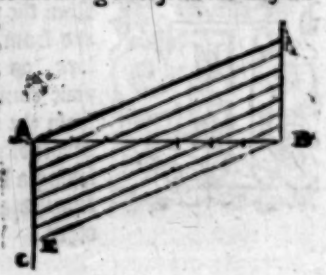
The 4. Chapter.

To diuide any limited right line into as many equall partes as ye liſt.

Diſmuch as hereafter in drawing of platters and menſuration of ground by inſtrument, the diuiſion of right lines into many equall partes is required, I thought good to giue inſtruction therof before I entreate of thoſe matters: yee ſhall therefore vpon either end or limite of the diuiſible line, ereare a Perpendiculare, the one vpward the other downeward, and opening your compaſſe at aduenture, meaſure out ſo many partes in either perpendiculare, as you would make diuiſions in your line, and drawing right lines from the points in one perpendicular to the points in the other, beginning from the firſt of the one, to the laſt of the other, ye ſhall diuide the line giuen, into ſo many equall partes as there be diuiſions in your perpendiculares.

Example.

ADmit the line A B which I would diuide into ſeauen equall portions, I ereare vpon A and B, the perpendiculars A C, B D, as you may behold in the figure: and opening my compaſſe at aduenture, I meaſure out ſeauen partes ending at E F. then drawing lines from the diuiſions in the other (beginning from the laſt in one perpendicular, to the firſt in the other) you may behold in the Figure the line A B, parted into ſeauen equall portions, in this manner may you proceed infinitely to diuide it into as many portions as you liſt.



The

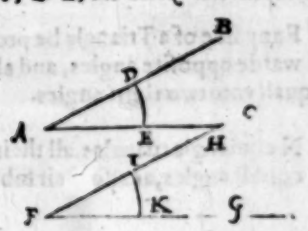
The fyft Chapter.

How equalll Angles are made.

Ire one foote of your compaſſe vpon the concourſe or meeting of thoſe two right lines that containe the Angle, whoſe like or equall you would make, and opening your compaſſe at pleaſure, deſcribe an Arke cutting the two containing ſides of the Angle: then drawe an other right line, and placing one foote of the compaſſe (remaining immooueable) thereon, with the other deſcribe an Arcke riſing from that laſt drawne line, then reſort to your Angle, and open your compaſſe to the wideneſſe or diſtance of the two interſections made by the Arcke in the two containing ſides, and tranſporting the ſame diſtance to your ſecond Arcke, ſet one foote of the compaſſe at the beginning thereof, I meane where it riſeth from the line, with the other cutte the laſt deſcribed Arcke, then laying your Rular to that interſection and to the centre of the Arcke, drawe a right line till it concurre with the other: thus haue you a newe Angle equall vnto the former.

Example.

Svppoſe B A C, the Angle whoſe like or equall I deſire, D E, the Arcke drawne with one foote of my compaſſe, whyle the other remained in A, the compaſſe immooueable, I ſet one foote in the line F G, drawing with the other the Arke K I, This done, I open my compaſſe to the diſtance of D E, and placing one foote in K, with the other I croſſe the Arcke, in I, finally, laying the Ruler to I and F, I drawe the line F H, and thus haue I made the Angle H F G. equall to the firſt Angle B A C.



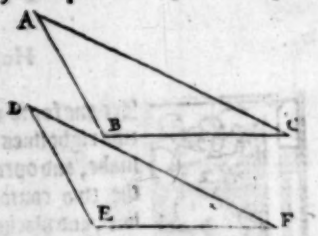
The 6 Chapter.

To make a Triangle equall to any other right lined Triangle aſſigned.

Irſt as yee were taught in the laſt propoſition make an Angle equall to ſome one Angle in that Triangle, it forceth not which of them it be. Then extend out thoſe ſtraight lines that containe this Angle, till they bee of equall length with the containing ſides of his correſponding Angle in the Triangle. This done, couple the endes of thoſe two right lines together with a thirde, and ſo haue you framed a Triangle equall to the former.

Example.

THe triangle aſſigned is *A B C*, to his Angle at *B* by the former probleme I frame an equall or like, conteined with the lines *D E F*, extending *E D* and *E F*, till they become of equall length with *A B* and *B C*, which I may eaſily do, by extending my compaſſe firſt to *A B*, *B C*, and after tranſferring thoſe diſtances or lengths to *D E*, and *E F*. Finally applying my ruler to the ends or limits of thoſe lines, I drawe the ſubtending ſide *D F*, and thus haue I framed a new Triangle *D E F*, equall to the other *A B C*.



I Thinke it not amiſſe before I entreate of Geometricall menſurations, to premiſe certaine Theormes whereby the ingenious may readily conceiue the ground, reaſon, and demonſtration of ſuch rules as ſhall enſue.

The firſt Theorme.

ENy two right lines croſſing one an other, make the contrary or verticall angles equall.

The ſecond Theoreme.

IF any right line fall vpon two Parallele right lines, it maketh the outward Angle on the one, equall to the inward Angle on the other, and the two inward oppoſite angles on contrary ſides of the falling line alſo equall.

The third Theoreme.

IF any ſide of a Triangle be produced, the outward Angle is equall to the two inwarde oppoſite angles, and all three Angles of any Triangle ioyned togither, are equall vnto two right angles.

The fourth Theoreme.

IN equiangle triangles, all their ſides are proportionall aſwell ſuch as containe the equall angles, as alſo their ſubtendente ſides:

The fift Theoreme.

IF any foure quantities be proportionall, the firſt multiplied in the fourth, produceth a quantitie equall to that which is made by multiplication of the ſecond in the third.

The ſixth Theorme.

THE viſible beames falling on plaine conuexe or concaue glaſſes, are reflected in equall Angles.

The ſeuenth Theorme.

IN right angled Triangles the Square of the ſide ſubtending the right Angle, is equall to bothe the ſquares of his conteyning ſides.

The eight Theoreme.

AL Parallelogrammes are double to the triangles that are deſcribed vpon their baſes their altitudes being equall.

The ninth Theoreme.

AL like or equiangle Figures retaine double the proportion of their correſpondent ſides.

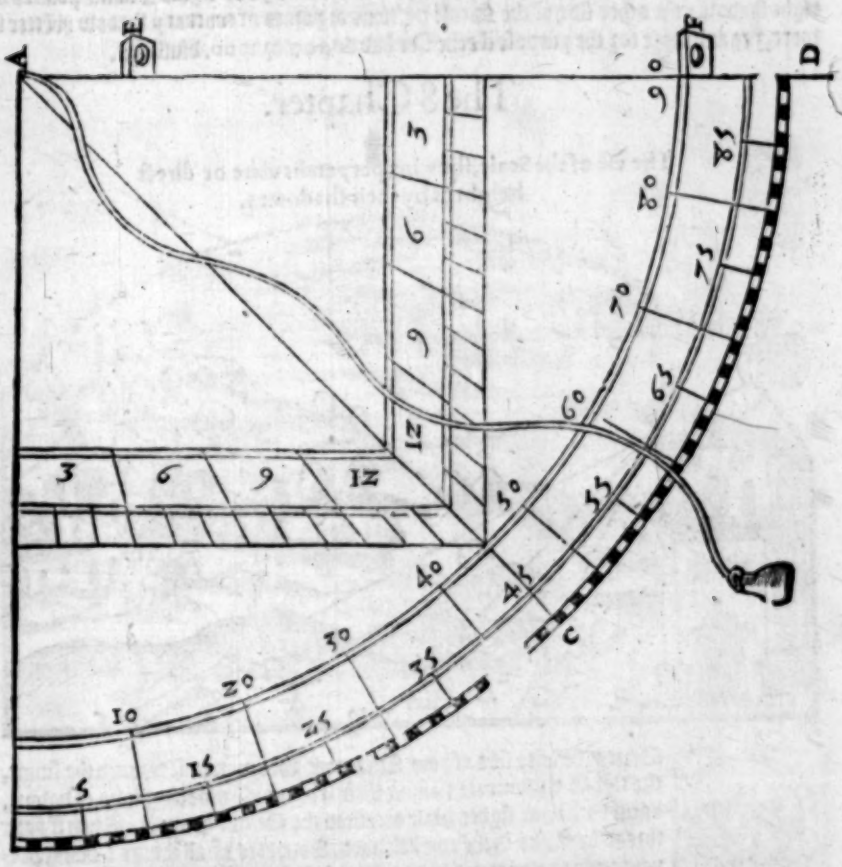
The tenth Theoreme.

IF from any Angle of a Triangle to his ſubtendent ſide, a perpendiculare deſcende the Square of that ſubtendente ſide, or baſis added to one of the containing ſides

ſides ſquare, ſurmounteth the Square of the third ſide, by a rightangled Parallelogramme contained of the whole baſe and double that his portion, which lyeth betweene the perpendiculare and the laſt named third ſides ſubtendent Angle.

The 7 Chapter.

The deſcription of the Quadrant Geometricall.



Irſt ye muſt make a common ſimple large quadrant thus, with your compaſſe drawe an Arke or Circumference, that may be more, or at the leaſt ſufficient for a quadrant, then put bothe the feete of your compaſſe in that Arke, making two Pricks. Now the diſtance of theſe two points diuided in two equall parts, adding one portion to the aforeſaid circumference or diſtance, ſheweth a preciſe quadrant, ye ought then to pull of each ſide from the centre a line to the vttermoſt points, which be the extreames of your quadrant. Againe drawe a line from your centre A to the mids of the quadrants circumference C, and if ye liſt ye may diuide that quadrant into 90. graues thus: Firſt in 3. parts, then euery third in 3. ſo haue ye 9 portions: Now euery of them in 2. rileth 18. Then each in 5. equall partes maketh 90. degrees.

Of the Scale.

FOr the Scale ye ſhall drawe from either ſide of your Quadrant a right hanging line (as is declared) touching the middle line in one pointe, ſo haue yee the ſides of your Scale each to be deuided in 12.60.100.1000. points all marked from the centre A, the more the more commodious. Forget not to haue two equall fine plates of braſſe perſed in the middes (for your ſightes) and placed on the ſide A D, as yee ſee E F, with a line and plummet falling out of the centre. A. I call the Scale in this Quadrant the two ſides within diuided in certaine portions or partes. And theſe 12. partes next to your ſightes I name pointes of right ſhadow: the other ſide of the Scale portions or points of contrary ſhadow: better it were, yea and more for the purpoſe if eche ſide had 60.100. or 1000. diuiſions.

The 8 Chapter.

The vſe of the Scale, ſhewing perpendiculare or direct heightes by their ſhadowes.

GOuerne the leſte ſide of your Quadrant Geometricall toward the ſunne, the thread & plummet hauing their free courſe mouing it vp or downe, vntill both your ſightes haue receiued the Sunne beames. Then if your thread be found in the twelfth part, ſhadowes of all things (being perpendiculare eleuated) be equall with their bodies, if the plummet with the thread be perceiued cutting the parts next to the ſightes, which I call right ſhadowes, then euery thing direct is more than his ſhadowe. By that portion which 12. exceedeth the partes where the thread was found: if it fall on the firſte parte of right, take the ſhadowe 12 times to make the heigth, if it chaunce on the ſecond portion ſixe times, on the third foure times, on the fourth thriſe, in the fifth twiſe and two fiftes of the ſhadow, in the ſixte pointe twiſe, in the ſeauenth once, and fiue ſeuenths of the ſhadow, in the eight portion once and a halfe, in the ninth once and the third parte, in the tenth once and the fift parte: in the eleuenth pointe yee ſhall take the ſhadowe once and the eleuenth part of that ſhadowe: or in fewe wordes, multiply the length of the ſhadow by 12. and the product diuide by the partes in which you found the thread, your quotient ſheweth the heigth: but and if it be in the partes of contrary ſhadowe: augment the length of the ſhadowe with the partes declared by the Plummet, and the encreaſe diuide by 12, ſo commeth the altitude alſo. Enſample, in the figure that goeth before it is plaine to be perceiued.

When

When the thread falleth on 12 portions, the shadowe is equall with the thing it selfe. In 6 of right it is but halfe, in sixe of contrarie it is twise the height: so to conclude, ye may see as the sixe in right exceedeth the partes, so dooth the altitude or bodie the shadowe, and contrarie in contrary shadowe, behold your figure how the thread cutteth 6 partes of contrary shadowe in the Quadrant next to the right hand, the shadow B C then being 210 foote, multiplie (as I haue sayd) the length of the shadow 210 feete, with 6 the parts cut by the thread, encreaseth 1260, that diuided by twelue, riseth 105: the altitude of such a bodie which had a shadowe then 210 feete. Thus of all such like.

The 9 Chapter.

Of Vigetius concerning heigthes

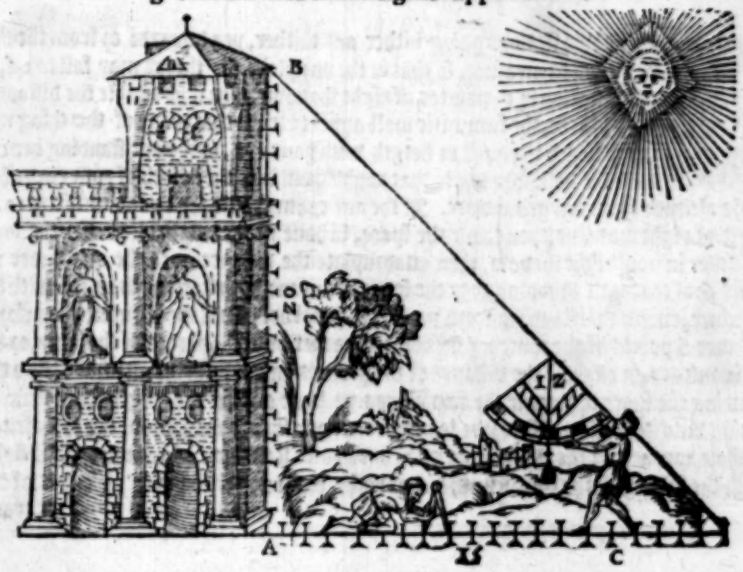
Affirmeth by any certaine measure directly standing (whose shadowe is knowne) the heigth of any other thing the shadowe then measured not to be hid, performed by the rule of proportion.

Example.

Suppose the shadowe of any thing 210 *feete. Now say* 20 *the shadowe of a thing knowne giueth* 10, *what shall* 210, *riseth* 105: *the heigth.*

The 10 Chapter.

Without shadowe or any supputation by your Quadrant geometricall to take heigthes approchable.

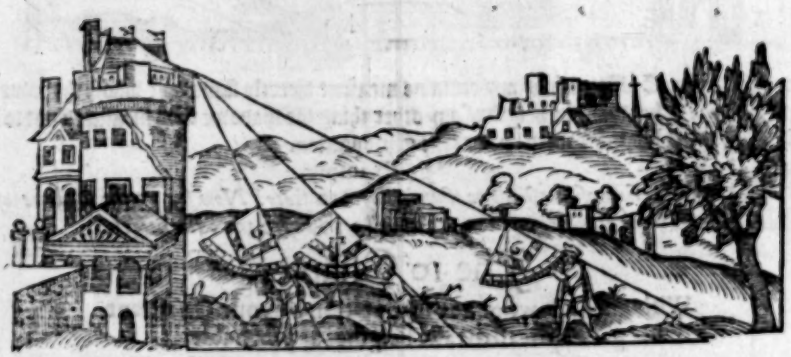


Lm

LIft vp ingeniouſly your quadrant exactly made toward the thing to be meaſured, looking diligently through both the ſightes, going backe or forward as occaſion is giuen vntill ye ſee the top, ſo that your line or ſubtile thread fall iuſtly vpon the twelfth point. Now if you meaſure the diſtance from you to the baſe (which baſe heere I call the pointe directly vnder the top, then haue ye the altitude of the higheſt ſummitie to the right pointe or baſe in heigth equall with your ſtanding, adioyning vnto it the heigth of your eye downeward. Enſample, The lyne and plummet in the figure aboue falleth preciſely in the twelfth portion, the ſpace then being from you to the baſe, which is from A to C 15 foote. To this ye muſt adde the heigth of your eye (heere imagined 5 foote) ſo haue ye 20 feete, the true altitude from A to B. As the length of the coyde from the eye to C ſheweth the meaſure to be layde backe: ſo doeth the touche of the line and plummet in C, declare where yee muſt beginne to laye the meaſure backe.

The 11 Chapter

With the ayde of two places to ſearch out improcheable heightes.



SEeke two ſtations going hither and thither, yea towarde or from the thing ye intend to meaſure, ſo that in the one place the thread may fall in 12, the other ſtation in 6. pointes of right ſhadowe, then if ye double the diſtance of both places, the ſummitie ſhall appeare from that parte of the thing meaſured which is equall in heigth with your eye, or if your ſtanding bee euen with the baſe, ioyning to that doubled diſtance the heigth of your eie, ye haue the whole altitude from the ground, &c. If the one roome cauſe the thread to fall in 12. the other in 8. of right ſhadowe, then triple the ſpace, ſo haue ye the heigth alſo. Or the one in 12. the other in 9. of right ſhadow, then quadruplate the diſtance: yea the one vnder 12. that other 6. of contrarie ſhadowe, then the ſpace betweene both ſtations is equall with that you meaſure, euer vnderſtanding from your eye vpwarde. Or if the plummet be inforced to fall vnder 6 points of the contrary ſhadowe, the other vnder 4 partes of the ſame, or in 4 and 3 of contrary, in all theſe the diſtance of the place is equall with the altitude. So then in meaſuring the ſpace betweene the two places ye haue gotten the heigth from your eye vp, putting vnto it (as I haue ſaid) the length from your ſight downeward, the iuſt altitude of the whole appeareth: the Baſe being euen with your ſtanding. Example. This figure declareth the falling of the thread vnder 6 of right, and 12. Alſo vnder 12 and 6 of contrarie,

trarie, by doubling the ſpace betweene the two firſt places, the altitude appeareth. In like manner the diſtance from the middle ſtanding to the laſt, bringeth the heigth from your ſight.

How lengths in heigth are knowne.

I Would not haue you ignorant heere howe to knowe lengths which be in heigth not eaſie to come vnto, firſt by your Arte afore mentioned get eyther heigth, ſubduce the leſſe height out of the more, of force your deſired length remayneth. Or thus, let the plummet fall vnder 12 portions, marke your place, goe in towarde the thing (the threade as it was) vntill ye ſee the baſe of that length, ſo the diſtance betweene the two ſtandings is vndoubtedly the length: heere needeth no Example.

The 12 Chapter

How by your Quadrant with calculation ſpeedely to knowe all heightes acceſſible.

YOur Quadrant as afore is ſaid handſomly eleuated againſt or towarde the thing to be meaſured, perceiuing thorow the ſights not more than the toppe. Marke well the diuiſions of pointes touched in your ſcale, if they bee of right ſhadowe, multiplye the diſtance from you to the baſe by 12, and diuide by the partes of your Scale which your thread made manifeſt. But and if they be of contrarie ſhadowe, worke contrarilie: that is augment by the partes, and make partition by 12, remembring euer to adde the heigth of your eye downewarde to your Quotient, ſo haue yee your deſire: the Baſe being equall with your ſtanding.

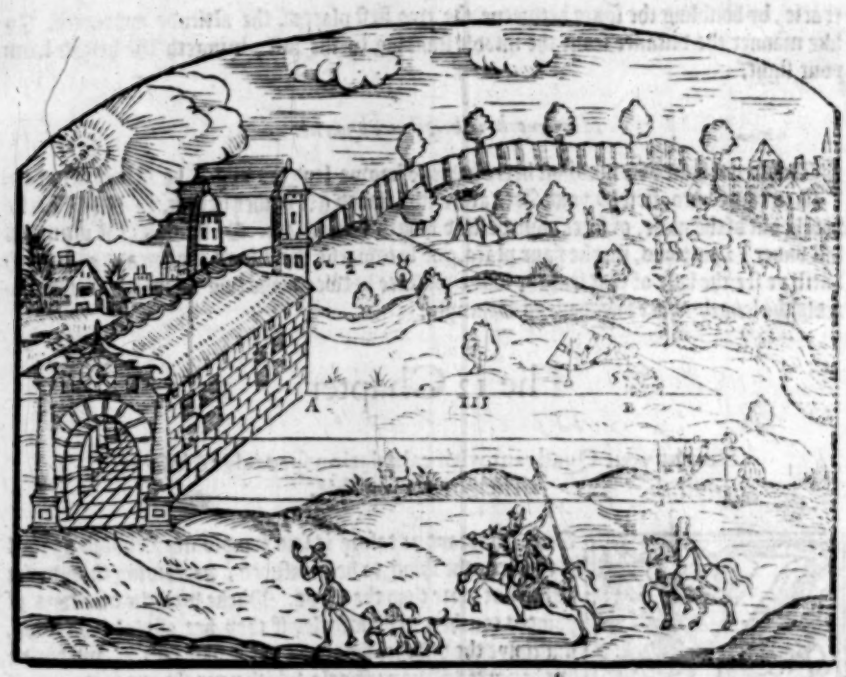
Example.

ADmit the thread with the plummet note 6 partes of contrarie, as yee may ſee in this Figure: the diſtance from the baſe A to your ſtanding B 115 foote, multiplie that by 6, ſo haue yee 690, the which diuided by 12, yeeldeth 57 ½ foote: to this adioyne fiue foote (being the heigth of your ſight to the grounde) conclude the Altitude 62 ½ foote.

Heere yee ſhall note that in menſuration of heightes with this inſtrument, it ſhall bee requiſite firſt to finde what parte of the Altitude is leuell with your eye, which you may thus doo:

CAuſe the plummet and thread to fall vpon the ſixe line of your quadrant where the degrees begin, and then ſearching thorough the ſightes, that part which you can eſpie of your Altitude, (the plummet hanging vpon the foreſaid laterall line) is leuell with your eye, the heigth whereof from the baſe compared with the altitude of your eye, diſcouereth the vnequalitie or difference of the grounde: that is to ſay, howe much higher or lower the baſe of the thing to be meaſured, is than the ground at your ſtation, which difference as yee ſhall ſee cauſe added or detracted from your heights found as is before declared, yeeldeth moſt exactly the true altitude, and thus ſhall you be aſſured neuer to erre, how vneuen or vnleuell ſo euer the ground be.

The



The 13 Chapter.

To get inacceſſible heights by ſupputation (with the helpe of two places) ſuppoſing either ſide of the Scale diuided, 100 partes.

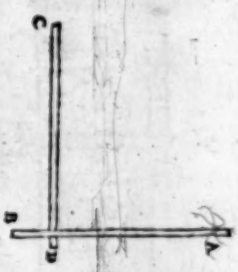
IF your thread in the firſt ſtation fall vpon 50 points of contrarie, with thoſe diuide 100, ſo haue ye 2. In the other place (going right backe or forward no way declining) admit it note 25 of contrarie, now 100 diuided with 25 riſeth 4, withdrawe 2 from 4, 2 is left your diuident, meere the ſpace betweene both ſtandings, and diuide that by 2, your diuiſor, ſo haue yee the heigth from the eye vp. Note, if the difference of the Quotient be 1, the ſpace betweene the ſtandings ſhalbe equall with the deſired heigth, adding your ſtature. If 2. the ſpace is double to the altitude, if 3, threefolde, &c.

Or thus worke: Reduce the parts of contrarie ſhadow vnto portions of right, and then doo as you would with pointes of right: that reduction is made thus, multiplie 100 in himſelfe, ſo haue ye 10000, the which diuided by euery parte of contrarie ſhadowe, ſo ſhall they be as points of right ſhadowe: And if yee haue made two ſtations, pull the leſſe Quotient from the great, the reſt waighe as you haue beene inſtructed. No end hath the Geometer in finding true meaſures, many I might ſaye infinite moe wayes heightes are founde, by anye two equall thinges orthogonally ioyned with Staffe, Corde, Squire, Triangle, Glaſſe, &c. as heerein followeth.

By any

By any two things of one length ioyned thus in right Angle, Altitudes are found.

THe end C applied to your eye, goe backeward or forward as you shall see cause, till you can espie the toppe and base of your Altitude, by the extreames A B: so doing the distance betweene your foote and the base, is equall to the heigth, without adioyning the altitude of your eye, which in all the rest before shewed is required, onely heere yee shall take heede so to couple A B and C D that in beholding the altitude, your line A B may depend perpendiculare or equedistant to the heigth. In like sorte may you meete the distance of anye two things in sight, and that exactly, if you vse discretion in placing A B, that it be alway parallele to the line measured.



The 14 Chapter.

Heightes are ingeniouslie searched out by a staffe.

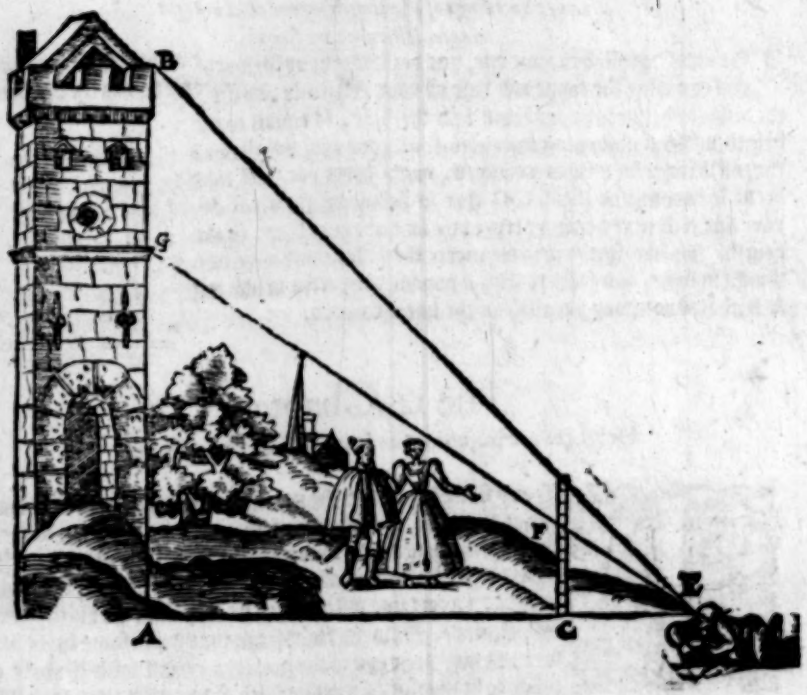
IF any staffe be erected, the measurer vpon his backe beholding the top of the thing, the distance of the eye from the foote of the thing sheweth the heigth. Or thus receiue my minde more largely, prepare a right staffe diuided in 12 or mo equall parts, that done, set it right vp a certaine distance (as ye list) from the bigth which you will measure. Now go right from that staffe some space at pleasure: laying your eye to the grounde equall with the base of the thing to be measured, mouing back or drawing neer to that staffe, vntill yee may rightly and plainly see the very summitie or vpmost part of the thing to bee measured, by the top of your staffe, which performed make a marke where your eye had his place: Now measure the distance or space from the staffe to your eye with the staffe it selfe, and note what proportion the staffe hath to the distance, the same shall your heigthe haue to the length from your eye to the base of that Altitude. Ensample. The staffe G D (in this figure) and the distance C E are equall, therfore affirme the heigth A B to be equall yea so long as the distance betweene your eye and the base of that required heigth which is A E, if otherwise according to the proportion aforementioned, yee may by the rule (called the golden precept,) bring the iust heigth thus, meete the grounde betweene your eye and the staffe suppose it 12 foote, then the distance from your eye to the base 200 foote, your staffe 5 foote, say of 12 commeth 5, what shall come of 200, so haue ye 83 feete and $\frac{1}{3}$ your exact heigth.

The 15 Chapter

Ye may also hereby readily measure all lengthes standing in heigth as the altitude of any Tower standing on a hill, or the length of a steeple; aboue the battlements, or the distance betweene story and story in edefices, your selfe standing on the ground.

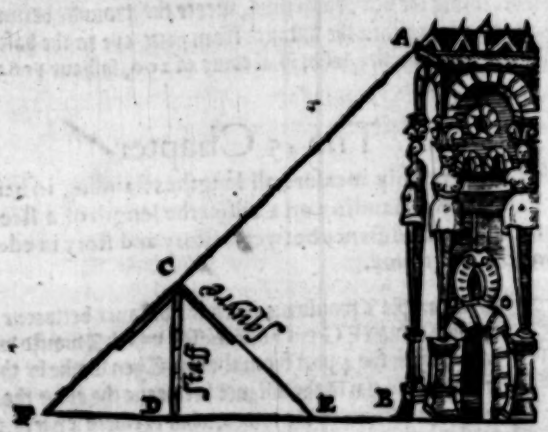
OR Example I would measure the distance betweene B and G, suppose the line visuall F G cut your staffe (which I would with diuided in 12 parts) vpon the 4 part from the top. Then worke by the rule of proportion: saying 20 parts the distance betweene the eye & the staffe geueth 200 foote, the distance of the tower, what geueth 4 parts, thus shal you find the fourth number proportionall 40 foote, which is the exact length of B G. And thus may you measure onely by a staffe diuided in 12 equall partes (without any other instrument) any altitude, howsoeuer it be situate:

Behold the Figure on the other side.



The like is brought to paſſe by the Squire, beholde the Figure.

ALwayes D C will retaine the ſame proportion to D E, that B F doth to A B, ſo that B F meaſured, you may by the golden rule attaine the heigth of A B, or contrarie if you know the Altitude A B, you may proportionally learne the longitude F B. Pleaſanter to practiſe is this than the former and moſt exact for Altitudes. But where as ſome doe vſe it as well for longitudes and diſtances, making the ſtaffe a ſide common



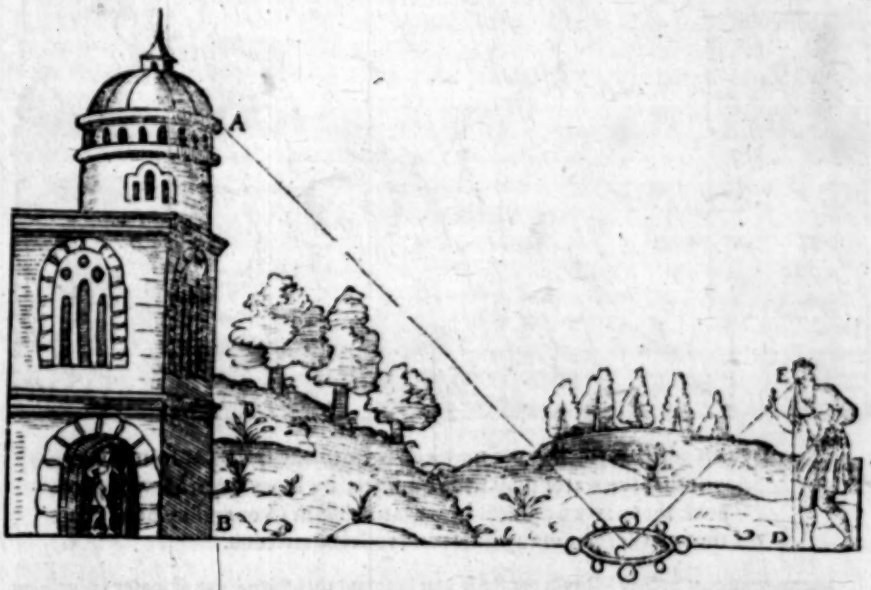
of the greater and leſſer triangles: though demonſtration beare it, I allowe not of it, the Angle made with the ſquire and ſtaffe, groweth ſo acute, and vnſenſible, that great errour enſueth the leaſt miſtaking.

The

The 16 Chapter.

By a Glaſſe heighthes may be pleaſantly practiſed and found on this wiſe.

CAſt a glaſſe on the ground, ſo it may lie equall, yea euen in heigth with the baſe of the thing to be meaſured, pour eye on the glaſſe fixed, goe from it vntill ye ſee no more than the very top of that thing of which ye require the heigth. Then let a Plummet with a line come from your eie to the ground, marke the fall of that Plummet, looke what proportion the diſtance beareth (from the ſaid Plummet to the Centre of the glaſſe) compared to the length of the cord, the ſame ſhall the ſpace betweene the glaſſe and the baſe of the heigth, haue to the altitude deſired.



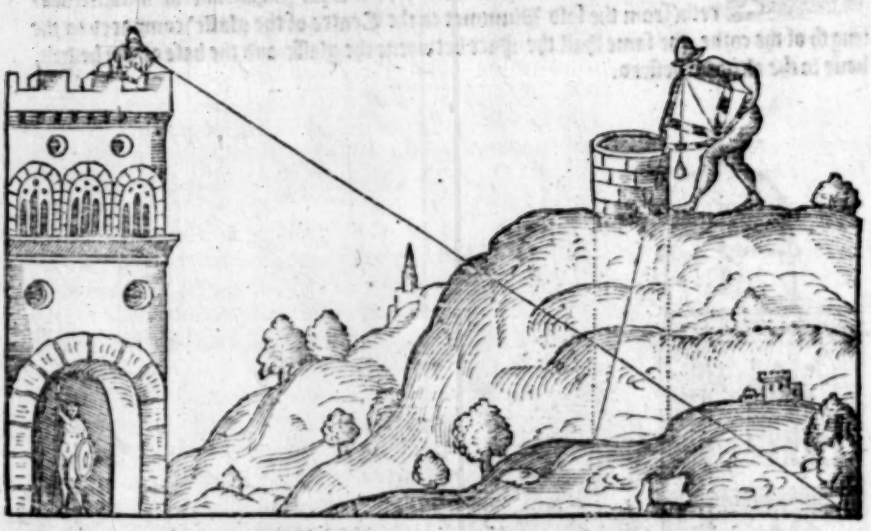
THe precept of proportion may be as well heere vſed as afore, and ſo to attaine heights. Behold the figure, as the diſtance D C from the fall of the Plummet to the centre of the glaſſe, is equall with the line E D, falling from the ſight E ſo the length C B from the glaſſe to the baſe of the tower, ſwarueth not from the deſired Altitude A B.

The 17 Chapter.

To meaſure the deepeneſſe of any well by the Quadrant Geometricall.

NOw to returne to the firſt inſtrument Geometricall and ſo to the ende, the breadth or Diameter knowne, ſet your Quadrant on the end of your Well in the very toppe, then lift vp or put downe this Quadrant, vntill ye ſee the bottome on the contrarie ſide, marke the points cut, looke what proportion the partes haue to the whole ſide, the ſame ſhall the Diameter haue to the length or deepeneſſe. Therefore by the number of the poines found diuide 12, ſo the Quotient vttereth how often the breath of the Well maketh the depth, or multi-

pipe the breadth by 12, the Product diuide by the points found, the depth alſo appeareth. Enſample. In the figures following there be 3 points of the Quadrant cut, &c. 3 in 12 is contained foure times, ſo is the Diameter in the length or profunditie. In like manner of proportion (as is declared) yee may gather the length of things aſcending ſome high Turret. Farther note whatſoeuer I haue ſaid of the Quadrant appertaining to heightes know that to be ſpoken of the Square Geometricall, which heere ſhall be put forth for lengthes onely, one reaſon one ground ſerued them bothe. As eaſie alſo is it to knowe howe much water, I meane howe manye quartes, gallons, or other meaſure are contained in theſe Welles, or in any other Regular excauate body, which ſhall plainely appeare in the laſte kinde of Geometrie in place due.



By this inſtrument alſo yee may know from any ſpring or fountaine, whether the water may bee conueyed by Pipes or otherwiſe, to any other place or places howe farre diſtant ſo euer they bee, yea though there be many hilles and valleis betweene.

I Would not haue you ignorant that the nature of water is ſuch, as by pipes it may bee rered aboue the fountaine head, and carried ouer hilles or mountaines howe high ſo euer they bee, ſo that the vent or end where the water muſt iſſue out of the Pipe be inferior to the Fountaine whence it is deriued: neither needs you care though there be many vallies in the waye, for experience teacheth that the lower your water falleth, the more freely it will run, and the more pure and holeſome it ſhall be. Alwayes it behoueth you to haue conſideration of the Fountaine whence it proceedeth, and the Seſtarne, Well, or other place whereinto it falleth, that this be lower than the other. It is alſo to be wayed howe this difference of highneſſe and lowneſſe is to bee accompted, ſome ſuppoſe that all places lying in a ſtraight leuell line from the ſpring head, are of one heigth, which opinion is erronious, becauſe the water (being an heauy body as the earth is) pieſſeth and tendeth alway to the center. And in all his courſes (being not violently forced contrary to his nature) mooueth downeward, or at the leaſt vniformly and equidiſtant the from that centre. Whereby it is manifeſt that no ſpring can of his owne nature runne in leuell or right line from his head, for this equidiſtant courſe to the centre, is an arke or portion of a circle. But euery leuell right line (conſidering it is a contingent or touch line) is

carued

caried aboue the circumference, and the farther it is extended, the farther distant it is from the Centre, so that either the water must make his course vpwarde vtterly contrary to his nature, or else it shall decline from his leuell right line. Indeed the Globe of the earth and water being so great, any small Arke or portion of their circumference, will not sensible differ from a straight line. And yet in conueying of waters any great distance, your experience will bewray an error. The meane therefore to attaine perfection heerein, is to finde the difference betweene this straight and circular line, whereof heereafter I shall intreat more at large: onely heere will I open how without any error sensible, (sufficient for any Mechanicall operation) by the aid of sundrie stations ye shall accomplish this purpose.

First it behoueth yee diligently to marke at the fountaine head the Superficies of the water, and if the ground so serue that yee may place your eye euen therewith, take your Quadrant, and turning your selfe towarde the place whither you meane to conuaye this water, (if it be within sight) espie the same through the sightes of your Quadrant, marking diligently the fall of the Plummet, which if it cutte any of the Degrees, ye may conclude it is not possible for the water naturally to runne thither. But if your Sestourne or place be not to be seene at the Spring head, then espie some other marke through the sight of your Quadrant on either side toward your place, alwaies causing the thread and Plummet to fal directly vpon the lateral line where your degrees begin, then removing & situating your eye at the point or marke which you last espied, find through your sights a new mark, causing your plummet & thread to fall vpon the aforesaid lateral line, and thus proceed from station to station, till you come to the sight of your last place, then if your Plummet & thread cut the Degrees, ye may conclud as afore: but if the ground at the Spring head be such, as you cannot conuenientlie place your eye as I haue said, then let fall from you eye, or some place of equall heigth with your eye, a string and Plummet to the brimme of the water, measuring the length thereof. If at your last station your Plummet and thread hanging as I haue before said, your visuall line passing through the sightes of your Quadrant fall aboue the Sestourne where this water should issue out, erecte a Pole or such like thing to the heigth or length that the string was at the Fountaine head: and if your visuall line reache higher than the toppe or summitie of that Pole also, yee may conclude that this water may bee deriued thither. And if from that parte of the Pole your visuall line cutteth, yee abate the Perpendiculer from your eye to the water at the Fountaine head, and for euery mile trauailing 4 inches, the pointe where you leaue is exactlie leuell with the Superficies of the water, and so high it may be brought, and not aboue. It behoueth ye also to take order that your stations be not aboue 200 or 300 pace at the moste a sunder, otherwise error sensible may ensue. Yee may also (if need require) at euery station erect an high Pole, and so may you passe ouer both Mountaines and Valleys, alwaies noting at euery station, what portion of the Pole your line visuall dooth cutte, reseruing them to bee added or subtracted as you see cause, at your last station. Superfluous, yea rather teadious should it bee to vse moe wordes in so plaine a matter, the ingenious Practisioner will finde sundrie wayes to helpe himselfe as occasion requires by sight of the ground, &c.

The 18 Chapter.

To get the length or distance of any place or marke in sight, be it neuer so farre, and that without instrument.

Among many practises I finde fiue wayes principally to bee had in estimation, the first ensuing without anye instrument, other requiring aide of instruments, whose names, compositions and vse followe, as seemeth meete. Although in measuring of lengthes after the mindes of manye writers the auoiding of hilles, and in fewe wordes moste plaines hath beene desired, least great errors should ensue, heere such thinges shall not be

be required: Onely it ſhalbe needfull at the time of your meaſuring to haue ground at libertie on the one ſide. This commoditie had the ground leuell or otherwiſe worke thus, at the beginning of your length ſet vp a ſtaffe or marke which may bee ſeene a farre off, then goe from it Orthogonally or Squirewiſe of which ſide yee will 200 foote, or as yee liſt, the more ground the better, put vp there a ſtaffe alſo: Now conuaye your ſelfe to the firſt ſtaffe or marke going back from it 300 foote more or leſſe at your pleaſure, ſet vp there the third ſtaffe, ſo that the firſt marke or ſtaffe and it agree all in a right line from your ſight to the fartheſt point of your length by the iudgement of your eye. Now goe ſidewiſe from thence as afore in a right angle vntill the ſecond marke offer it ſelfe aright betweene the extreame part of your length and ſight, there put vp the fourth ſtaffe. All this performed, ſeeke out the diſtance betweene the firſt ſtaffe and the ſecond that name your firſt diſtance, then the length betweene the firſt and third ſtaffe, call that the ſecond diſtance. Againe the ſpace betweene the third and the fourth ſtaffe is the third diſtance, ſubduce the firſte diſtance from the third, ſo remaineth your Diuiſor, then multiplye the third diſtance by the ſecond, and the product diuide by your diuident or Diuiſor, the Quotient ſheweth the true length from the third ſtaffe to the fortreſſe or marke deſired. For more plainneſſe behold the figure.

Example.

HEre this letter A repreſenteth the fortreſſe Caſtle or marke which is the extreame or fardeſt part of the length to be meaſured, B is your firſt ſtaffe, C the ſecond ſtaffe, differing from B the firſt orthogonally 100 *foote, D the third ſtaffe being diſtant backe from the firſt in a right line with the marke* 133 *foote. E is the fourth ſtaffe running ſidewiſe orthogonally or in a ſquire from the third, vntill the fardeſt part of your length or marke is perceiued in a right line with the ſecond ſtaffe, being diſtant from D the third ſtaffe* 120 *foote. Now by ſubtraction ſubduce* 100 *from* 120, *there remaineth your diuiſor* 20. *Then multiplie* 133 *with* 120, *ſo ariſeth* 15960, *which diuided by* 20 *commeth* 798 *foote the true diſtance betweene D and A.*

But for aſmuch as this concluſion is to bee done without inſtrument, and heere orthogonall motion ſidewiſe is required, it ſhall be requiſite alſo to declare howe an orthogonall or right angle is vpon the ſodaine to bee made, yee ſhall therefore (according to Pythogoras *inuention mentioned among the diffinitions at the beginning of this Booke) take* 3 *ſtaues, cordes or ſuch like, making the one* 4 *ſuch partes as the other is* 3 *and the third* 5: *This done conioyne their endes togither and the angle ſubtended of the longeſt ſtaffe is aright, which firſt placed at B, and after at D, directing one of his comprehending ſides to A, the other ſhall guide you to C and E, or if you deſire with more expedition to diſpatch and not tarrye the proportioning of the cordes or ſuch like to this Pythagoricall rule, take any* 3 *ſtaues, ſtickes or threads, and conioyne them, making a triangle, it forceth not of what forme or faſhion it bee: then placing one angle thereof at B, turning one ſide to A, direct your ſelfe ſidewiſe by the other, alwayes remembring to place the ſame Angle at D, and departing ſidewiſe againe in like manner, in all the reſt doe as before is declared. Thus vſing any meane diligence, yee ſhall moſt exactelye meaſure anye diſtance.*

The

The 19 Chapter.

With Halbardes, Pikes or ſtaues hauing no other inſtruments, you may meaſure the diſtance betweene any two markes lying in a right line from you, not approching any of them.

You ſhall firſt (as was declared in the laſt Chapter) prepare an Angle with topning any three ſtaues or ſuch like together, which you muſt (at your ſtanding) place in ſuch ſort that one of the ſides containing the Angle, may lie directly towarde the marke: then ſetting vp a ſtaffe, pike or other marke there, departe ſidewiſe as the other ſide of your Angle ſhall direct you, ſo far as you liſt, the more ground the better. And there ſet vp your ſecond ſtaffe or marke, then goe directly backe from your firſt ſtaffe (alwaies keeping it exactly betweene your ſight and the markes) as many ſcore againe, or pike length as yee liſt, ſetting vp a third ſtaffe. This done, you ſhall place the ſame Angle you vſed at your firſt ſtaffe, nowe againe at your third ſtaffe, in all pointes as it was before: The one ſide of the Angle lying directly towarde the firſt ſtaffe, the other ſide will ſhewe you whither you ſhall goe to place your fourth ſtaffe, for paſſing on ſtill in a right line with that ſide of your angle, you ſhall at the laſt find the ſecond ſtaffe iuſtly ſituate betweene you and the fardeſt marke, there ſet vp the fourth ſtaffe, then remoue your Angle againe to the ſecond ſtaffe, and placing there as before the one ſide euen with the firſt ſtaffe paſſe on in a right line with the other till you come directly betweene your neereſt marke and the fourth ſtaffe, there pitche vp the firſt. Nowe ſhall you meaſure how many pace, halberd or pike length is betweene your firſt and ſecond ſtaffe, deducting that from the diſtance betweene the third and fourth, and this remaine you ſhall reſerue for a diuiſor. Then multiplie the diſtance betweene the ſecond and firſt ſtaffe in the diſtance betweene the third and fourth ſtaffe, the product diuided by your reſerued diuiſor, yeeldeth in the Quotient the true diſtance betweene theſe two markes.

Example.

A B the two markes whoſe diſtance I would meaſure, C my ſtanding place where I ſet vp my firſt ſtaffe, I my triangle made of three ſtaues, halberds, billes, or any ſuch like things, K L M, the Angle which I will now vſe in this practize N, firſt placed at C, ſecondly at D, thirdly at E, at C and D, the ſituation is all one, but at E, it ſomewhat differeth as you may beholde in this figure, which I would haue you note, leaſt happely yee bee deceiued in your practiſe, the firſt ſtaffe C, the ſecond E, the third ſtaffe D, F the 4, G the fift ſtaffe, C E the diſtance betweene the firſt and third deducted from D F, the diſtance betweene the ſecond and fourth there remaineth H F your diuiſor, which meaſured, I admit 50 *halberd lengths. The diſtance betweene G E* 30 *halberd lengths, the ſpace betweene D F* 100 *pike length. Now* 100 *multiplied in* 30 *produceth* 3000, *which diuided by* 50 *leaueth in the quotient* 60, *I conclude therefore the diſtance betweene A B* 60 *pike lengthes. This one thing is ſpecially to bee noted, that whatſoeuer you meete the ſpace G E withall, whether it be halberd, bill, arrow or ſtaffe that yee vſe the ſame in meaſuring H F, and as for D F, it forceth not what you meaſure it withall, marry whatſoeuer it be, your quotient ſhall beare the ſame denomination: preciſenes is to be vſed in placing of your triangle, and in meaſuring E G, and H F, otherwiſe error may enſue, eſpecially if D F bee but a ſmall diſtance, and the Angle at B very ſharpe, there needeth in this matter no furder admonition, ſmall practiſe will reſolue all doubtes.*

The

The 20 Chapter

To meaſure the diſtance betweene any two markes howſoeuer they be ſituate, though there be riuers or ſuch like impediments betweene you, as ye cannot approche nighe any of them, and that without inſtrument alſo.

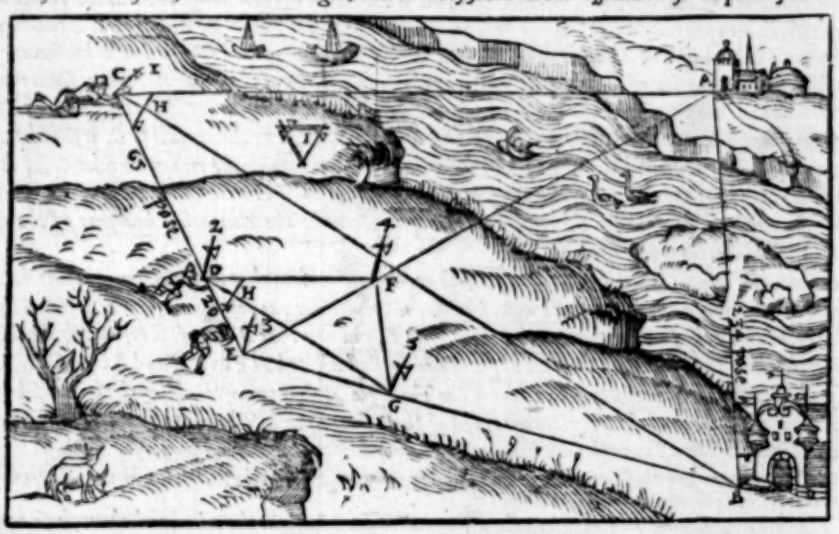
YOur Angle as before hath beene ſayde, prepared of anye three ſtaues, you ſhall firſt at pleaſure ſet vp one ſtaffe, and applying thereunto your Angle in ſuch ſort, that the one containing ſide lie directly to one of your markes (which here for diſtinction I will call the firſt) go backward to and fro til you find your ſecond marke preciſely couered with your ſtaffe, noting what part of the ſide ſubtending the Angle is cut by your line viſuall and there make a fine notche, or marke vpon that ſubtending ſtaffe, which doone you ſhall goe ſidewiſe from the firſt erreared ſtaffe, as the other containing ſide of your Triangle will direct you ſo farre as yee liſt, and then ſet vp your ſecond ſtaffe, yet paſſe on from thence in a right line as many foote pace or other meaſure, as you will, ſetting vp againe the third ſtaffe, now at your ſecond ſtaffe ſituate your Triangle in all reſpects as it was at the firſt, and paſſe on from thence in a right line with that containing ſide of your Angle that riſeth from your ſtaues, and commeth ſomewhat toward the marke going ſo farre till you eſpie your ſelfe iuſtly betweene your third ſtaffe and the firſt marke, there ſet vp the fourth ſtaffe, then reſort to your Angle againe, and ſtanding behinde that ſecond ſtaffe, note whither a right line from the Angle to that notche (before made on the ſubtendent ſtaffe or ſide of the triangle) will direct you, for that way preciſely ſhall you go on till you come in a right line with the ſecond marke and third ſtaffe erecting there the fift ſtaffe. This doone you muſt meaſure the diſtance betweene your ſecond and third ſtaffe, reſeruing that for a diuiſor, then multiply the diſtance betweene your firſt and third ſtaffe in the diſtance betweene the fourth

and

and fyfe ſtaffe, the product diuided by your reſerued Diuiſor, yeeldeth in the Quotient the true diſtance betweene the two markes.

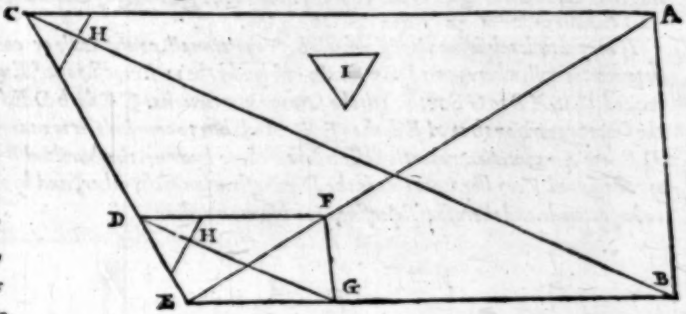
Example.

A B the two markes whoſe diſtance I would mete C, my firſt ſtaffe. I my Triangle made of 3 ſtaues placed therat, as you may ſee in the figure directing with the one containing ſide to A, the firſt marke and with the other to D and E my ſecond and third ſtaues. H is the fine notche or marke vpon the ſide ſubtendent to my Angle, where the line viſuall from C to the ſecond marke B paſſeth. my Triangle I ſituate now at D as it was before at C, the one conteining ſide lying euen with the ereared ſtaues the other directeth to my 4 ſtaffe F, placed in a right line with E, the third ſtaffe, and A the firſte marke. Againe my line viſuall proceeding from D to H the ſubtill notche in the ſubtendent ſide of the angle, extendeth to my 5 ſtaffe G, ſituate exactly beeweene E the 3 ſtaffe, and B the other marke: This done, I meaſure the Diſtance betweene my 2 and 3 ſtaffe, finding it 20 foote, likewiſe betweene the 4 and 5 ſtaffe 72 foote, finally betweene the firſt and third ſtaffe 65 pace, ſo that according to the rule before giuen, multiplying 65 in 72, I haue 4680, which diuided by 20 yeeldeth in the quotient 234 ſemaine pace is there betweene A and B. I haue not here ſet out the figure in iuſt proportion, anſwering to theſe numbers, for that is not requiſite, but in ſuch forme as may beſt open and make manifeſt the ſituation of the ſtaues and triangle, wherein conſiſteth all the difficultie of this practiſe.



A proofe or Demonſtration Geometricall of the former concluſion

TO auoide tedious recitalles of the premiſſes, I ſuppoſe the figure in all reſpects made as it was before: the propoſition that I will here demonſtrate ſhall be, that D E retaineth the ſame proportion to E C, that G F doth to A B, which admitted or aſſumed as a Lemma, the rule before giuen of multiplying F G in E C, and diuiding by D E, to produce A B may two wayes be approued geometrically by the 16 propoſition of the 6 booke of Euclides elements, and arithmetically by the 19 propoſition of the 7 booke of the ſame elements, wherein I mind not to vſe mo words, conſidering it is nothing els but the calculation by the rule of three, whoſe demonſtration hath bin handled by diuers, and well knowne to any, meanely in theſe matters tranailed. But how theſe D E and E C become to bee proportionall with G F and A B hath not bin by any hitherto proued, this Lemma therefore or pro-



poſition I minde to demonſtrate. Firſt it is apparant that D F is equidiſtant to A C by the 28 propoſition of the firſt of Euclides Elements, the wordes are theſe.

Ἐὰν εἰς δύο εὐθείας εὐθεῖα εἰσπίπτουσα, τὰς ἐναλλὰξ γωνίας τὰς ὑπ' αὐτῆς ἴσας ἀλλήλαις ποιῇ, παράλληλοι εἰσὶν αἱ εὐθεῖαι.

And may thus be Engliſhed, If any right line falling or paſſing through two other right lines, making the outward angle equall to the inward oppoſite on the ſame ſide, or the two inward angles ioyned together equall vnto two right angles, thoſe two right lines are parallele: but heere the line E C paſſing through the lines A C, D F maketh the outward angle F D E equal to the inward oppoſite on the ſame ſide A C D by ſuppoſition, bicauſe they were both made with one angle of the triangle, I may therefore conclude by this Theoreme, that D F is equidiſtant to C A, & farther infer by the ſecond Theoreme of the 6 booke, that A F to E F hath the ſame proportion that C D hath to D E, the propoſition is this.

Ἐὰν τριγώνου μιᾶς τῶν πλευρῶν ἀχθῇ τις εὐθεῖα παράλληλος, αἱ λοιπαὶ τοῦ τριγώνου πλευραὶ, ἢ ἐὰν αἱ τοῦ τριγώνου πλευραὶ εὐθεῖαν τμηθῶσιν, οὐδὲ πρὸς τομὰς οὐδὲ ξυμπεσοῦνται εὐθεῖα, παρὰ τὸν λοιπὸν ἔσται τῇ τοῦ τριγώνου πλευρᾷ παράλληλος.

If a Parallele line be drawne to any ſide of a Triangle it ſhall proportionally cut the two other ſides, and if two ſides of a Triangle be proportionally diuided the line that coupleth thoſe diuiſions ſhall be parallele to the other ſide. It is manifeſt by the firſt part of this theorem that D F being as it was before proued parallele to A C one ſide of the triangle A C E that it doth proportionally diuide the two other ſides C E, E A, in the points F D the like ſhall be proued of D G, for ſeeing the right line C E falleth on the two right lines D G, C B making the outward angle H D E equall to the inward and oppoſite angle H C D on the ſame ſide of the line C E which in the conſtruction of the Figure was ſuppoſed, it muſt needs follow by the 28 propoſition of the 1 booke of Euclide tofore recited, that D G is parallele to C B, and forasmuch as in the Triangle B C E, D G is drawne parallele to the one ſide C B, it ſhall by the ſecond propoſition of the 6 booke of Euclide (before alſo recited) deuide the two other ſides C E, E B proportionally, ſo that B G ſhall retaine the ſame proportion to G E that C D doth to D E, and ſo conſequently the ſame that A F doth to F E, as it is plaine by the eleuenth Theoreme of the fift booke of Euclide: his wordes be theſe.

Οἱ τῷ αὐτῷ λόγῳ οἱ αὐτοί, καὶ ἀνάλογον εἰσὶν οἱ αὐτοί.

The ſence thereof is this, that anye two proportions being equall or agreable to any one are alſo equall betweene themſelues, as heere firſte it was proued that A F to F E bare the ſame proportion that C D to D E, and now that B G to G E retaineth the ſame proportion that C D to D E: therfore by the Theorme laſt recited A F to F E and B G to G E, ſhall be proportionall: ſo haue you now two ſides of the Triangle A E B proportionall with the two ſides of the Triangle F E G, and the Angles conteyned of thoſe ſides equall. Therefore by the 6 Theorme of the 6 Booke thoſe two Triangles are Equiangle, the words of that Theorme are theſe.

Ἐὰν δύο τρίγωνα μίαν γωνίαν μιᾷ γωνίᾳ ἴσην ἔχῃ, περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον, ἰσογώνια ἔσται τὰ τρίγωνα, καὶ ἴσας ἕξει τὰς γωνίας, ὑφ' ἃς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν.

If two Triangles haue an Angle in the one equall to one Angle in the other, and about thoſe equal Angles the ſides proportionall, thoſe two Triangles ſhall be equiangle, and thoſe Angles, equall whoſe ſubtendent ſides are proportionall. Euclide alſo in his 4 propoſition of the ſame booke ſaith thus.

Τῶν ἰσογωνίων τριγώνων, καὶ ἀλλήλοις τὰς πλευρὰς περὶ τὰς ἴσας γωνίας, ἥ τε ὁμόλογος τῇ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσα πλευρά.

In Equiangle triangles aſwell the containing as the ſubtending ſides of equall Angles are proportionall, I may therefore affirme (ſeeing the Triangles A B E, E F G are Equiangle) that A B hath the ſame proportion to F G, that A E hath to F E, but before it was prooued that A F and F E were proportionall to C D and D E, conioynedly therefore A E to F E, ſhall retaine the ſame proportion that C E doth to D E, by the 18 propoſition in the fift booke of Euclide, ſaying thus:

Ἐὰν ὦσι μεγέθη ὁπωσοῦν ἀνάλογον, καὶ συνθέντι καὶ ἀνάλογον ἔσται.

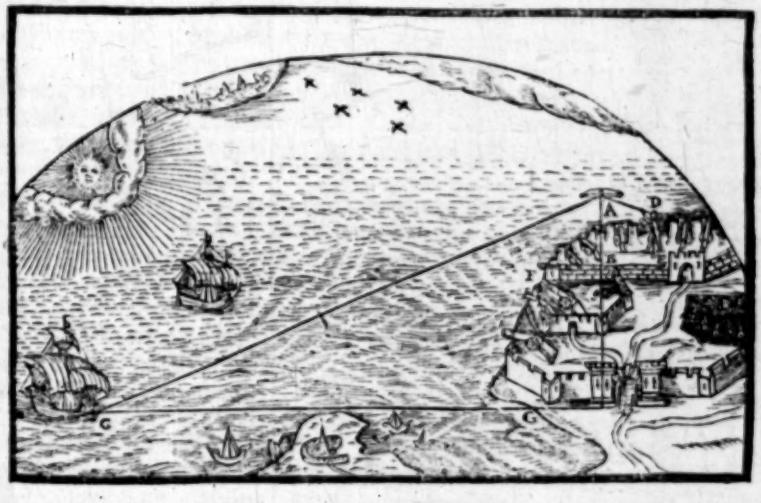
If magnitudes diſioynedly or ſeperatly be proportionall, conioynedly or compounded they ſhall alſo be proportionall, whereupon I may finally inferre by the 11 Propoſition of Euclides fifte Booke tofore recited, that A B to G F retaineth the ſame proportion that C E doth D E, bicauſe they both obſerue the ſame proportion that A E doth to E F. And thus to conclude, it is manifeſt that A B, F G, C E, D E, are 4 quantities proportionall: where of three knowne, the fourth A B by the Rule of proportion is to be found. Very like vnto this is the Domonſtration of the other, and by the ſelfeſame propoſitions to be prooued, and therefore ſuperfluous to vſe mo wordes.

The

The 21 Chapter

How ye may moſt pleaſantly and exactly with a plaine glaſſe from an high cliffe, meaſure the diſtance of any Shippe or Ships on the Sea, as followeth.

THe beſt kinde of Glaſſe for this purpoſe is of ſteele finely pulliſhed, ſo that the Superficies thereof bee ſmoothe, neither conuexe nor concaue, but flat and plaine as may be poſſible. This Glaſſe it behoueth yee to hang vp aboue the top of the cliffe with the pulliſhed face downeward equediſtant to the Horizon wherein you muſt vſe great diligence, for if there happen any erroꝛ in the ſituation thereof, great inconuenience may followe in your menſuration. This done, let a plumbe line fall from the Centre of your Glaſſe to the Superficies ground platte on the top of the cliffe: (which grounde platte) alſo you muſt vſe ſome diligence in the choyſe thereof, that it be as leuell and plaine as ye can finde any, but if it be not altogither euen oꝛ exact leuell, yee ſhall ſupplie that wante as I ſhall heereafter ſhewe you: But to returne to the purpoſe, your Glaſſe thus ſituate, turning your face toward the ſhip oꝛ other marke on the ſea, whoſe diſtance yee deſire, goe backward, alway hauing your eye fixed on the Glaſſe till ſuch time as ye can ſee the Ship, oꝛ rather the very Hull nerte to the water therein, that done, let another plumbe line fall from your eye to the ground, then circumſpectly meaſure both the length of theſe plumbe lines, and alſo the diſtance betweene this plumbe line and the other that fell from the Centre of the Glaſſe, this done, yee ſhall deduct the length of the perpendicular from your eye, out of the length of the other perpendiculare from your Glaſſe, and the difference reſerue for a Diuiſoꝛ, then multiplie the diſtance of the two perpendiculare lines, by the heigth of the Cliffe, I meane from the water vpward to the Glaſſe, and the product diuide by your reſerued Diuiſoꝛ, the Quotient will ſhew you the exact diſtance to the Ship oꝛ marke. But if your ground be not leuell, ye ſhall by your Quadrant ſearch the difference oꝛ inequalitie thereof, and if it bee lower at the Glaſſe than at the viewing ſtation, you ſhall deduct the difference from your Diuiſoꝛ, but if contrariwiſe, the difference ſhall be added to the Diuiſoꝛ, and then ſhall yee woꝛke as I haue before declared.



Example.

Suppoſe a Ship on the ſea C, whoſe diſtance I deſire to knowe ſtanding on the cliffe G F E, your glaſſe A equidiſtantly elenate, A B the perpendiculare lyne 72 inches, E your viewing ſtation, D E the altitude of your eye 69 inches, B E the diſtance betweene your two perpendiculare lines 20 foote, F G the heigth of the Cliffe from the ſea 96 pace. Now by deducting 69 inches the altitude of your eye from 72 the lengths of A B, the plumbe line from the Glaſſe, there remaineth three your Diuiſor, if the ground be leuell, otherwiſe ye muſt adde or detract the difference or vnequalitie according to the rules tofore giuen, but heere ſuppoſing your grounde leuell your Diuiſor remaineth 3. then reſolue 20 foote into inches, and multiply the ſame in the altitude of the cliffe vp to the Glaſſe 98, ſo haue ye 23520, which diuided by 3 yeeldeth in the quotient 7840 which is the exact diſtance of that parte of the Ship which yee did eſpie from the Centre of your Glaſſe.

YOu may on this manner from an high hill or mountaine, hauing any plaine or leuell ground on the top, not onely meaſure the diſtance of any marke that ye can ſee, but alſo ſet foorth the true plat and proportion of an whole Countrey, with all the Townes, Coaſts, Harborroughes, &c.

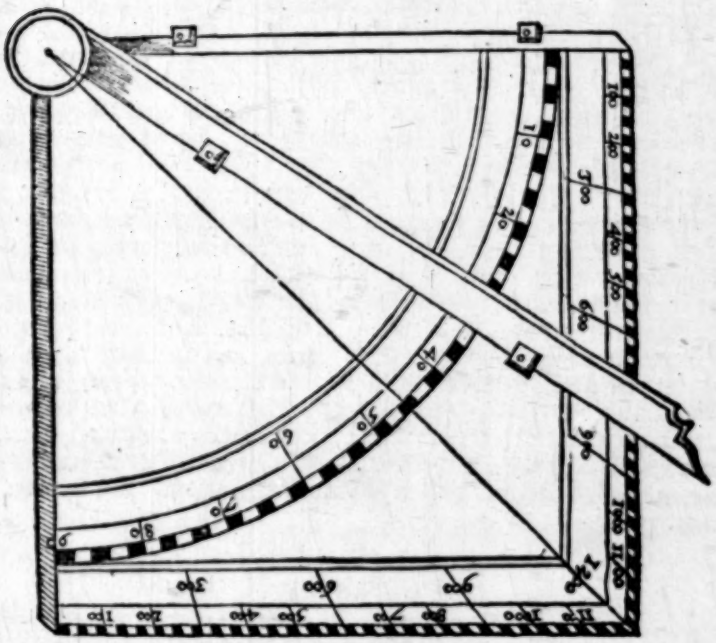
For if you mone circularely about your glaſſe, alway when you eſpie any marke, ſetting vp a ſtaffe, writing thereupon the name of the place yee ſee, whether it bee Village, Porte, Roade, or ſuch like, yee ſhall in the end ſituate as it were the whole countrey in due proportion vpon your platforme, ſo that meaſuring the diſtance of euery ſtaffe ſet vp from the middle line Perpendiculare falling from the Glaſſe, and the diſtance likewiſe of euery ſtaffe from other, yee may (working by the Golden rule) finde out the exact diſtance of euery Towne, Village, Porte, Roade, or ſuch like from your platforme, and alſo how farre euery one is diſtant off from other. Thus much I thought good to open concerning the effects of a plaine Glaſſe, very pleaſant to practiſe, yea moſt exactly ſeruing for the deſcription of a plaine champion countrey. But maruelous are the concluſions that may be performed by glaſſes concaue and conuex of Circulare and parabolicall formes, vſing for multiplication of beames ſometime the aide of Glaſſes tranſparent, which by fraction ſhould vnite or diſſipate the images or figures preſented by the reflection of other. By theſe kinde of Glaſſes or rather frames of them, placed in due Angles, yee may not onely ſet out the proportion of an whole region, yea repreſent before your eye the liuely image of euery Towne, Village, &c. and that in as little or great ſpace or place as ye will preſcribe, but alſo augment and dilate any parcell thereof, ſo that whereas at the firſt apparance an whole Towne ſhall preſent it ſelfe ſo ſmall and compact together that yee ſhall not diſcerne anye difference of ſtreates, yee may by application of Glaſſes in due proportion cauſe any peculiare houſe, or roume thereof dilate and ſhew it ſelfe in as ample forme as the whole towne firſt appeared, ſo that ye ſhall diſcerne any trifle, or reade any letter lying there open, eſpecially if the ſunne beames may come vnto it, as plainely as if you were corporally preſent, although it be diſtante from you as farre as eye can diſcerie: But of theſe concluſions I minde not here more to intreate, hauing at large in a volume by it ſelfe opened the miraculous effects of perſpectiue glaſſes. And that not onely in matters of diſcouerie, but alſo by the Sunne beames to fire, powder, or any other combuſtible mater, which Archimedes is recorded to haue done at *Syracuſa* in Sicilie, when the Romane Naule approched that Towne. Some haue fondly ſurmiſed he did it with a portion of a ſection Parabolicall artificially made to reflect and vnite the Sunne beames a great diſtance off, and for the conſtruction of this Glaſſe, take great paines with high curioſitie to write large and many intricate demonſtrations, but it is a meere fantaſie and vtterly impoſſible, with any one Glaſſe whatſoeuer it be to fire any thing, onely one thouſand pace off, no though it were a 100 foote ouer, marry true it is, the Parabola for his ſmall diſtance, moſt perfectly doth vnite beames, and moſt vehemently burneth of all other reflecting Glaſſes. But how by application of mo glaſſes to extend this vnitie or concourſe of beames in his full force, yea to augment and multiplie

the

the ſame, that the farder it is caried the more violently it ſhall pearſe and burne. *Hoc opus hic labor eſt*, wherein God ſparing life, and the time with opportunitie ſeruing, I minde to imparte with my countriemen ſome ſuch ſecrets, as hath I ſuppoſe in this our age beene reuealed to very few, no leſſe ſeruing for the ſecuritie and defence of our naturall countrey, than ſurely to be meruailed at of ſtrangers.

The 22 Chapter

The making of an Inſtrument named the Geometricall ſquare.



I ſhall prepare a fine plaine Plate, or a cleane foure ſquare planed board, yea, for want of them, foure equal, ſmooth, and well tried rulers, of what length, breadth or thicknes yee liſt, the longer the better, yet in my fantaſie to auoyde painefull cariage, it is moſt commodious that euery of them be but a foot and an halfe in length, one inche in breadth, a quarter or more thick, ye muſt ioine them by the helpe of ſome Artificers ſquirewiſe, vpon thoſe rulers, plate or boorde iuſtly ioyned, draw foure lines perpendicular or ſquire the one to the other. Now deuide the two fardeſt ſides frō the centre, each hauing 1200 portions at the laſt, marke all from your centre, forget not to haue an Index, not with commune ſights, but thus, let the neereſt be a thin plate, halfe an inch broad, and 3 inches in height, and in the middes a fine ſlit, the ſecond and fardeſt from the centre of that length, a ſtreight pin with a little knob in the top. Theſe ſights muſt be iuſtly ſet vpon the line fiduciall of your Index. This Index I would wiſh alſo marked with 2000 ſuch diuiſions as the ſcale ſide hath 1200, it hath place in the centre, and there made to tarry, ſo that with eaſe it may be turned from the firſt to any point. The exact handling of this Inſtrument with moſt comlineſſe framed, I commit to the diligent maker. For more inſtruction, behold the Figure, ye may commodiouſly deſcribe a quadrant in your ſquare, the making of which is declared.

The

The 23 Chapter.

You may redely hereby without Arithmetike mete the diſtance of any marke.

THis Inſtrument handſomlie placed vpon his ſtaffe or otherwiſe, laye the line Fiduciall of your Index vpon the beginning of the Degrees in your Quadrant, and turne your whole Inſtrument (the Index not mooued) till ye maye eſpie through the ſights your marke then remoue the Index to the contrary ſide of the quadrant, placing the line fiducial on the ſide line where the degrees end, & looking through the ſights, ye ſhall eſpie a marke ſidewiſe, ſome certaine number of ſcores, the more the better. This done, ſet vp a ſtaffe where the Centre of your Inſtrument was, and placing it againe at the marke laſt eſpyed, ſet your Index on the beginning of the Degrees, mouing your whole Inſtrument, till you finde through the ſightes the ſtaffe at the firſt ſtation, then remoue your Index (the Quadrant keeping his place) till yee may againe eſpie through the ſightes your marke, which done, note the degrees cut by the line fiduciall: and then worke thus, vpon ſome euen ſmothe Superficies whether it be boord, plate, or paper: draw firſt a ſtreight line, and opening your compaſſe to ſome ſmall diſtance, call that ſpace a ſcore and make ſo many ſuch diuiſions in your line as there is ſcores betweene your ſtations. Then create vpon the one end of your line a perpendicular, and fixing the one foote of your compaſſe at the other ende, opening it to what wideneſſe ye thinke good, drawe an Arke riſing from the ſame line that repreſenteth your ſtationarie diſtance, and diuiding it into degrees (as you were taught in making the Quadrant) extend from the Centre to the number of Grades cut by your line Fiduciall a right line, till it concurre with the perpendiculare before erected. Then ſee howe oft that ſpace (which repreſented the Score in diuiding your Stationarie diſtance) is conteyned in the perpendiculare, ſo many ſcore is the marke off from your firſt Station, and by diuiding the Hypothenuſall line, you may in like manner finde the diſtaunce from the ſeconde Station.

Example.

A *the firſt Station*, C *the ſecond*, D *the marke*, A C *foure ſcore paſe: degrees of the Quadrant cut at the ſecond Station*, 71 ½: H *the vnitie or meaſure repreſenting one ſcore*, E F 4 *partes*, G F 12, G E 12 ⅓, *or neere thereabout. Thus may you conclude the diſtance of the marke from the firſt Station* 12 *ſcore pace. The Hypothenuſal line or diſtance of the marke from the ſecond Station* 12 *ſcore and* 13 *pace.*

The

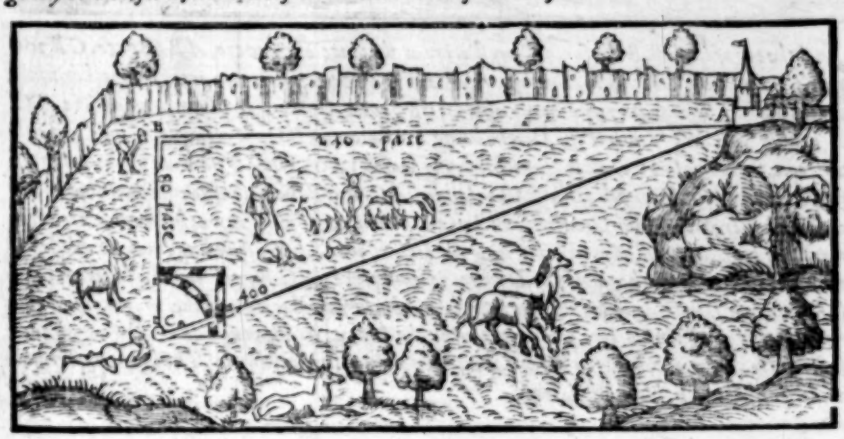
The 24 Chapter.

How with your Square Geometricall to tell any length so farre as ye may see by supputation.

YOu shall disagree from those writers that haue declared the vse of the Square in the Latitudoong, other than this which I shall now open vnto you. So order your Geometricall square, that all sides may be of like heigth from the ground, to auoide Grasse, mole hilles and such other impediments, precisenes in this ordering is not so requisit, call to remembrance that your Square hath two principall lines, one squire vnto the other running frō the centre of either side to ꝑ beginning of your points: set your Index, vpon the line Fiduciall vpon one of those lines, that side lying along with the Index towards the marke. Nowe, the extreame parte of your length perceiued through your sightes, turne the Index (the Square not moued) to the other principall line, squire to the first, looking againe through the sightes, and noting some marke a good distance from you the more ground the surer: This done, and a staffe pitched vp where the Centre of your Instrument stood, conuey that Instrument to the second marke, turning it and your Index to the place where you firste were, the Index being in the principall line as afore, euen so soone as yee can espie your first Station through the sights, remooue that Index vntill you may see the extreame part of your length, your sight receiuing it diligently note the points touched: Now if the Index fall on the left side of your Scale, I meane the side which falleth Perpendicular to that side of the Square issuing from the Centre, whereon your Index was first placed, then must ye multiplie the space between the first and second place by those partes cut, and diuide by 1200, the Quotient is your desire. But if the Index fall on the right side of the scale, then shall you worke contrarie, multiplying the space betweene your Stations in 1200, and diuiding by the partes cut, or yee may reduce the partes of the right side, to parts proportionall of the left, and worke with them according to the first rule thus: Diuide the Square of 1200 by the parts cut in the right side of your Scale, the Quotient is the parts proportionall, which increased by the distance of your Stations, making partition by 1200, the Quotient is the true distance of the marke from your first Station.

Example.

A is the place to be measured, B, the marke where I first disposed mine Instrument, from it I go orthogonally to C the Index cutting there 400 *in the right side of your Square, the distance between B & C, I haue supposed* 80 *pace, wherfore multiple* 1200 *by* 80, *so there commeth* 96000, *which diuided by* 400, *declareth vnto me* 240 *pace, the true length from B to A. Or by diuiding* 1440000 *the Square of* 1200, *with* 400 *the partes cutte, you shall produce in the Quotient* 3600, *your proportionall partes found by the Rule of Reduction, which augmented in* 80, *yeeldeth* 288000, *and that diuided by* 1200, *bringeth in the Quotient* 240, *which is the Longitude A B agreeing with the former operation. This manner of reduction, I would wish you diligently to note, for it shall hereafter in diuers conclusions be vsed.*



The 25 Chapter.

How to mete any line Hypothenuſall as the diſtance from your eye ſtanding in a valley to the top of an hill or high turret, &c.

FIrſt, if the Hill or Turret be ſteepe vp, ſo that the foote be viſible lying perpendicularly vnder the top, yee ſhall firſt meaſure the diſtance of the baſe, either by this inſtrument or otherwiſe as was before declared: and alſo the heigth of the top or ſummitie of the ſame hill or tower: which done, ye ſhall ſquare aſwell the longitude as the altitude, ioyning together the products, the roote quadrate of the whole number, is the deſired diſtance or line Hypothenuſall: you may in this manner (approching nighe any towne of warre) tell the iuſt length of the ſcaling ladders that ſhall reache from the brim of the ditch or edge of the counterſcarfe, to the top of the wall or curteine, by adding the Square of the Ditches latitude, to the heigth of the curteine aboue the leuell of the outward bancke, for the roote of the product will be the true length of the ſcaling ladder.

Example.

A the top of the hill, B the foote, C my ſtation or the place of mine eye, A B 60 pace, C B 200 pace, the Square of 60 is 3600, the Square of 200 is 40000, theſe two ioyned together make 43600, whoſe Quadrate roote being about 208 pace 3 foote is the Hypothenuſall line A C. Likewiſe A B the breadth of the Ditche being 30 foote, and B C the altitude of the Curtaine 30 foote, there two ſquares added together bring foorth 1300, whoſe Quadrate roote being 36 foote very nigh, is the length of the ſcaling ladder A C. But if the baſe of your mountaine be not viſible, then ereare vp your Geometricall Square, the index placed (as was before declared) toward the top of the Hill A, and remooning the Index (your Square ſtanding immoueable) eſpie your ſecond ſtation Orthogonally at D where ye muſt place the Centre of your Inſtrument, and ſo ſituate your ſquare againe, that you may behold both your ſtation and the mountaine toppe without ſtirring of the Square, onely remooning the Index: in all the reſt do as is before already ſufficiently declared, behold the Figure, there needeth no other Example. The laſt Chapter well underſtood, openeth this moſt plainely.

To

To meaſure the diſtance betweene any two markes that lie in one right line from your eye.

YOu may reſolue this by the former, meaſuring how far either is diſtant from your ſelfe, and then deduct the one from the other, Or thus an other waye, the ſide of your Geometricall ſquare directed towards them, depart orthogonally (as is to-fore declared) 100 or 200 paces as ye liſt, the more the better, then place your Inſtrument againe, turning the ſide toward your firſt ſtation, remouing the index to either of the markes, noting what parts at either place the index dooth cutte of the ſcale. And if the index at both times fal on the left ſide, deduct the leſſe from the greater, with the number remaining, augment the diſtance betweene your ſtations, and diuide by the whole ſide of the Scale, your Quotient is the diſtance. If the index at either time fall on the right ſide, then muſt you by the rule afore giuen, reduce them into partes proportionall, or if at one time the index fall on the left, at an other time on the right, then ſhall you onely reduce the partes cut on the right ſide, which done, deduct as before is ſayd the leſſer from the greater, and with the remainder multiply your diſtance ſtationary, the product diuided by 1200 yeeldeth how far one marke is beyond the other.

Example.

Admit A B the markes in a right line from C your firſt ſtaeion, D the ſecond ſtation Orthogonally ſituate from C, where your ſquare being placed, ſuppoſe your Index firſt cut 800 parts on the left ſide, and after 900 partes on the right, theſe 900 of right ye muſt reduce, diuiding the ſquare of 1200 by 900, as was taught in the former Chapter: ſo will your Quotient amount to 1600, from which if yee withdrawe 800 the partes cut on the left ſide, there will remaine 800 which multiplied in 200 pace the diſtance ſtationary C D, there amounteth 160000. This diuided by 1200 yeeldeth in the Quotient 133 $\frac{1}{3}$ the diſtances therefore of A B your markes is 133 pace, one foote 8 inches.

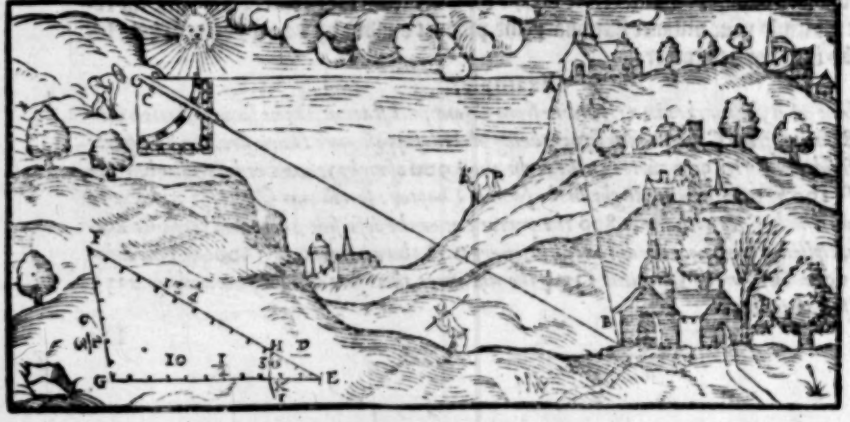
The 26 Chapter.

To measure the distance betweene any two marks lying in one plaine leuell ground with your eye or station howsoeuer they be situate without supputation.

MEasure by the rules before giuen, howe far either marke is off from you, then placing the inder vpon the side of the square, turne your instrument till you can espie through the sights one of your marks, the square so remaining stedy, moue your inder toward the other marke, and when you haue found that marke also through your sights, note what degrees of the Quadrant, the line fiduciall cutteth, this done, ye shal vpon some plaine boorde, plate, or such like, draw a straight line, then open your Compasse some meane wideneſſe, and fixing one foot at the beginning of the line, with the other make an intersection. Now if the markes be many Miles off, you may terme that portion a mile, or if the distance of the marks be smal, as here, but if they be very neer at hand, this little line shall represent a pace onely, heerein you must vse discretion, respecting the distance of the marks, & so proportioning your line which is the vnity of the worke, that your platt or boord may receiue the rest of your operations: then proceede with your compaſſe, making so many diuiſions in your drawne line, as there are miles, leages or paces in ỳ distance of one of your marks from your standing, this finished, open your compaſſe at pleaſure, fixing the one foot at the end of your line, with the other draw a circumference or arke, & this arke you shall diuide into degrees, as was taught before in the making of the Quadrant, beginning at your drawne line, & so paſſing on, till you come to such number of degrees, as was cut by the line Fiduciall, then laying a Ruler to the center, applying it to ỳ end of the arke, draw an other straight line, & your compaſſe againe opened to the length of your little line, (which I terme the vnitie of your worke) begin at the foresaid centre, making againe so many diuiſions in that latter line, as there was Miles, leages or paces in the distance of the other marke from your standing: And if it fall out that in those distances there be any odde leage, pace, or feet, ye may diuide one of those litle lines or vnities of your worke into mo parts accordingly, & so set foorth proportionally the exact distances of the two marks according to the measure first found: this done, ye shall couple together the ends of the two straight lines with an other straight line, finally opening your cõpaſſe to the length of the vnitie, beginning at the one end of this last drawne line, measure how many of these vnities is therein contained, for so many miles, leages or pace (according to the denomination of the vnity) may ye say there is certainly betweene those two marks. But if at the end of this latter measuring, there be any portion left leſſe than the vnitie, you must as I haue before sayd by diuiſion of the vnitie search out what portion it is. For more plainnes behold the example.

Example.

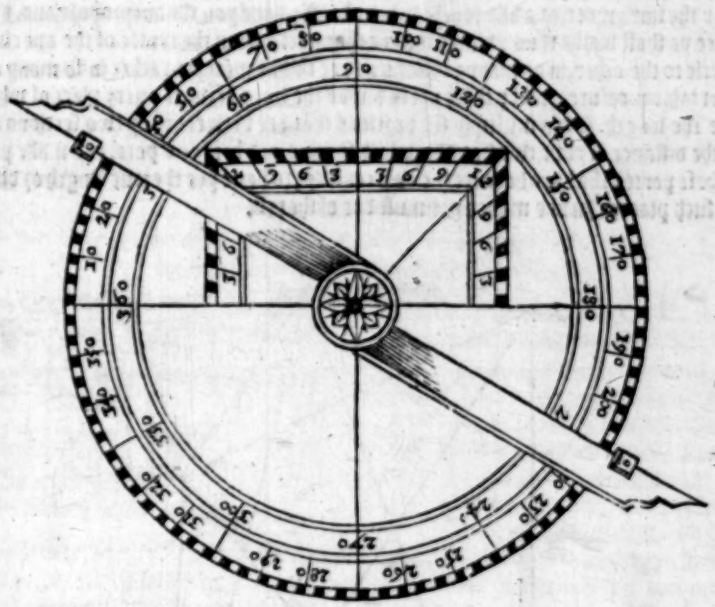
Admit I would measure the distance betweene A B two churches my ſelfe ſtanding on a hill at C. firſt I ſuppoſe C A 10 mile $\frac{1}{2}$. C B 13 $\frac{1}{4}$ the arke of the quadrant cut by the line fiduciall 30 degrees, then reſorting to some plaine boord or ſuch like, I draw the right line E F: My vnitie or mile I make D, & opening my compaſſe to that meaſure, I tell on 13 $\frac{1}{4}$ making an end at F, then opening my compaſſe at pleaſure, I make the arke H I, one foote of my compaſſe faſtned in E, and beginning at H, I number toward I 30 grades, ending at K: then draw I the line E K, forth to G, which with my compaſſe extended to the length of D, I diuide into 10 parts, and $\frac{1}{2}$ finiſhing at G, & with a right line conioyning G F, I meaſure againe how many vnities is therin conteined. I find 6 $\frac{3}{4}$ or there about, wherefore I conclude betwe e theſe two churches 6 $\frac{3}{4}$ miles, this kind of meaſuring is good for the vnlearned, but ſuch as haue arithmetike, other rules moſt exact ſhall enſue.



The 27 Chapter.

The compoſition of the ieſtrument called Theodelitus.

IT is but a circle diuided in 360 grades or degrees, or a ſemicircle parted in 180 portions, and euery of thoſe diuiſions in three or rather 6 ſmaller partes, to it yee may adde the double ſcale, whoſe ſingle compoſition is mentioned. The ſides of that ſcale diuided in 12, 60, or 100 partes. The inder of that inſtrument with the ſightes, &c. are not vnlike to that which the ſquare hath: In his backe prepare a vice or ſcrue to be faſtned in the top of ſome ſtaffe if it be a circle as heere: let your inſtrument be ſo large that from the centre to the degrees may be a foote in length, more if ye liſt, ſo ſhall you not erre in your practiſes, the backe ſide muſt bee plaine and ſmoothe to drawe circles and lines vpon, as ſhall be declared: for a farther declaration of that I haue ſayde, beholde this figure following.

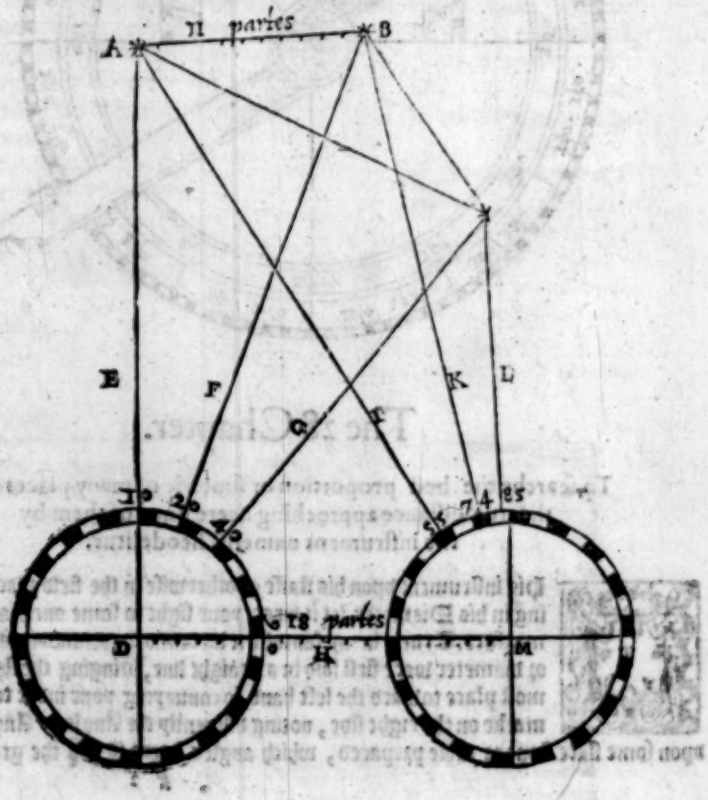


The 28 Chapter.

To ſearche the beſt proportion or ſimetrie of many places with the true diſtance approching neere none of them by the inſtrument named Theodelitus.

THis inſtrument vpon his ſtaffe or otherwiſe in the field placed the inder being in his Diameter, let it direct your ſight to ſome one place which ye will meaſure. Truely in my fantaſie it were more commodious if his dimetient or diameter were firſt laid in a ſtraight line, bringing the ſight to the vttermoſt place toward the left hand, ſo conueying your inder to euery place or marke on the right ſide, noting diligently the Angle or Angles of poſition vpon ſome ſlate ſtone or table prepared, which angles heere I call the grades or degrees

from the dimetient apparantly cut by the line fiducial whilst he is brought to euery marke. This performed, resorte to the backe side of your instrument where necessitie requireth a circle or a semicircle to be made, deuided exactly in 360, or if it bee an halfe circle in 180 degrees or portions, euen as your Theodelitus here is, from whose center ye must finely draw those angles of position noted before in your table taken by your Instrument, so that after your purpose had, they may be cleane put out, then pull the index the instrument vnmoued toward the right hand, at pleasure obseruing through the sights some marke 300 yardes from you or lesse as ye list. There shall be your second place, noting vpon your slate the angle of position from the dimetient to the line fiductall, directing to the second place or mark, which Angle ye must drawe in the backe side from the Centre at large euen as you found it in your instrument, then conuey your Theodelitus from thence to the second marke or standing place, causing the diameter iustly to note your first abiding. And here euen as before ye must search Angles of position againe, and marke them in the table or slate, which done, resort to the back, & vpon the last angle being the line directing to the second standing place, draw a circle as far or neer to the other as ye list: or a semicirle diuided in 180 degrees. Whose diuisions must take their beginning at the line which is diameter of the semicircle. Now draw your visuall lines or angles of position last taken by your instrument at large, & see wher the lines meet, or alike toucheth his like. So haue you the dueyporions. For the Distance ye shall worke thus, diuide the line that goeth from the centre of the one circle or semicircle to the other, in as many portions as yee thinke meet, or rather in so many as yee finde certaine measure, and by those parts diuide the lines betwixt euery place of which ye require the length. Then multiply the portions that are betweene any two sections or places in the distance of your two stations, which I imagined here 300 pace, and make partition by those partes that are betwixt the two centres, so haue yee the true length or distance of two such places. In like manner yee must doe of the rest.



Example.

Whereby all thing may better appeare, A B C are the markes in the field to be measured, D the first abiding or standing place, where ye shall set the centre of your instrument, his diameter lying directly against A the first marke E F G H the foure visuall lines running by the Angles of position of the instrument vnto all the marks: the first noting no degree or portion: the second 20, the third 40, the fourth 90 degrees of the instrument, which directeth to the second station M, where ye shall now set the centre of your instrument, the diameter lying right against your first abiding: here the lines visuall I K L running to the markes: cut new Angles of position againe. The first noteth 55 degrees, the second 74, the third 85 grades. Now, if yee marke diligently where these lines crosse the other, there is the true proportion of such places, from those sections or crossings drawe right lines as appeareth by the Figure. Now to get the distance, ye shall seeke out the space betweene the two stations D M: being 300 pase or yardes, notwithstanding it is diuided but into 18 parts: also betweene the markes or places A and B, are contained 11 suche partes. Seeing that I am ignorante what number of yardes bee conteyned in those 11 portions, I am compelled to worke by the rule of proportion thus, 18 bringeth 300 yardes, what shall 11 bring me? your quotient sheweth 183 and $\frac{1}{2}$ that is $\frac{1}{4}$ which maketh a foote: so betweene A and B are contained 183 paces and a foot. Thus of all other aswell of D A, D B, D C, M C, M B, M A, as of C B, C A, the Reader must not be ignorant that euen as I haue supposed by this figure, the instrument in the field placed and all things performed by it with all diligence marked vpon a slate or such like, so ought it to be exactly drawne on the backeside of your instrument, euen as the Figure afore declareth.

Certes most excellent and farre passing all other is this kinde of measuring, requiring great exercise, the distance of one place had I am brought into the knowledge of many with the best proportion.

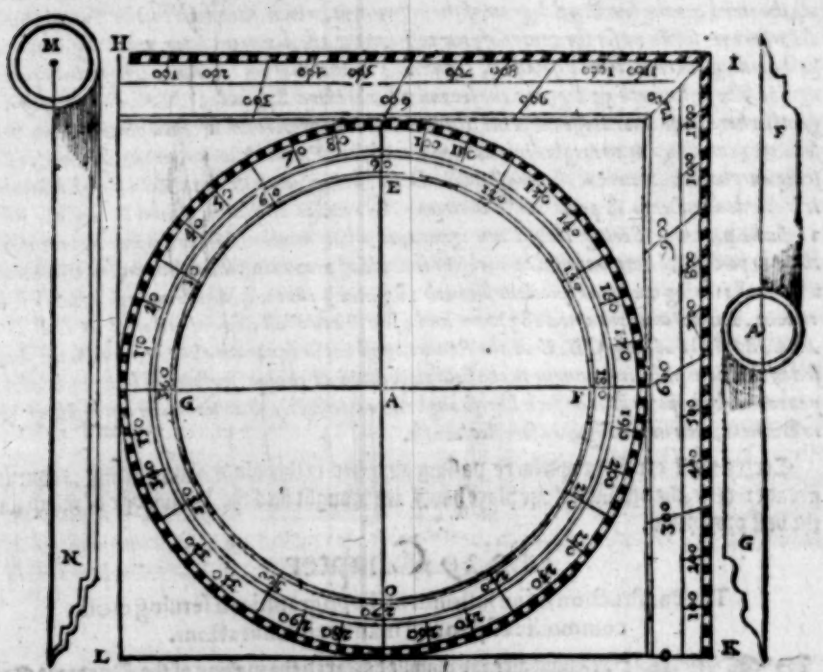
The 29 Chapter

The construction of an instrument Topographicall seruing most commodiously for all manner mensurations.

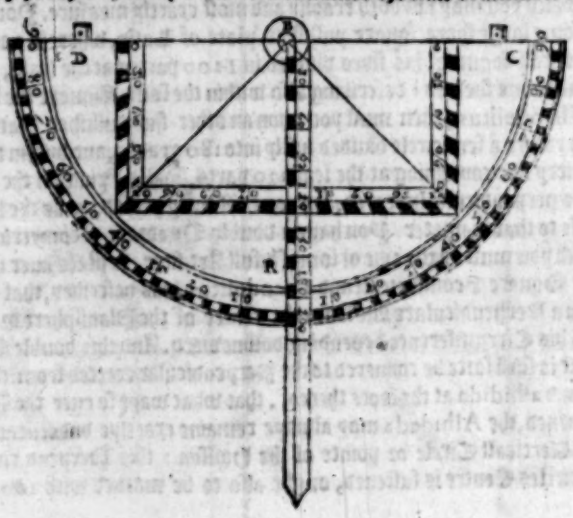
HAuing already plainlie declared the making of the Quadrant Geometricall with his Scale therein contained, whose vse is cheefelie for altitudes and profundities: the composition also of the Square and Planisphere or Circle named Theodolitus, for measuring lengthes, breadths, and distances. It may seeme superfluous more to write of these matters, yet to finish this treatise, I thinke it not amisse to shew how you may ioyne these three in one, whereby you shall frame an instrument of such perfection, that no manner altitude, latitude, longitude, or profunditie can offer it selfe, howsoeuer it be situate, which you may not both readily and most exactly measure. You shall therefore first prepare some large foure square pullished plate of Latin, wherein you may describe your Geometricall Square, his sides diuided in 1200 partes at the least, with inder and sightes as was before shewed: describing also within the same Square the Planisphere or circle called Theodelitus, then must you vpon an other fine pullished plate, drawe your Quadrant, or rather a semicircle diuided iustly into 180 grades, and within the same a double Scale: euery side containing at the least 120 parts, finally, fixing on the dimetient thereof two sightes perpendicularly reared, and equedistantly persed, so as the line visuall may passe parallele to that diameter. You haue a double Quadrant Geometricall with a double scale, which you must by the aide of some skilfull Artificer, so place ouer the other plate, wherein your Square Geometricall and Theodolitus was described, that his centre may exactly rest in a Perpendiculare line from the centre of the Planisphere or Circle named Theodelitus his Circumference depending downeward. And this double Quadrant or semicircle, must in such sorte be conuered to the Perpendicular erected from the centre of the planisphere, and alhidida at the foote thereof, that what waye so euer the Diameter with sightes bee turned, the Alhidada may alwaye remaine exactlye vnderneath it, directing bothe to one Verticall Circle or pointe of the Horison: this Perpendiculare whereunto the Semicircles Centre is fastened, ought also to be marked with 200 parts equall to

the

the diuiſions of the ſcale beginning at the centre, ſo proceeding downeward till you come to the end of thoſe 200 portions: more I need not ſay of this inſtrument, conſidering the conſtruction, if euery parte hath beene ſeuerallie declared ſufficiently before, for the placing and comioyning of them behold the Figures.



IKLH the Square Geometricall, MN his inder with ſightes, GEFO Theodeſitus, GF his Alhidada or inder with ſightes AB the line perpendiculare from B downward noted with 200 partes, equall to the diuiſions of the Scale, DRC the ſemicircle hauing on his Diameter two ſightes fixed as was tofore declared. This is alſo to be noted, that the double ſcale is compound of two Geometricall ſquares, the one ſeruing for altitudes, the other for profundities. The ſquare which the line perpendicular cutteth when the Diameter is directed to any markes lying lower than your ſtation, I cal the ſcale of profundities, the other ſhall for diſtinction be named the ſcale of altitudes.



This

THis semicircle ought so to be placed that the centre B hang directly ouer the centre A, and that the diameter D C with his sightes may bee moued vp and downe, and also sidewise whither you list, alwaies carying G F about directly vnder it. You must also prepare a staffe piked at the end, to pitche on the ground with a flat plate on the top to set this instrument vpon. It is also requisite that within Theodelitus you haue a needle or flie so rectified, that being brought to his due place, the crosse diameters of the Planisphere may demonstrate the foure principall quarters of the Horizon, East, West, North and Southe: and this may you doe by drawing a right line making an angle (with that one diameter of your instrument representing the meridiane) equall to the variation of the compasse in your region: which in England is $11\frac{1}{4}$ grades or neere thereabout, and may be readily obserued in all places sundrie wales. But thereof I minde not here to entreate, forasmuch as it appertaineth to *Cosmographie*, and Nauigation, whereof I haue compiled a treatise by it self, touching the fabrication this may suffice. Now for the vse great heed must you take in pitching of the staffe whereupon this instenment is placed, that it stand perpendicularly which by a line and plummet ye may trie, also when the instrument is placed thereon, yee shall by a line and plummet fixed on the centre of the semicircle discerne whither it be rightly situate: for if the thread and plummet hanging at libertie fall close by the perpendiculare, then is it well: otherwise ye must moue the staffe to and fro till ye find it so. This done, it behooueth you also to set this instrument on your staffe, that the needle haue his due place, so as the semidimetientes of Theodolitus may direct vnto the foure cardines or quarters of the Horizon, then what soeuer marke you espie, whose distance, altitude, or profunditie you desire, turne the dimetient of the semicircle to and fro, vp or downe, till through the sightes thereon fixed you haue espied it, alway the circle of Theodelitus remaining immoueable: finally you shall note both what degrees the Alhidada cutteth of the circle, and the perpendiculare of the semicircle, and also what parts of the perpendiculare is intercepted with the Scale, these numbers thus found, you shall diuerslie vse as shall hereafter be declared. This is farder to be noted, that the double scale is compound of two geometricall squares, the one seruing for altitudes, the other for profundities. The Square which the line perpendicular cutteth when the diameter is directed to any marke lying lower than your station, I call the Scale of profundities, the other shall for distinction bee named the Scale of altitudes.

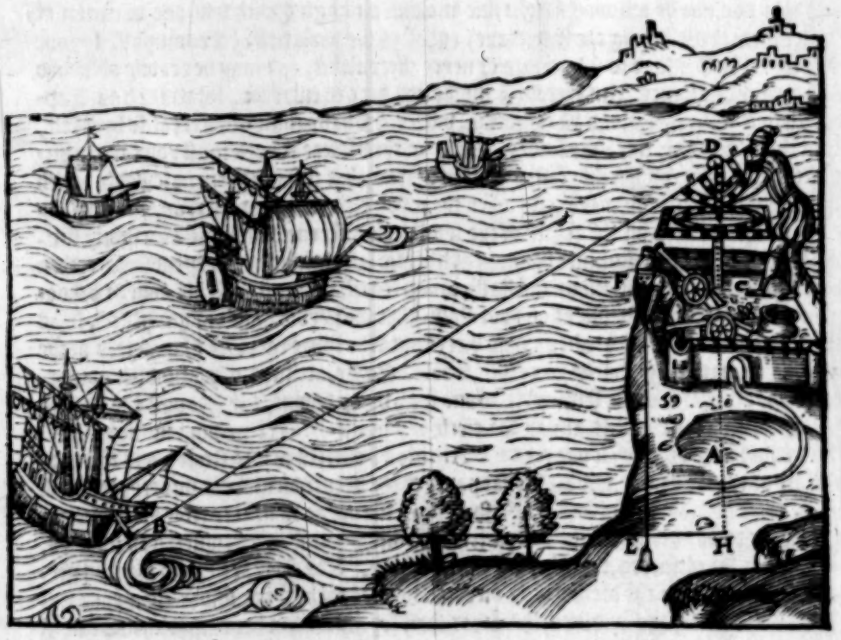
The 30 Chapter

By this instrument to knowe how many miles or pace any Shippe is distante from you, your selfe standing vpon an highe cliffe or platforme by the sea coast.

YOur Topographicall instrument equedistantly situate to the Horizon (as was before declared) turne the diameter of the semicircle towards the ship, and when you haue espyed through the sights the wale or lowest parte of the hull nexte the water, note exactly what part of the Scale is touched with the line perpendiculare, then measure the height of the cliffe, or rather the centre of your semicircles altitude aboue the sea, & multiply the same in the hole side of your scale, diuiding by the partes touched of the perpendiculare line in your Scale, the quotient is the distance of the ship from the [illegible] of the cliffe lyinge perpendicularlye vnder the centre of your instrument, but if you multiplie the partes of the perpendiculare intercepted with the Scale in the cliffes altitude before measured, and diuide by the partes of the scale cutte, the quotient will

will ſhew the lyne Hypothenuſa, or diſtance of that parte of the Shippe which your line viſuall touched from your eye, or adioyning the Square of the longitude firſt found to the Square of the altitude, the roote quadrat of the product is alſo the true length of the line viſuall.

Example.



Admit I ſtand on the cliffe A, and ſee the ſhip B lying at rode in the ſea, I deſire to know howe farre of ſhe is from me, mine inſtrument conueniently placed at C (as is tofore declared) I turne my dimetient of my ſemicircle towards the ſhip, mouing it vp and downe till I eſpie through the ſightes of the wale of the hull next the ſea, and therewithall I finde my perpendiculare cutting the thirde parte of my Scale, then meaſure I the altitude of the cliffe aboue the ſea E F, finding it 59 paſe, whereunto I adde one for the altitude of my inſtrument D C, ſo haue I 60 paſe, the heigth of the ſemicircle aboue the ſea, which multiplyed in 120 the ſide of the Scale, there amounteth 7200, which diuided by 3 yeeldeth in the Quotient 2400 paſe, that is two milet, 3 furlongs, 25 paſes, the longitude H B. Now ſquare 2400 paſe, ſo haue you 5760000, whereunto if you adioyne the ſquare of H D the product will amount to 5763600, whoſe quadrat roote is the longitude of your line viſuall, repreſented by D B, beeing not fully 4 foote more than H B.

THis concluſion ſerueth moſt commodiouſlie for all ſuch as ſhall haue committed to their charge any platforme with ordinaunce, for hereby you may exactly at the firſt viewe, tell the diſtance of any ſhip or marke, ſo that hauing a table of Randons made, mounting your peeces accordingly, no veſſell can paſſe by your platfourme (though it be without pointe blancke) but you may with your ordinance at the firſt bouge hir and neuer beſtowe vaine ſhotte. This inſtrument ſerueth alſo no leſſe aptly to bee ſituate on the Rampire or Mounte within any towne of warre, whence you deſire with ſhotte to beate the enemie alooſe of, before he ſhall approche nigher. But to giue full inſtruction for ſhooting great ordinance, and for

for the variation of Randons, hauing respect to the length of the peece waighte of the Bullet, force of powder, proportion of the Concaue *Cylinders*, and Distance of the marke, it would require a long discourse: I haue thereof in a booke by it selfe very largely, and I dare be bould to saye, sufficiently entreated: for conferring and conioyning Geometricall demonstration with my long continued painefull practises, I haue at the last reduced that most irregulare course & circute of the Bullet (framed and compounded of violent and naturall motions) within the boundes of numbers and Arithmeticall rules, which whether it be difficulte or no, they onely knowe that haue or shall attempt to doe the like. This surely I will say, that as the ignorant in Geometricall and Arithmeticall proportions, shall neuer attaine perfection, though hee turmoile in powder and shot all the dayes of his life: so the Geometer, how excellent so euer he be, leaning onely to discourse of reason, without practise (yea and that sundrie wayes made) shall fall into manifolde errors, or inextricable Labirinthes. Among many that I haue read concerning that matter, I note one Nicholas Tartalia an Italian, who surely for his singular inuention and perfect knowledge in Geometricall demonstrations, few or none in our time or many ages before may bee compared with him: and yet handling this Argument, he hath erred euen in the principall, and as I might terme them the veriest trifles: I meane touching the vttermost Rando and circute of the Bullet, which he affirmeth to be made of a Circular and right line: others haue supposed it to be compact of many right lines, making seuerall Angles proportionally to the seuerall mountes of the peece, or hollow *Cylinders*, and many like opinions and manifest errours haue diuerse well seene in Geometrie for wante of experience admitted and maintained. But of these in due place. Now to this purpose.

The 31 Chapter

To know how much higher or lower any marke is than the leuell from your eye, although there be such impediments betweene, that you can neither approche nighe vnto it, nor see the base.

HEre shal you vse great diligence in the placing of this instrument, that it be situate precisely equedistant to the Horizon, which done you shall turue the Diameter of your mooueable Semicircle to and fro, till you can through the sightes espie the marke, noting therewithall the pointes or partes cut in the scale and perpendiculare, then measure the distance of the same marke from your eye, as you were taught before, which multiplied in the partes of the scale, if you diuide by the partes of the Perpendiculare, the Quotient is the difference howe much the marke is higher or lower than your eye. Also if you adde the Square of the partes cutte in your Scale, to 14400 reseruing the product for a Diuisor, and multiplie the Square of the distance Hypothenusall in the Square of the partes cutte in the Scale, diuiding the offcome by the reserued Diuisor, the roote quadrates of the Quotient is the foresaid difference or vnequalitie of leuelles. And this may yee also finde howe much any one marke is higher or lower than an other, although they be farre a sunder, and either of them remote from you, by comparing their altitudes found in this sorte together.

Example.

Admit B *my ſtation where I place mine Inſtrument* A *the marke whoſe altitude I deſire aboue the leuell of mine eye, though I may not by reaſon of ſundry impediments approch nigh vnto it, nor ſee the baſe. Firſt I meaſure the Hypothenuſall line* A B, *by the precepts tofore giuen, which I ſuppoſe* 500 *pace, then perceiuing through the ſightes of my Semicircle the marke* A, *I finde* 10 *partes in my Scale of Altitudes, intercepted with the Perpendicular line, the ſquare thereof ioyned to* 14400 *produceth* 14500 *my Diuiſoure. Then doe I multiplye the Square of* 500 *in* 100, *the Square of the partes cut in the Scale, ſo haue I* 25000000, *which diuided by* 24500, *the Diuiſor before reſerued, your Quotients Quadrate roote will bee very nigh* 41 *pace* 31 *Inches, and this is the true difference or vnequalitie of leuell betweene the marke* A, *and the Centre of your Semicircle* B, *ſo that if a Well be ſoonke of ſuch deapth that the bottome thereof were lower than* A 41 *pace* 31 *Inches, as I admit the line* A C, *then may you certainly affirme, that* C *the bottome of that Well is leuell with* B, *and yet may you not thereby, inferre that from a Fountaine head, lying of equall heigth with* B, *you may naturally derine water to* C, *for the leuell of waters is circulare, as I haue before in the Booke declared: And heere I thinke it not amiſſe to giue you a precept how to finde the diuerſitie of theſe leuelles, whereby yee may exactly reſolue ſundrie queſtions pertaining to Water woorkes, wherein diuers haue greatly erred, obſerning not this difference.*

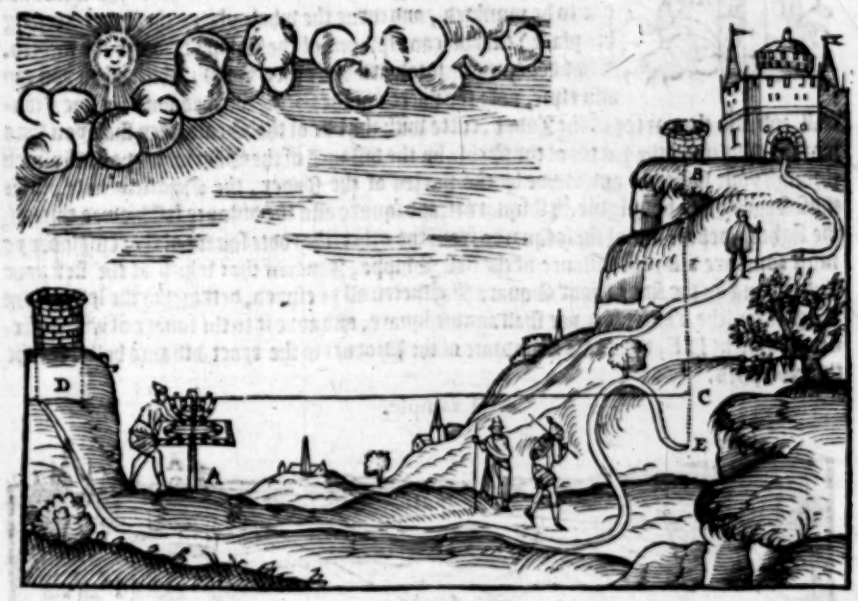
The 32 Chapter.

To finde the difference betweene the ſtreight and circular or true water leuell from a Fountaine at any place appointed.

This difference is nothing elſe but the length of a Pendiculare line falling from the leuell right line of a Fountaine to the water leuell of the ſame, as in the figure following A is your Station, D the Fountaine whence I would conuey water to the Foote B ſtanding on a hill, C a poynte by imagination conceyued directely vnder B within the earth in a leuell right line from the Fountaine D, or rather the Superficies of the Water therein

conteyned. E the water leuell, that is to saye, the highest pointe that any water will naturally runne at, being conueyed by pipe from D. The line C E is this difference of leuels which you shall thus attaine. First it behoueth you to get the Distance of the Fountaine from the place whither you would conueye your water, which Distance you shall multiply by it selfe, adding the offcome to the Square of the earths Semidiameter, and out of the product extract the roote Quadrate, from which roote if you withdrawe the foresaid Semidiameter, the remainder is your desired differences of line C E.

Example.



Admit the Distance D C 10 *Myles, the Semidiameter of the earth,* 5011 *Italyan Myles, euery Myle conteyning* 1000 *pace Geometricall, the pace being* 5 *foote, the Square of this Semidiameter is this number of pace* 25110121000000. *Likewise* 10 *Myles the Distance squared yeeldeth* 100000000 *paces, this added to the Square of the earths Semidiameter produceth* 25110221000000 *paces: Nowe if from the roote Quadrate thereof ye subtract the Semidiameter, there will remaine* 9 *pace,* 4 *foote and* 11 *Inches: so much you may assuredlye say, that the water leuell E is vnder the other leuell at C. Nowe if you would know standing at A by the Fountaine not approching nighe the Castle howe deepe it were requisite to sinke a Well, there to receiue this water you may thus doe, first measure the line B C, that is to saye, howe high the ground platte of the Castell is above the leuell right Line of the Fountaine D, for this you are taughte howe to doo before, then searche out the difference betweene the straight and water leuell of the same Fountaine by the rule giuen in the last Chapter, these two ioyned together, doo produce the profunditie B E, that is howe many pace, foote and inches you shall sinke a Well at the Castell, to receiue Water from that Fountaine. Herein there neede no Example, the premisses well vnderstand, this conclusion is manifest.*

The 33 Chapter.

You may alſo by this Inſtrument meete the diſtance betweene Shippes on the Sea, or other markes on the lande, howſoeuer they be ſituate, with the aid of Arithmeticall ſupputation.

FIrſt meaſure the diſtance of either Shippe from your ſtation, howſoeuer it be, which you may do by ſundrie meanes before declared then taking of the Perpendicular and Semicircle, and firing the Index of the Square Geometricall vpon his due Angle firſt place it on the ſide of the Square, whence the diuiſions of the Scale begin to be numbred, and turne the whole plate (the Index keeping his place) till you can eſpie one of the Ships through the ſightes. That done, moue it toward the other Ship, which when you can alſo eſpie, note the partes of the Scale touched with the line Fiduciall, and alſo the partes of the Index, cutte with the ſide of the Scale, then ſhall you thus worke. Multiplie the partes of the Scale by the diſtance of the Ship which you laſt eſpied through your ſightes, and diuide by the partes of the Index, the Quotient note, for it muſt ſerue you to double vſe, firſt ſquare it, and ſquare alſo the diſtance laſt viewed through the ſightes, deduct one of theſe ſquares from the other, the roote ſquare of the remainder ye muſt compare with the diſtance of the firſt Shippe, I meane that which at the firſt your Index lying on the ſide of your Square Geometricall ye eſpyed, detracting the leſſer from the greater, the remainder yee ſhall againe ſquare, and adde it to the ſquare of your reſerued Quotient D E, the roote Quadrate of the Product is the exact diſtance betweene the two Shippes.

Example.

I ſtand in the Caſtle A, the two Shippes, whoſe diſtance I require B and C, A C meaſured as is tofore declared 2000 *pace, A B* 2500, *the partes of the Scale touched with the Fiduciall line of the Index* 50, *the partes of the Index cut with the Scale* 130, *B the Ship firſt viewed, while the Index reſted on the ſide of the Square Geometricall, C the Ship laſt eſpied: when the Scale was cut with the line Fiduciall of the Index, I multiplie therefore according to the Rule aboue giuen,* 50 *in* 2000, *ſo haue yee* 100000, *which diuided by* 130 *yeeldeth* 769 $\frac{1}{13}$ *whoſe ſquare detracted from the Square of* 2000, *leueth* 3408284 $\frac{4}{169}$, *whoſe Quadrate roote is* 1846 $\frac{6}{13}$ *which deducted from* 2500 *the diſtance of the latter Shippe from the Caſtle, there remaineth*

653

653 $\frac{11}{17}$, whose square added to the square of 765 $\frac{3}{17}$, the Quotient before reserued, produceth a number, whose roote is neere vnto 1009 $\frac{1}{2}$, these two Shippes therefore ye may conclude 1009 $\frac{1}{2}$ pace a sunder.

The 34 Chapter

To draw a platte of any coast or country, containing the true proportion and Symetrie thereof, in such sorte that you may readily tell how farre any place is distant from other, and that without Arithmetike.

YOu shall ascende on some high Tower, Hill, Cliffe, or other place, from whence you may commodiously beholde on euery parte the whole Countrie rounde about adiacent in your Horrizon, there set vp your Instrument Topographicall on his staffe, and in such sorte place it by aide of the needle, that the foure Semidiameters may lie East, West, North and Southe, euery one answering his like quarter of the heauen, then turne the Diameter of your Semicircle, to euerye Towne, Village, Hauen, Rode, or such like, espying through the sightes, the middle or moste notable marke in euerye of them, noting therewithall in some Table by it selfe, the Degrees cut by the Alhidada in the Circle, which I call the Angles of Position, and so make you a Table of your first station. Then searche out your eye, viewing round about, some other loftie place, from whence you may behold againe all these places, for that shall be your second station: and turning thereunto the Diameter of your Semicircle, note also what parts of the Circle is touched with the line Fiduciall of the Alhidada: This done, situate your Topographicall Instrument, in all respects as was before said, and turning the Diameter of your Semicircle, espying through the sightes, all such markes as you sawe before, note againe the Degrees cut, or Angles of position, writing the name of euery place, and his Degrees by it, so haue you an other table of your second station: with these tables you shall resorte to some plaine smoothe Superficies of borde, parchement, paper, or such like, and thereon describe a large Circle, dividing it as you were before taught, into 360 partes, like to the Circle in your Instrument. Then from the Centre thereof to euery degree noted in your first Table extend straight lines, writing vpon euery of them the name of his place, and vpon that line that representeth your second station fixe the one foote of your compasse, opening the other at pleasure, drawe an other large Circle, dividing it into 360 Grades, and from the Centre thereof, extend right lines to euery Degree noted in your second Table, writing as before vpon euery of them the names of their places or markes, wherfore they are the sight Angles. Finally you shall note diligently the concourse or crossing of euery two like lines, making thereon a Star or such like marke, with the name of the place correspondent. Now if you desire to know how far euery of these Townes, Villages, &c. are distant from other, you shall thus doe, measure the Distance betweene your stations by instrument or otherwise, as you haue bin before taught, and diuide the right line between the Centres of your Circles into so many equall portions, as there are Miles, Furlongs, or Scores betweene your stations, then opening your compasse to one of those partes, you may measure from place to place, alway affirming so many Miles, Furlongs or Scores (according to the Denomination of that one parte whereunto you open your Compasse) to be betweene place and place, as you find by measuring there are of those parts. Some consideration you must haue in placing the Centre of your second Circle, so conueniently distant from the other, that the concourse or meeting of semblable positionall lines, may bee within the Compasse of your paper, &c.

Example.

There is a Sea coast hauing sundry Harboroughes, Townes, Villages, Castles, and such like situate thereon, whose Platte in due proportion I require, with the exact distance of euery place from

from other, hauing therefore elected a loftie ſeate, from whence I may behold all theſe places, (mine inſtrument ſituate as is declared) remouing the Index to the fardeſt being a Caſtell ſtanding in the mouthe of an Hauen, hauing receiued it through my ſights, the fiduciall line of mine Index, cutteth 30 Grades, thence remouing it to the next, being a Village or Fiſher towne, the Index cutteth 50 degrees, againe at the next being a Cittie, it cutteth 75 grades at the fourth being a great Baye I note both the entrances, at the Eſterne ſide of the Baye it cutteth 95 Degrees, at the Weſterne 100 Degrees, finally at the fardeſt place Weſtward, being a Forte within the land, the Index dooth cut 130 grades. Thus haue I the Table of my firſt ſtation, as followeth.

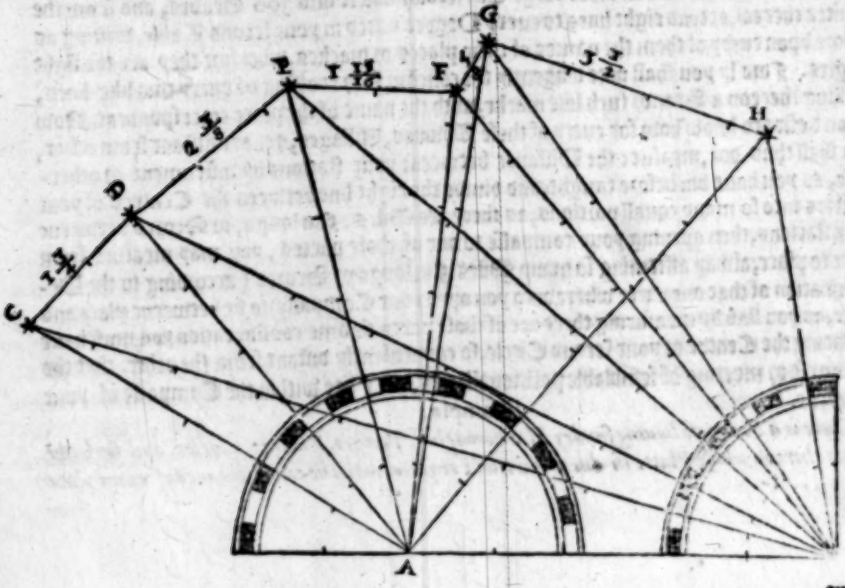
The table of my firſt Station.

The Caſtle	30 Degrees
The Village,	50 Degrees
The Cittie,	75 Degrees
The Eſterne head of the Baye,	95 Grades
The Weſterne head of the Baye,	97 $\frac{1}{2}$ Grades
The Forte within the lande.	130 Grades

This done, I behold an other highe Hill, from whence I may in like manner view all theſe places, & turning my Index thereunto, I finde the line Fiduciall lying vpon 180 degrees. Then carying my inſtrument thither, and placing it in all points there, as it was at the firſt ſtation, I turne my Index againe to the firſt marke or Caſtle, finding it to cut 15 grades, at the ſecond 25 degrees, at the third 40 grades, & ſo to the reſt as appeareth in this Table following. And as I haue done of theſe few, ſo might I how many noble markes ſo euer there were, writing their names ſeuerally with their Degrees noted, as enſueth.

The table of my ſecond ſtation.

The Caſtle,	15 Degrees
The Village,	25 Degrees
The Cittie,	40 Degrees
The Eſterne head of the Baye,	50 Grades
The Weſterne head of the Baye,	55 Grades
The Forte within the lande.	80 Grades



With

Example.

With these Tables repaire to a plaine smooth Superficies, drawing therein a faire Circle as you see diuided in 360 partes, and from his Centre A, I pull out right lines to euery Grade noted in my first table A C the line of the Castle, A D of the Village, A E of the Cittie, and so fourth of the rest, ending with the line A B, cutting 180 Degrees in my Circle, then making B a Centre I describe a Circle diuided as the other at A, and from his Centre pull out streight lines to all those Grades that were noted in my second Table: now marking the concourse of semblable lines, that is to saye, where the line of the Castle issuing from A meeteth with the line of the Castle proceeding from B, I note it with a starre as you may beholde at the letter C, and thus prosecuting the like in the rest, alwayes setting a Starre or like marke vpon the concourse of correspondent right lines) all other intersections not regarded) I haue finally situated all these places in due proportion, noting them with these letters C D E F G H: as in the figure you may see.

Now to learne the distance betweene euery of them, I diuide the space betweene A B into 5 partes, for so many Myles by mensuration I finde betweene my two stations, then drawing right lines from C to D, from D to E, and so from euery one to other, opening my compasse to one of these partes, I measure howe many times it is conteined in euery of these right lines: whereby I conclude the number of Myles betweene euery of them, as appeareth in the table folowing.

The Table.

The distance of euery marke from the stations.	The distance of euery marke from other.
A C 5 *Myles*, 1 *Furlong*.	C D 1 *Myle* 6 *Furlongs*.
A D 5 *Myles*, $\frac{1}{2}$ *Furlongs*.	C E 4 *Myles* $\frac{1}{2}$ *Furlongs*.
A F 5 *Myles*, 3 *Furlongs*.	C F 5 *Myles* 5 *Furlongs*.
A G 6 *Myles*, 1 *Furlong*.	C G 6 *Myles* 1 $\frac{1}{2}$ *Furlongs*.
A H 6 *Myles*, 3 *Furlongs*.	C H 9 *Myles*.
B C 9 *Myles*, 6 *Furlongs*.	D E 2 *Myles* 3 *Furlongs*.
B D 9 *Myles*, 1 *Furlong*.	D F 4 *Myles* 1 *Furlong*.
B E 8 *Myles*, 3 *Furlongs*.	D G 4 *Myles* 4 *Furlongs*.
B F 7 *Myles*.	D H 7 *Myles* 4 *Furlongs*.
B G 7 *Myles*, 2 *Furlongs*.	E F 1 *Myle* 7 $\frac{1}{2}$ *Furlongs*.
B H 5 *Myles*.	E G 2 *Myles* 2 $\frac{1}{2}$ *Furlongs*.
	E H 5 *Myles* 5 *Furlongs*.
	F G 5 *Furlongs*.
	F H 3 *Myles* 6 *Furlongs*.
	G H 3 *Myles* 4 *Furlongs*.

Thus passing or chaunging your Stations, you may make seuerall plattes, conteyning the true proportion and distances of Townes, Villages, Partes, Rodes, Hilles, Riuers, and all other notable places through an whole Realme, but then how to reduce them all into one faire Carde or Mappe you shall be taught hereafter.

The

The 35 Chapter.

How to reduce many Plattes in one, and to make a faire Carde or Mappe of an whole prouince or region, and that in as large or ſmall roome as you will aſſigne, without anie Arithmeticall calculation.

I Minde not heere to ſet foorth the manner howe to ſituate places in their due Longitude and Latitude, neither howe to furniſhe your Mappe with Meridians, Paralleles, Zones, Climates, and other Circles correſpondent to the heauenly Sphere, for that apperteineth to Coſmographie, onely in this place ſhall you learne Chorographically to make a Carde, whereby you may readily know the true Diſtance and ſituation of places one from an other, hauing therefore as is before declared (by ſundry Plattes made in euery ſeuerall Horizon or Kenning) found out the true diſtance of euery notable place from other, you ſhall make one entire table of all, conteining the number of Miles, Furlongs, &c. betweene place and place, beginning at the Eaſt, and ſo proceeding on till you come to the fardeſt Weſtwarde. This Table thus finiſhed, you ſhall vpon your Parchement, Paper, or other plaine whatſoeuer it be, fitte for this purpoſe, drawe one ſtreight line directly through the middle thereof, croſſing the ſame with an other perpendicularly, againe as you were taught at the beginning of this Booke, then write at the endes of theſe lines the foure principall quarters, Eaſt, Weſt, North and Southe. Nowe it behoueth you to conſider (conferring your plattes together) how farre diſtance the moſt Eaſterne place is from the Weſterne, and likewiſe the Northerne from the fartheſt Southwarde, that you may accordingly ſo proportion your Mile, as all theſe places fall within the compaſſe of your Carde: Nowe by the one ſide of your Superficies, drawe a ſtreight line of competent length, then opening your compaſſe to the wideneſſe of that meaſure which yee call your Mile, marke out 20 of them in your laſt drawne ſtraight line, which you may garniſh with other Paralleis, diuiding euery Mile into his Furlongs, this ſhall be called your Scale. Nowe muſt you againe geſſe (as neere as you can by comparing your plats) which is the middlemoſt place of this countrey that you deſcribe, and the ſame ſhall you ſituate vpon the interſection of the two former drawne lines, making there a ſtarre, and writing the name thereof, whether it be Cittie, Village, Caſtle, or ſuch like, then ſearch out in the ſame platte what other notable place or marke lyeth Eaſt, Weaſt, Northe, or Southe from it. And if you eſpie anye reſorte to your Table prepared as I haue before ſaid, ſearching out the Diſtance betweene theſe places, and to ſo many Miles or Furlongs extend your compaſſe in the Scale laſt made, then keeping your compaſſe immoueable, ſet one foote thereof vpon the interſection or middle place, extending the other to that quarter wherein you founde the place ſituate, make an interſection with the ſtreight line, and there likewiſe make a ſtarre or other fine marke, writing the name of the place: But if you finde no notable marke lying preciſely Eaſt, Weaſt, North, or Southe, from that aforeſaid middle place, then take ſome other what you liſt in your platte, and note bothe the diſtance and alſo the Angle that it maketh, with the middlemoſt alreadye marked in your Carde, that is to ſay (if it lie not in one of theſe principall quarters from it) how many degrees it declineth, and deſcribing vpon the interſection a Circle diuided into Grades, pull out from the Center a right line to the like number of degrees that you founde in your platte: finallye opening

pour

your compaſſe to the like number of Miles in your Scale that you perceiue in the Table to be betweene thoſe places, fixing one foote in the interſection with the other, cutte this laſte drawne line, and at that ſection make a marke, writing ther by his name, thus haue you two places. Now for all the reſt one rule ſhall ſuffice, whatſoeuer it be that you will next marke, ſearch out in your table his diſtance from both theſe already noted, and opening your compaſſe to the like number of Miles in your Scale, placing the one foote in either of the places already deſcribed, make an Arke with the other, and where thoſe arkes croſſe there is the ſituation of the third place. And thus may you proceed to all the reſt, onely taking heede that hauing opened your compaſſe to the number of Miles you fixe the foote in his correſpondent place, and ſo ſhall you (paſſing on from one to another) exactly ſituate them all, as by the example following ſhall more plainly appeare.

Example.

B D C E the Parchment, paper or other plaine, whereon I would deſcribe the country whereof I haue alreadie taken the plat or plattes, as was before taught, B C and D E the two croſſe diameters, making interſection at A, I the line which I terme a mile, H G the Scale conteyning 20 of them, this mile if I liſt to make a large carde ſhould be diuided into Furlongs, yea and euery Furlong into partes, and therein you muſt vſe conſideration, appointing the length of your Mile according as you deſire to haue your charte great or ſmall. Now it behoueth you to reſort to your plattes, ſearching out as nigh as you can by eſtimation the middlemoſt place. And that in my laſt platte (which heere I minde to vſe for Example ſake) was my firſt ſtation, heere placed at A, then finde I directly Weſt from thence 5 Myles my ſecond ſtation. Extending therefore

my Compaſſe to ſo many in the Scale, and ſetting the one foote in A, with the other, I make a ſection in A C at K, and that is my ſecond ſtation. Now will I place the Caſtle which I finde in my platte or Table made by it, diſtant from my firſt ſtation 5 ½ Myles, from the ſecond 9 ¼ Myles. Firſt therefore I extend my Compaſſe in the Scale to 5 ½ Myles, and fixing the one foote in A, with the other I drawe an Arke, then opening my compaſſe to 9 ¼ partes in the Scale, and ſetting one foote in K, I croſſe with the other the Arcke tofore drawne in L, in like manner opening my compaſſe againe to 5 ½ in the Scale, which is the number of Myles betweene the Village, and my firſt ſtation. The one foote in A, I drawe an Arke, then extending the Compaſſe againe to 2 Myles lacking ¼ parte, the diſtance betweene the Caſtle and Village, I croſſe the foreſaid Arcke in M, there is the Village. In this manner you ſhall proceed to the reſt, making fine prickes or other little markes for the true places of them, then bleniſhing all the arkes, Circles, and other lines, which I ſuppoſe drawne with blacke lead or ſuch like, that you may eaſilye put out or raſe awaye, you ſhall finallye beautifie it with Images and Figures, as you thinke moſt agreeable and fit to expreſſe and repreſent the patterne, I meane the Countrey it ſelfe that you deſcribe, as by drawing a Caſtle at L, a Village at M, a Citie at N, a Harborough at O P, and ſo of the reſt. Euen in like manner may you deſcribe any Towne, Forte, Campe, or Pallace, and ſet out the true ground Platte of all manner Edifices, altering your Scale, and in ſtead of Myles, dividing it into Scores, Pace, foote, and ſuch like ſmall meaſures.

A note for Sea Cardes.

YOu maye alſo if you will diuide the Circle at the middes of your Map into 32 partes, pulling out ſtraighte lines from the Centre to the vttermoſt bounds of the Charte, repreſenting the 32 windes, and vpon the ſame Centre deſcribe an other Circle ſo large as your Map can conteine, the Circumference thereof will be parted into 32 equall portions with the foreſaid lines repreſenting the windes. Now, if vpon euery of thoſe interſections as a Centre, you deſcribe a Circle diuiding euery of their circumferences into 32 winds, extending from their Centres ſtreight lines throughout the whole Map as before you ſhall make a Sea Carde, wherin you may by the former rules, place Coaſts, Harboroughes, Rockes, Sandes, Shelues, Chanels, Rodes, with their ſoundings and depthe of Ancorage, &c. But of theſe matters here I meane not particularly to diſcourſe, referring the more ample declaration of them to an other treatiſe of Colmographie and Nauigation, wherein I ſhall not only diſcouer the manifold errors that Mariners fall into by vſing their commune Chartes and Rules, but alſo ſet fooorth true exact and eaſie precepts for them, with ſundrie rare concluſions hither to not practiſed of any. Whereby alſo the ſkilfull and more learned ſorte of Marriners ſhall vnderſtande howe to make ſuch kinde of obſeruations of the variation of their Compaſſe in their Indian Nauigations, eſpecially in Circulating and enuironing the Earthe, as may reduce that moſte ſtrange and irregulare alteration of the Nauticall Compaſſe to a Theorike certaine, and thereby as readily in all Altitudes to finde out the true Longitude, as they haue meanes already in all Meridians, to finde out the Parrallele or Latitude, the want whereof hither to is the onely maime of that Arte and Science, which in our age hath diſcouered greater matters then in a thouſand yeares before.

The

The 36 Chapter.

Concerning Mines made vnder the Earthe.

OF Mines there are diuerſe ſortes made vnder the Earthe, ſome for Diſcouerie of Mineralles and Mettalles, others to paſſe ſecreatlye from place to place, or to ſurpriſe Townes or Caſtles of force, or to make a Fornace to plante Barrels of Powder vnder any Curtaine or Bulwarke to blowe it vp. In all theſe kinde of Mines it is a matter of cheefeſt importance, to know alwayes vnder what place directlye the Caue is brought, and how to guide it to come and approache directlye vnder, or to the place yee affect, I will therefore deuide this Chapter into three Problemes, adding to euerie one his practicall reſolution.

The firſt Probleme.

How to guide any direct Mine leuell toward any place in ſighte ſituate on a Hill or Mountaine And to knowe when yee are come directly vnder that place, and how much ye are vnder the ſame.

The ſecond Probleme.

Howe to guide any Mine enclining vpwarde or downeward directly vpon any point yee ſhall be aſſigned viſible at the firſt ſtation, and to knowe at all times how much yee are vnder the leuell thereof.

The third Probleme.

Howſoeuer yee ſhall bee inforced by Rockes, Waters, or other Obſtacles, to carrie the Mine either vpward, downewarde, or laterally declining, to knowe ſtill the certaine ſituation how farre yee are diſtant from the firſt enterie of the Mine, and likewiſe the diſtaunce from the place yee deſire to approche.

FOr reſolution of all theſe three Queſtions, ye muſt firſt by the 23. 25, and 31 Chapters, finde out the true diſtance of the place whither you meane to carrie the Mine. I meane the diſtance aſwell Hypothenuſall as Fundamentall, and alſo the heigth of the ſame place aboue the Fundamentall or leuill line. That doone you are alſo by the helpe of the Needle or Compaſſe, placed in the Topographicall Inſtrument, to finde out the true Angle of Poſition, Viz. how many Degrees, from the Eaſt, Weaſt, or other principall quarters of the Heauens, the iourney lyeth. This doone, yee are by the ſame Inſtrument ſtill to guide the Mine directly vpon that line, and then by the

Semicircle ſo ſcituate as the Perpendicular cut no grade or part of the Quadrantes by the ſightes obſcruing alſo that it be carried leuel, you haue no more to do but exactly to meaſure the length of the mine, and when you find it ſo many paces long as by inſtrument you found the Diſtance Fundamentall, yee are aſſured that ye are come directly vnder the place propoſed. But the beſt meanes to direct theſe obſcure mines made in the bowels of the Earth is to haue a ſtaffe of the ſame height with the ſtaffe of the Topographicall inſtrument, and vppon it a Percher or other finer candell of waxe, that may be as high as the Perpendicular of the Inſtrument, ſo as the light of the candel may alwaies bee as high from the flower of the mine as the center or ſightes of the Semicircle for Altitudes is, and then ſetting vp the ſtaffe, and light at the one end of the mine, and the Inſtrument placed as hath bene taught at the other, the ſightes will direct either directly to the ſaid light, and then is the mine perfectly leuell, or certain Feet or Inches ouer or vnder, which by an other like or leſſe light aduaunced or abaſed ouer or vnder, the other yee ſhall readily finde: And ſo haue you the trew meaſure in feet or inches, that the end of the Mine is higher or lower then the entrie or trew leuell line Fundamentall, which deducted from the Altitude of the marke firſt obſerued, if the Mine did aſcende, or adding the ſame to the firſt Altitude if the Mine did deſcende, you haue exactly how many foote and inches, the Mine is vnder the firſt aſſigned marke. But conſidering in theſe obſcure voyages which are made through Mines wrought vnder ground, the cheefe direction growes from the Needle or Compaſſe, I thinke it fit to adioine heere a breefe note of the *Magnes Stone*, whereby that vertue is imparted, and what errors may growe by diuers accidents, if ye ſhould not be forewarned of them.

A note concerning the vertue of the Magnes ſtone, his variation naturall and errours accidentall.

The Magnes ſtone hath ſo rare a property impreſſed through the influẽce of the heauens, as by onely touching the point of the Needle, hanged on his perfect Centre of Counterpeize, it deliuereth a vertue which preſently turneth the Needle to the Meridian, but not ſo exactly, but that in moſt places of the world, it hath a certaine variation or difference, the which although in ſeuerall Climats and Meridians it alter and vary, yet in any one place the variation is ſo certaine as it neuer chaungeth: and therfore once truely obſerued or corrected ſerueth perpetually for that Region.

I meane not heer to diſcourſe of the cauſes of this variation, whether it be by reaſon of any Mine or Rockes ſcituate in any parte of the Earth, which by their attractiue power worke that effect, or whether any ſuch attractiue point or points be in the heauens, or whether the direction of the Needle be alwayes Parallelle, and the variation growe by the alteration of our Zenithe or Horizon, but ſeeing it is moſt certaine euer and immutable in any one Region, it ſufficeth for our preſent purpoſe to knowe that with vs in England the variation is one pointe of the Compaſſe, that is 11 $\frac{1}{2}$ Grades, the which is the true Angle of Variation or Deuiation, whereupon your Needle muſt be ſcituate, and being ſo corrected yee can commit no error, by reaſon of that naturall Parallaxe or Diuerſitie, but there are other cauſes accidentall of errour, as if by hap ye ſhall encounter in the Mining, with any vre of Iron, or in the Caues any Pikaxes or other inſtruments of Iron happen to lye nigh vnto your Compaſſe, that alſo may diuert your needle from his true ſcituation, and cauſe great error in the Angles, and ſo conſequently in the whole account, but this ye ſhall eaſily finde, if you worke with two Inſtruments, the one hauing a needle or Compaſſe, and the other none but directed onely by the ſtations themſelues and their Angles of Deuiation, for if both concur in one, ye are aſſured ye haue done right, if otherwiſe, yee muſt the more carefully *ab Origine* examine your Angles, whereof I thought it fitte the more at large to admoniſh you, becauſe in theſe matters Militarie of ſo great importance, the inguiding of the Mine, may in ſteed of deſtroying your enemies ſpoile great numbers of your owne people, waſt your Boulder, and dishonor and foile the enterpriſe.

The ſecond Probleme.

How to guide any Mine enclining vpward or downward directly vpon any point yee ſhall be aſsigned viſible at the firſt ſtation, and to knowe at all times how much you are vnder or ouer the leuell thereof, or of the firſt enterye.

IN this caſe yee are to proceede in all reſpects as in the former, ſauing that at euery ſtation where the Myne is to aſcend or deſcend, yee muſt obſerue the difference of the aſcent or deſcent how much it is higher or lower then the line Fundamentall by the Arte before taught, keeping a table of all the Aſcentes by it ſelfe, and likewiſe of the Deſcentes. Then at what place ſoeuer yee would know the ſituation, and all the precedent Aſcentes, and likewiſe the Deſcents, deducting the leſſe out of the greater, the remaine diſcouereth the diſtance from the leuell of the entrie vpwardes, if the Aſcentes were greater or vnder, if the Diſcentes did exceede, and the ſame deducted from the Altitude of the propoſed marke firſt diſcouered, if the Scituation be lower, or added if it be higher then the firſt entery, diſcloſeth exactly how many foot or inches yee are vnder the place propoſed: neither need you care which way your Myne is carried, whether it be directly vppon the propoſed place, or Collaterally declining backward or forward, for this nothing altereth the aſcent or deſcent, but the angle of Aſcent or Deſcent is onely varied by the length or ſhortnes of the line of direction, and therefore as yee note the ſeuerall Aſcentes and Deſcentes vpon euery alteration of the angles of Poſition or Direction, ſo are you at the ſame time alſo to meaſure exactly the trewe length of euery of thoſe lines of Poſition, noting euery Longitude with his correſponding Altitude: And it ſhall not bee amiſſe alſo at euery chaunge to marke the degrees of Aſcent or Deſcent cut by the Perpendicular of the Inſtrument in his ſemicircle, and ſo at euery ſtation and alteration to ſet downe theſe three, the longitude, the Altitude, or Profundity, and the Angle or grade cut by the Perpendicular. And your Table thus framed, yee are prepared to plat down the tourneies by the Art which ſhall now enſue.

The third Probleme.

Howſoeuer yee ſhall bee enforced by Rockes, Waters, or other impediments to conuey the Myne either vpwarde, downewarde or collaterally declining to finde out ſtill the certaine ſituation, and how farre yee are vnder or ouer the leuell Fundamentall and how farre diſtant from the entrey or place propoſed.

FOr all Aſcentes or Deſcentes of any Myne, ye are already ſufficiently inſtructed in theſe two former Partitions or Problemes, to knowe at all times and in all places of the Myne howe much yee are ouer or vnder the line or leuell Fundamentall, &c. But if by any Obſtacles that yee ſhall vnder the Earth reincounter, yee bee inforced to carrie the Myne ſidewiſe from the direct line of poſition, you muſt note the Angle of the ſaid Deuiation from that line. That is to ſay how many Degrees the Myne declineth from it, & accordingly plat it downe vpon ſome faire Paper or Parchment as hath been ſhewed in the 34 Chapter, drawing firſt the line of Direction, and then the line of Deuiation, and in the ſame meaſuring out alſo by the ſcale of paces in the ſame line of Deuiation the length of the Myne ſo farre as it proceedes in a ſtraight line: but if by any new occaſion, yee ſhall againe be inforced to diuert either farther off or neerer, to the firſt line of Direction, Platte it ſtill downe on the Paper or Parchment exactly both in meaſure and Angle, till ye can come to croſſe

the

the firſt line of direction, there ſtaye, and then examine by the Platte how many pace that Interſection or croſſing is diſtante from the pointe of the firſt Entrie, the ſame deducted from the firſt obſerued Diſtance Fundamentall, ſheweth howe farre yee are ſhorte of the deſired place, whereby yee may accordingly againe guide the Mine to come directlye vnder it, And by the Arte taught in the two firſt partes of this Chapter, knowing howe much yee are lower alſo then the place you are appointed, yee may in diſcretion before hande either Aſcende or Deſcend, in ſuch ſorte as yee ſhall arriue as nigh or farre diſtant from it as you liſt. A matter of no ſmall importance, for conueying of Mines, and making of Fornaces, of Powder to teare vp any Bulwarkes, Gates, or Curtaine of any Forte or Towne, and no Inſtrument that euer was inuented for this purpoſe comparable with this Topographicall, the which by his Theodolitus deliuereth exactly all Angles of Deutation Collaterall. And at the ſame inſtant alſo by his Perpendiculare and Semicircle, dooth diſcouer the Angles of Aſcent and Deſcent, and ſo moſt exactly and ſpeedely perfourmeth as much as can be required for that ſeruice. But the more ample vſes and great benefite of this kind of Inſtrument in martiall ſeruice, ſhall more at large hereafter appeare in my Treatizes of Architecture Militarie, for ſetting downe and ſtaking out ſpeedily the Curtaines of Townes, Fortes, and Bulwarkes, their Diamond pointes, Flankers, Shoulders, Eares Counterfortes, Counterſcarpes, Mountes, Caualliers, and all other partes in their due proportions, to bee perfourmed with leaſt waſte and expenſes, and to the greateſt aduantage for annoyance of the Ennemie. Wherein alſo more particularly ſhall be deſcribed, the ſundrie varietie of Mines and Countermines of all ſortes, as well thoſe that are wrought vnder the Rampires of Townes, Fortes, Caſtels, and Bulwarkes, to reſiſt Forren violence, as of thoſe that are vſually made or may bee made by any Armie beſeeging or approaching, to ſurprize them by ſecret Caues, or by Cauernes reinforſed with Barrels of Powder to teare vp any Gates, Flankers, or other principall Members of them, for in all theſe Actions, as there is great varietie and difficultie, ſo is there very great need and vſe of this Inſtrument. But whoſoeuer throughly acquainteth himſelfe with this preſent Treatize, I meane not onely Theorically but by Practize alſo, ſhall bee fullye prepared for thoſe my workes of greater difficultie, and ſhall thereby finde thoſe Myſteries plaine and facile, that otherwiſe would ſeeme moſt obſcure and difficile. Hauing of purpoſe at theſe latter Impreſſions of my Arithmeticall Stratioticos, and this preſent Geometricall Practize, augmented either of them with ſuch Additions, as ſhall prepare a readye waye to the vnderſtanding of all ſuch rare Theorikes and other Concluſions of great Artillerie, as haue not hitherto beene publiſhed in any language.

THere are many other Inuentions alſo and Mechanicall rules taught by Italians and ſome Germans, without any Geometricall Inſtruments to meaſure the Altitude of Towres, the breadth of Ditches and Diſtance of Markes, and in the Field vpon a Drumme head to make Plattes, and ſet downe the proportion of anye Sconces, Fortes, Bulwarkes, or Townes, yee ſhall approache the view of, but they are meanes ſo vncertaine and ſo ſubiect to error vpon any ſmall miſtaking, as I hold them vtterly vnworthie to be coupled with theſe Propoſitions, which are all in the higheſt degree of certeintie confirmed with inuincible demonſtrations Geometricall, onely I meane to giue a breefe Note of one kinde of plaine Inſtrument for the ignorant and ruder ſort not inconuenient, and ſo cloſe vp the firſt Booke of this Geometricall Practize.

A note

A note touching a Platting Inſtrument

for ſuch as are ignorant of Arithmeticall *Calculations.*

SOme take the Semicircle of my Topographicall Inſtrument, ſetting the Perpendiculare thereof vpon a ſtraighte long Rular deuided into a thouſande or more equall partes, and in ſteade of the Horizontall Circle vſe onely a plaine Table or boarde: whereon a large Sheete of Parchment or Paper may be faſtened. And therevpon in a fayre daye to ſtrike out all the Angles of Poſition, euen as they finde them in the Field without making Computation of the Grades and Scruples.

Or if you liſt to deſcribe any Wood, or the Circute and Proportion of any other place, where yee cannot beholde the whole or the greateſt parte at once, ye may with this plaine Inſtrument beginne at anye one poincte where yee liſt. And directing the ſightes of the Semicircle to the nexte pointe or Angle ſo farre as yee may goe in a right line by the Subiacent Rular drawe out the line of Poſition, And meaſuring howe manye Score or Paces, or Perches the ſame is by the aunſwerable Scale, to pricke it out on the paper accordingly, as hath beene ſhewed already at large in the 34. Chapter. In this manner yee may proceede on from Angle to Angle, ſetting downe ſtill as you goe vpon the Paper, the perfect Platte thereof: But of one thing I thinke fitte to admoniſhe you, that you muſt in this kinde of Platting proceede wonderfull warelye, for if you erre neuer ſo little in the ſeuerall Stations, when yee come to cloſe vppe the Platte, you ſhall finde a great and apparant Error. But by practiſe as yee finde the inconuenience thereof, yee ſhall alſo learne by croſſe Angles of Poſition to examine and rectifie thoſe errors, wherem I ſpend the leſſe time becauſe it is nowe ſo vſuall, and the reaſons of the Premiſſes well vnderſtanded will well cleare all doubtes in this rude and plaine kinde of meaſuring. This being an Inſtrument onelye for the ignorante and vnlearned, that haue no knowledge of Noumbers, and not to bee practized but in fayre weather, and where yee may haue time ſufficient euen in the Fielde to pricke and ſet downe the Charte.

I had

I had thought heere to haue adioyned an other Inſtrument, with Rules to meaſure exactlye the heigth of any Tower, or length of any Curtaine, or breadth of any Ditche from any Hill or Mountaine, as farre as you can diſcerne them. Albeit it were 2, 4 or 6 Myles diſtant vtterlye without ſhotte or annoyance of the Enemie: But becauſe it ſerueth moſt fitlye for martiall ſeruice, I haue reſerued that inuention for my other Treatiſe of Fortification or Architecture Militarie: and meane onely in this Treatiſe to annexe ſuch additions as may prepare a waye to the vnderſtanding of ſuch rare concluſions concerning aſwell the Science of great Artillerie, as the other of *Architecture Nauticall* and *Militarie*, which I haue not yet reduced to ſuch Methode and perfection as can content me. But for that parte of Geometrie Practicall that dooth concerne Menſuration of lines, whether they be Altitudes, Longitudes, Latitudes, or Profundities: theſe Problemes may ſuffice. And nowe to the Second parte, concerning the Menſuration of Superficies of ſome termed *Planimetra*.

The end of the fyrſt Booke called *Longimetra*.

THE SECOND KINDE of Geometrie called *Planimetra.*

HAVING accompliſhed the firſt parte called *Longimetra*, contayning ſundrie rules to meaſure lengths, breadthes, heigths, depths, and diſtances: I thinke it meete nowe to proceed to the ſecond kinde named *Planimetra*, wherein yee ſhall haue rules for the menſuration of all manner plaine Figures, to know their content ſuperficiall. And foraſmuch as there is no Superficies, but is enuironed with lines either ſtreight or curue. And all Figures comprehended with ſtreight lines may bee reſolued into Triangles: It ſeemeth moſt meete, firſt to teach the meaſuring of Triangles as followeth.

The fyrſt Chapter

Of Triangles.

AS there are three kinde of Angles, ſo is there alſo three kind of Triangles: the firſt is called *Orthogonium*, hauing one of his Angles a right: the other *Ambligonium*, containing one obtuſe angle: the third kinde is called *Oxigonium*, whoſe three Angles are all acute. Of right angled Triangles alſo there are two kindes, for either it hath two equall ſides, and then is it called *Iſoſcheles*, or three vnequall, and that is *Scalenum*.

The right angled Iſoſcheles is thus meaſured.

MVltiplie one of the equall ſides in it ſelfe, the halfe of the product is the Area or Superficiall content.

Example.

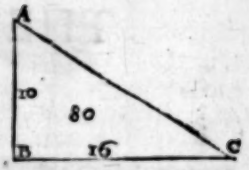
Admit A B C an Iſoſcheles right Angled Triangle, hauing the ſides A B and B C that contayne the right Angle B equall, either of them being 10, *I multiplie* 10 *in it ſelfe, thereof ariſeth* 100, *the halfe* 50, *is the Area of that Iſoſcheles right angled Triangle.*

A right angled Scalenum you ſhall thus mete.

MEaſure the longitude of two ſides containing the right Angle, augmenting the one by the other, the halfe thereof is that Triangular Superficiall content.

Example.

Suppoſe the right angled Scalenum A B C, his two ſides that containe the right Angle A B 10, *B C* 16, *the one augmented by the other, yeeldeth* 160, *whoſe halfe being* 80, *is that triangulare Area.*



In euery of theſe right Angled Triangles you ſhall not neede to meaſure more than two of their ſides: for any two knowne by ſupputation the third may be found; whereof I ſhall giue you rules with examples.

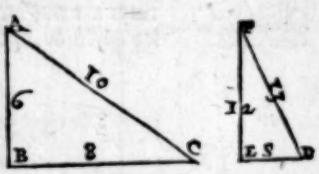
The 2 Chapter.

Any two ſides of right angled Triangles knowne by calculation to finde the thirde.

Vclide in the 47 propoſition of his firſt booke of Elements, hath prooued by demonſtration that the Squares of the two containing ſides ioyned together, are equall to the Square of the *Hypothenuſa*, or third ſide ſubtending the right Angle: whereby we may readily (any two ſides being giuen) finde the thirde thus: If the two ſides comprehending the right Angle be knowne, adde their Squares together, and the Radix Quadrate of the Product is the Hypothenuſa, but if yee knowe the length of that Hypothenuſa, and one other ſide, you ſhall ſubtract from the Square of that Hypothenuſa, the Square of that other giuen ſide, and the roote Quadrate of the remainder is the thirde ſide deſired.

Example.

In the Triangle A B C, A B is 6, *and B C* 8, *their Squares ioyned make* 100, *whoſe roote is* 10, *and that is the Hypothenuſa. In the Triangle D E F, the Hypothenuſa is* 13 *D E* 5. *I deſire the length of E F*, 25 *the Square of D E, ſubtracted from* 169, *the Square of the Hypothenuſa, leaueth* 144, *whoſe Quadrate roote* 12, *is the ſide E F.*



Of Obtuſe angled Triangles, there are alſo two ſortes, Iſoſcheles and Scalenum.

The 3 Chapter.

Ambligonium Iſoſcheles is thus meaſured.

You muſt firſt finde out the Perpendicular line, from the Obtuſe Angle to the contrarie ſide, by deducting the Square of halfe the ſide ſubtending the Obtuſe angle from the Square of one whole ſide containing the ſame angle, the roote Quadrate of the Product is the Perpendicular, which multiplied in the halfe of the foreſayd ſubtending ſide, produceth the Area.

Example.

Example.

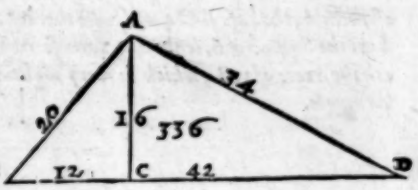
A B C the Iſoſcheles Ambligonium, A B 10. *B C* 16. 10 *ſquared is* 100, 8 *Squared yeeldeth* 64, *which deducted from* 100 *leaueth* 36, *whoſe Quadrate roote is* 6, *the Perpendicular A D. which multiplied by* 8. *produceth* 48 *& that is the Area of this Obtuſiangle Iſoſchelos.*

Ambligonium Scalenum, you ſhall thus meaſure.

FIrſt you muſt ſearch out his Perpendicular line in this maner, ſquare euery ſide, then add the ſquare of the ſide ſubtending the Obtuſe angle, to one of the other ſquares, from the offcome abating the third ſquare, the halfe of the remainder diuide by the foreſaid ſubtending ſide, the ſquare of the Quotient you ſhal deduct from the ſquare, which before you did add to the ſquare of the ſubtending ſide, and from the remaine, extract the root Quadrate, for that is the line Perpendicular, which multiplied by halfe the forenamed ſubtending ſide, wil produce the content Superficiall.

Example.

A B D, the Triangle, A B 20. *A D.* 34, *D B* 42. *the ſquare of A B* 400, *of A D* 1156, *of B D* 1764. 1764 *added to* 400 *maketh* 2164, *from which ſubtracting* 1156, *there remaines* 1008. *the halfe being* 504. *diuided by* 42. *the ſide ſubtending the Obtuſe angle, yeldeth in the Quotient* 12. *ſo much is the line B C, which ſquared maketh* 144, *that ſubtracted from* 400, *leaueth* 256, *whoſe roote Quadrate being* 16, *is the perpendicular A C. Now multiply* 16 *in* 21, *the halfe of the foreſaid ſubtending ſide B D, ſo haue you* 336 *the Superficies of that Ambligonium Scalenum.*



The 4 Chapter.

Of Acutiangle Triangles called *Oxigonia*, there are three kindes.

Or either the three ſides are equall, and then is it an Equilater Triangle, or two ſides onely equall, which is Iſoſcheles, or all three vnequall, and that is a Scalenum.

Of Triangles Equilater.

You ſhall multiplie the Square of the ſide in it ſelfe, and the offcome in 3. the Product diuide by 16 the roote Quadrate of the Quotient is the Area, or multiply the Cube of halfe the Equiangle Triangles ſide in the Semipertimetrie of the Triangle, the Quadrate roote of the Product is the foreſaid Area alſo.

Example.

Admit the Triangle ſide 6, *the ſquared ſquare is* 1296, *which augmented by* 3, *yeeldeth* 3888

and that diuided by 16, bringeth in the Quotient 243. whoſe roote Quadrate being 15, and betwene $\frac{1}{4}$ and $\frac{12}{11}$, is the Superficiall content of that Equilater Triangle. The ſame number is found by the ſecond rule, for the Cube of 3 is 27, which multiplyed in 9 the Semiperimetry yeeldeth 243, whoſe roote Quadrate is the Area of that Triangle agreeing with the fomer ſupputation.

Of *Iſoſchele* Acutiangle Triangles.

FRom the Square of one of the equall ſides ſubtract the Square of halfe the baſe or vnequall ſide, the Quadrate roote of the remainder multiplied in halfe the baſe, produceth the content Superficiall.

Example.

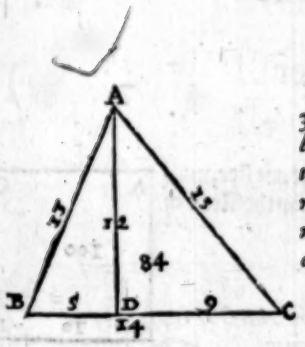
AB and AC the equall ſides of the Iſoſchele Triangle, either of them 6, the baſe BC 4, the halfe thereof 2, whoſe Square deducted from the Square of 6, leaueth 32, whoſe roote multiplyed in 2, bringeth the roote of 128, which is very nighe 11 $\frac{7}{13}$ the Area of that Triangle.

Of Oxigonium Scalenum.

TO finde the *Area* of Acutiangle Triangles that haue no ſide equall to other, it behoueth you to ſearche out the line Perpendicular from one of the Angles to the contrarie ſide, and multiply the ſame in halfe the baſe or ſide whereon it falleth, the Product is your deſire. But to get the line Perpendiculare, you ſhall thus worke: Square euery ſide, then adde the Square of the baſe, to the ſquare of one ſide, deducting fro the Product the third Square, halfe of the Remainder diuided by the baſe, the Quotient you ſhall againe Square and deduct this number from that Square, which you adioyned to the Square of your baſe, the roote Quadrate of the Remainder is the line Perpendicular.

Example.

The Acutiangle Triangle Scalenum ABC, AB 13, BC 14. CA 15, the Square of AB



169, of B C 196, of C A 225, 169 added to 196, produceth 365, from which 225 detracted, leaueth 140, whose medietie being 70, diuided by 14, yeelderh 5 the line B D, the Square thereof abated from the Square of A B, leaueth 144, his roote Quadrate is 12, the Perpendiculare A D: which augmented by 7 the medietie of the base, bringeth 84, the Area or Superficiall content of that Triangle.

Finally I thinke it not amisse to giue you one Rule Vniuersall to measure all manner Triangles of what sorte so euer they bee, and that without anye regarde of the Perpendicular.

The 5 Chapter.

A rule generall to measure all manner Triangles according to their plaine.

Vide all the sides of that Triangle together, taking halfe of the number which surmounteth. Now out of this halfe, you must pull by Subtraction euery side by it selfe, noting diligently the differences, that is, howe much euery side differeth from that halfe, multiply the differences the one in the other, and the Product in the thirde, with that which riseth augment the halfe aboue mentioned, then seeke of that summe the Quadrate roote, so haue you the true content Superficiall of that Triangle, the Example shall ensue, whereby all that I haue said, shall the better appeare.

Example.

Admit the Triangle whose sides you haue measured A B C. of whome the left side A B is 12, *B C* 16, *A C* 20, *these sides ioyned together, make* 48, *whose halfe is* 24, *from whome B C* 16, *differeth* 8, *A B* 12, *A C* 4. *so the difference are* 8, 12, 4. *Now multiply* 8 *with* 12, *riseth* 96, *the which augmented with* 4, *commeth* 384, *that multiplied in* 24, *the halfe of* 48, *surmounteth* 9216, *of this summe the Radix is* 96, *which is the verye content of this Triangle.*

Of quadrangles (that is to saye plaine Figures, hauing foure Angles and foure sides,) there are fiue sortes, as appeareth in the Diffinitions, the Square, the Rectangle, Rombus, Romboides, and Trapezia.

The

The 6 Chapter

Of Squares.

FOr the Square you shall onely measure one side, multiplying the same in it selfe, so haue you the Area or content Superficiall.

Example.

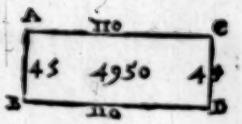
I finde A B the one side of the Square A B C D 10, *which augmented by it selfe riseth* 100 *the Area of that Square.*

Of Rectangles or right angled *Paralelogrammes.*

IN right angled Paralelogrammes ye must measure the two vnequall sides, multiplying the one in the other, the Product is the content Superficiall of the Figure.

Example.

I suppose the two sides A B, A C of that right Angled Paralelogramme. A B C D the one 45, *the other* 110, *these two multiplyed togither yeelde* 4950 *the Area.*

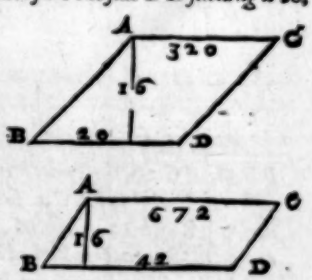


Of *Rhombus* and *Rhomboides.*

THese two figures haue one rule, it behoooueth you to measure one side, and the Perpendicular falling from one of the opposite Angles to the same side, these multiplied the one in the other produceth the Area.

Example.

Admit A B C D the Rhombus, whose Area I desire: I measure the side B D finding it 20, *then must I measure also the length of a line Perpendicular falling from A upon B D, which I suppose here* 16, *these two multiplyed the one in the other bring* 320, *the Superficiall content of the Rhombus. Likewise in the Rhomboides suppose I finde by mensuration the side B D* 42, *the line Perpendicularly falling from A upon B D* 16, *multiplye these numbers, I produce* 672, *the Area of that Romboides A B C D.*



But bicause it may seeme somewhat difficult to get the length of those Perpendiculars, bicause it is vncertaine on what pointe of *B D* the Perpendiculare line shall fall, I thinke good to prescribe you a rule how you may exactly and readily get all such lines Perpendicularly falling, and that not onely in these figures, but also in Triangles, which shall be no small ease and discharge of laborsome trauailing, when you shall measure great fieldes, or Champion plaines, &c.

The

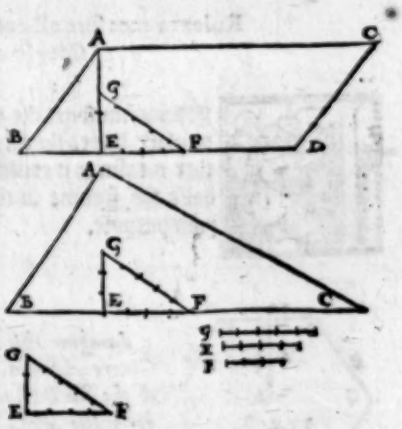
The seauenth Chapter

For measuring of lines perpendicular.

YOu shall prepare a right Angle by coniopning three staues proportioned according to these numbers, 3 4 and 5, as you were taught in the former Booke, the one of the staues conteyning the right Angle you must place directly vpon that side of your plaine Figure that subtendeth the Angle from whence your Perpendicular should fall, marking therewithall whither the other conteining side directeth, if it lie euen with the foresaid opposite Angle, then is the right line betweene your station, and that opposite Angle, the Perpendiculare. But if it direct not iustlye to the Angle, you shall moue to and fro in that subtendent side, till you finde it so, then by the precepts giuen in the former Booke, you may sundrie wayes measure the distance of the Angle opposite from your station, whereby you are brought in knowledge of this Perpendiculars length.

Example.

Admit A B C D the Paralelogramme, or A B C the Triangle, whose Perpendiculares falling from A to the contrarie side I desire to measure, but because I knowe not to what pointe in B C, or B D, these Perpendiculare lines from A will fall, I take my right Angle made of the three staues G E F, placing E F in the sides B C, and B D, passing too and fro in those lynes tyll I finde E G lyeth directlye with the Angle A, then saye I that a straight line from A, to my station or Angle E, is the Perpendiculare, whose length I may measure either by Quadratum Geometricum, *or otherwise, without Instrument, as in the first Booke is declared. Or thus deducting the Square of E B, the distance of my Station fro the ende or Angle of either field at B, from the Square of the line A B, rising from that subtendent side to the opposite Angle, the roote Quadrate of the remainder shall be the lyne Perpendicular, which multiplyed in B D, yeeldeth the Area of that Paralelogramme: or in halfe B C, so haue yee the Superficiall content of the Triangle A B C.*



The

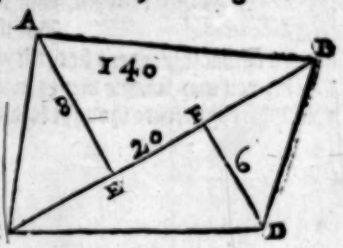
The 8 Chapter

To meaſure Trapezia.

YOu ſhall meaſure the length of a diagonall or croſſe line extended to oppoſite Angles through the Superficies of the Trapezium, and likewiſe the length of the two Perpendicular lines, falling from the other Angles vpon the ſaid croſſe line as you were taught in the laſt chapter, then adde thoſe two perpendiculars together, multipling the halfe of the product in that diagonall line, ſo haue yee the Area of that Trapezium.

Example.

A B C D the Trapezium, C B the Diagonall or croſſe line extended fro the Angle C, to B E and F, the two points where the perpendiculars ſhall fall: from the other Angles A, D, vpon the croſſe line C B, theſe pointes I finde with my right angled Triangle, as was taught in the laſt Chapter. Now to attaine the Area, I meaſure the length of B C 20, A E 8 F D 6: 6 and 8 ioyned make 14, the halfe is 7, multiplied in 20, the line Diagonall produceth 140, and that is the content ſuperficiall of that Trapezium.



The 9 Chapter.

Rules to meaſure all equiangle Superficies how many ſides ſo euer they haue.

FIrſt you muſt get the Centre of your figure, then from it pull a Perpendiculare line to the middes of ſome ſide, ſee howe many Perches or other meaſures it conteineth, adde all the ſides together, multiplying halfe the ſumme in the Perpendiculare or hanging line, ſo haue yee your purpoſe.

Example.

Imagine this Figure B C D E F G, and euery ſide length 12, the Centre A found, drawe a line Perpendiculare from it to the middle of the ſide B C, this line being 10 2/7 multiplyed in 36 the halfe number of the ſides, bringeth 374 2/7 the Superficiall contente of that Figure.

An

Another Example.

FGH is heere a Superficies being 5 square, euery side 10, Now from your Centre to the middes of the side pull the Perpendicular line which is 6 22/35 this multiplied in 25. the Semiperimetrie of that Figure, yeeldeth 172, and that is the Area of this Pentagonall Superficies.

A note to finde the Centres of those Equiangle Figures.

THe centre is found drawing lines from one angle to the contrarie, or from the middle of some side being odde to the opposite angle, the cutting or concourse of those streight lines sheweth the place of the Centre: and thus as is declared, you may readily measure all Equiangle Figures, what capacitie or number of sides soeuer they be of.

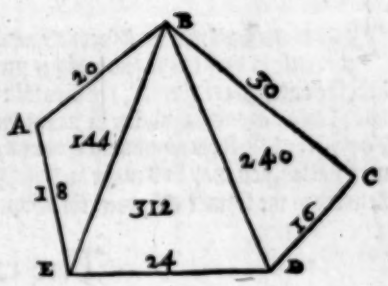
The 10 Chapter.

To measure the Superficiall content of any rightlined Figure of what forme so euer it be.

THe best rule I can prescribe, is to resolue it into Triangles, by drawing or imagining lines from Angle to Angle, and so measure euery Triangle seuerally: Finally, adding all the products together, ye shall haue the Area or content Superficiall of that whole Figure, which although it bee of it selfe (the premisses well vnderstande) plaine inoughe, yet to auoide all doubtes, I shall adioyne one Example.

Example.

Admit ABCDE an irregular Pentagonum, whose side AB is 20, BC 30, CD 16, DE 24, EA 18 this Figure you may diuide into three triangles, by drawing the two lines BE BD, thus by the rules before giuen, I measure first the Area of the Triangle ABE, which I finde 144, likewise BED, 312, the third Triangle BDC 240: these three numbers ioyned together, produce 696 which is the true Area or content Superficiall of that irregular Superficies.



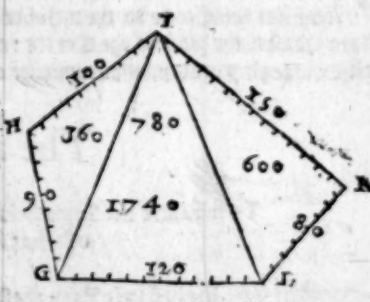
The 11 Chapter

A readie meane to find the content Superficiall of any great field, or champion plaine, howe irregular of forme or fashion soeuer it bee, without painfull trauailing about it, onely by measuring one side.

I Shall as I haue taught in *Longimetra* (either with your Theodolitus or your Topographicall Instrument) drawe vpon some leuell, smooth, plaine Superficies, the exact plat and Simetric of that field, then either by your Instrument

or measure how many rodde or Perche, is contained in some one side of that plaine, so many equall diuisions shall you make, opening your compasse accordingly in his correspondent side of your platte. This done, note well the fashion of the field, and drawing lines from Angle to Angle, or otherwise, diuide it into Triangles, or other regulare figures, as you shall see cause: then opening your Compasse to one of those diuisions, you may therewith measure euery side, and line, in that figure as exactly as with Corde, or Pole, ye should painfully passe it ouer, whereby with the aide of these former precepts, you shall seuerally measure euery Triangle or other regular figure, and ioyning together their contents, the resulting summe is your desire.

Example.



Suppose G H I K L that irregular Pentagonum, the true platte of some great field or plaine, made by one of the Instruments Geometricall, as is before in the first Booke declared, and that by mensuration I find the side G L 120 Perches in length, opening therefore my Compasse accordingly, I diuide G L into 120 equall portions, and proceeding on to euery side, I finde G H 90, H I 100, I K 150, K L 80, then beholding the forme thereof, I see it maye aptely be parted in three Triangles by lines drawne from I, to G and L. In like manner I seeche howe many such Diuisions there are in them, finding either of them 170 Perches in length. Finally by the rule generall of Triangles, I search the Area of euery Triangle, finding the first that is I H G 360, I G L 780, I K L 600, all three ioyned together, maketh 1740, so many Perches you may conclude the Area of that Figure, which reduced to Acres, diuiding by 160 bringeth 10 Acres 3 1/2 Roodes.

A note for Wooddes.

This waye shall you most speedely and exactly measure all manner Woodland, which otherwise is very harde and tedious precisely to doe, forasmuch as yee cannot at one view beholde euery parte thereof, nor measure such Diagonall lines, passing thorough it in sundrie places, as were requisite in irregular Figures, but with the aide of your Instrument Topographicall encompassing it round about, alway noting the Angles of Position at euery station, you may first make an exact Plat thereof, and after measure as you were instructed in the former Chapter, the Superficiall capacitie.

The 12 Chapter

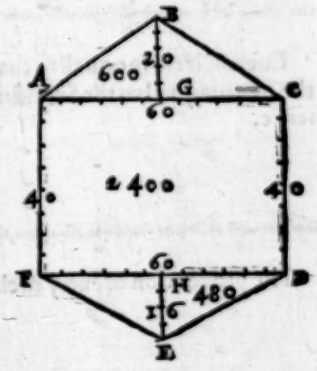
How you may from an high Hil, or Cliffe, measure how many Acres, Roodes, or Perches, is contained in any Field, Parke, Wood, or other plaine Superficies, in the countrie round about you, not approching nighe them,

Call to remembrance how you were taught in the first Booke by the Instrument Topographicall to set foorth the true Platte of an whole Countrie, and euery parte thereof, which, forasmuch as it is there at large set out, it were heere superfluous to recite againe, admitting therefore, (by the Arte there taught) an exact Platforme of the Field, Wood, or other plaine made, yee

yee ſhall with your Compaſſe diuide ſome one ſide thereof into 40, 60, or 100 equall partes, as you liſt, and keeping your Compaſſe immoueable, meaſure all ſuch other lines Perpendiculares, &c. as ſhall ſeeme requiſite to attaine the *Area* thereof, and by the former precepts, diuiding it into Triangles, Rightangled Parallelogrammes, or other regular figures, ye ſhall meaſure the Contents Superficiall thereof, that is to ſay, howe many of thoſe ſmall Squares, whereof euery little diuiſion was a ſide, is contained in that Superficies or Platforme. This done, yee muſt alſo with your Square Geometricall or other Inſtrument from the hill or Cliffe, meaſure the length of that ſide in the field, that the firſt diuided ſide in your Platte did repreſent, I meane how many rodde or Pearche it is long, then Square aſwell the number of Perches in the ſide of the field, as alſo the number of diuiſions in his correſponding ſide of your platte, the number proceeding of the Perches ſquared, ye ſhall multiplie in the Superficiall content of your platte toſore found, and the Product diuide by the Square of the Diuiſions in the ſide of your platte, the Quotient will be the number of Perches, which diuided by 160, and the remaine by 40, the Quotient will ſhewe the Acres and roodes contained in that field, Parke, or Wood, you meaſure.

Example.

Admit A B C D E F the platte or proportionall Patterne of a Parke which from ſome Hill or Cliffe a farre off I haue drawne by the aide of mine Inſtrument Topographicall, as was declared in the firſt Booke, and for that it is a Figure of many ſides, I ſearche how it may beſt be reſolued into regular Figures, which (as you may perceiue) is readely done withdrawing the two lines, A C & F D, which parts the whole figure in two Triangles & one right angled Parallelograme, then opening my Compaſſe to ſome ſmall diſtance, I diuide ſome one ſide (for Example the ſide A F) into 40 partes, and keeping my Compaſſe immoueable, I meaſure how many of thoſe Diuiſions are contained in A C and F D, finding either of them 60. Now, to meaſure the Triangles A B C, D E F, I drawe Perpendiculares from B to A C, and from E to F D, finding B G 20, and H E 16, Now 40 multiplied in 60, bringeth 2400, the right Angled Parallelogramme A D, 20 the Perpendicular in 30 halfe the Baſe yeeldeth 600, the Triangle A B C, & halfe the other Perpendiculare, in 60, the baſe ariſeth 480, the Triangle D E F, theſe three ioyned together, produce 3480, the whole content of A B C D E F, now as was taught in Longimetra, with my Square Geometricall, I muſt meaſure from the hill or cliffe the length of that ſide of the Parke repreſented here by A F, which I admit found as is before ſaid 356 rodde or Perche, whoſe Square being 126736 augmented in 2400, the content of my Plat bringeth 304166400, this diuided by 1600, the Square of the ſide A F yeeldeth 190104, and this diuided by 160 produceth in the Quotient 1188, and the remaine is 24. I conclude therefore that there is in that parke 1188 Acres, and 24. Perches.



In this ſort with ſmall exerciſe vſing induſtrie, in making your Plattes you may moſt exactly and ſpeedely ſurueye an whole countrey, with all his Paſtours, medowes, Marſhes, Woddes, and euery perticulare incloſure, whereof although I might propound an infinite number of examples and fill many leaues with varietie of rules, yet conſidering the Premiſſes to the ingenious will ſuffiſe, I thinke good to paſſe them ouer, referring the reſt to the practiſioner, who ſhall by his experience (well vnderſtanding the premiſſes) inuent manyfold mo, euen as occaſion ſhall be offred in viewing the ground with ſituation of places.

The 13 Chapter.

A note how to suruey an whole Region or plaine Champion Countrey by the aide of a plaine pullished Glasse.

IT behooueth you to resorte vnto the first booke where you were taught by the aide of a plaine pullished glasse of steele) vpon an high hill or leueled platforme) to set foorth the Symetrie or proportion of an whole countrie with all his partes. Ye shall therefore by the Arte there shewed, get the proportion or plat of that fielde, Wood, Marshe, or other inclosure, which you desire to suruey, and therewith worke euen in like manner as you were taught in the last chapter, to doe with the platte drawne with your Instrument Topographicail, one ground, one reason, and like operation serueth them both, the former well vnderstand maketh this so manifest as it needeth no other Example, onely this I must giue you warning of, that you attempt not this kinde of measuring, but onely in Champion and plaine Leuell Countries, for in vnleuell and hilly places, without most exquisite obseruation and learned handling great errour may ensue.

Thus hauing declared seuerall rules for euery kinde of rightlined Superficies, it seemeeth meete somewhat to say of such plaine Superficies as are enuironed with curue lines or mixte of bothe, and first of the Circle and his partes.

The 14 Chapter

Of Circles.

BEfore I entreate of the Mensuration of Circles, it shall be requisite to declare Archimedes rules, concerning the proportion of the Circumference to his Diameter, and of the Superficies to his Dimetients Square. The rules ensue.

The first Theoreme,

Πας κύκλος ἴσος ἐστὶ τριγώνῳ ὀρθογωνίῳ, οὗ ἡ μὲν ἐκ τῆς κέντρου ἰσημία τῆς περὶ τὴν ὀρθὴν, ἡ δὲ περιμέτρος τῇ βάσει.

Euery Circle is equall to that rightangled Triangle, of whose conteyning sides the one is equall to the Semidiameter, the other, to the Perimetrie or Circumference.

The second Theoreme.

Οὐκ ἔλαττος πρὸς τὸ ἀπὸ τῆς διαμέτρου τετράγωνον, λόγον ἔχει, ὃν ἔ ἄ πρὸς ἰ δ.

The proportion of euery circle to the Square of his Dimetient is as 11 to 14.

The third Theoreme.

Παντὸς κύκλου ἡ περιμέτρος τῆς διαμέτρου τοῦ πλασίου ἐστὶ, καὶ ἔτι ὑπερέχει ἐλάσσον μὲν ἢ ἑβδόμῳ μέρεσι διαμέτρου, μείζονι δὲ ἢ δέκα ἑβδομηκοστομόνοις.

The Circumference of a Circle is more than triple his Diameter by such a part as is lesse than $\frac{1}{7}$, and more than $\frac{10}{71}$ thereof.

How

How the Area of a Circle is found.

BY the former Theoremes ye may collect theſe Rules, Multiplie the Circumference of any Circle by 7, & diuide by 22, your Quotient is the Circles Diameter, whoſe medietie multiplied in halfe, the Circumference yeeldeth the Area, or multiplying the Square of the Circles Dimetient by 11, and diuiding the offcome by 14, your Quotient will declare the ſame.

Example.

Admit the Circumference of the Circle 44, which increaſed by 7. yeeldeth 308, this deuided by 22, bringeth in the Quotient 14, the Diameter, whoſe medietie being 7 augmented with 22 the halfe of the Circumference, produceth 154. Likewiſe the Square of 14 multiplyed in 11, maketh 2156, which diuided by 14, bringeth 154, the Superficiall capacitie of that circle.

Of the halfe Circle.

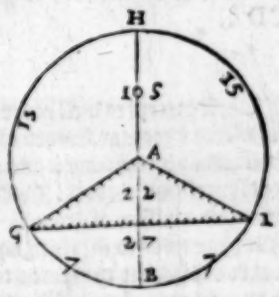
EVen as I haue declared that in multiplying the halfe Diameter of anye Circle in his halfe Circumference, the product to vtter his content, ſo by augmenting the halfe Diamer in the fourth parte of the Circumference, that is in halfe the Arke of the Semicircle, yee haue the whole ſumme of that halfe Circle.

Example.

Admit the Semicircle D E F were to be meaſured, whoſe Diameter D F drawne by the Centre C is 14. as afore, the Arcke D E F 22. Now multiplie the Semidiameter 7 in 11 riſeth 77, the plaine of that Semicircle, euen ſo of all portions or parts of a Circle. Although to the wittie this may ſuffice, yet to ſatiſfie alſo the meane witted, I ſhall not thinke it tedious to propound an other Example.

Of the Fragments or partes of a Circle.

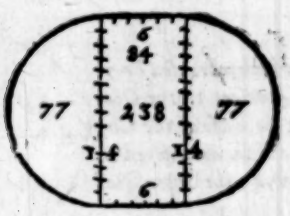
SVppoſe *G H I A* were a portion of a Circle to bee meaſured, the whole Circumference of the Circle whereof this is a portion is as afore 44. Nowe ſhall you multiplye 7 the Semidiameter *H A* in the halfe arke 15, ſo haue ye 105, which is the Superficiall content of that portion *G H I A*. If you deſire to know the Area of *G I B*, conteyned of the Corde *G I*, and Arcke *G I B*, yee muſt adde to the number before found, the Area of the Triangle *G I A* (which I ſuppoſe found by the rules tofore giuen) 22, that maketh 127 the Area of the Segment *G H I* to the corde *G I*, which ſubtracted from 154, the whole Circle leueth 27 the Area of the Segment *G I B*.

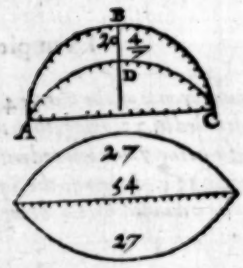


I thinke none will doubt how theſe Figures following are meaſured, bicauſe they are made of parts or portions of a Circle, whoſe plaine is gathered as I haue already declared.

In

In the left figure there is at eche end a ſemicircle, euery of them conteyning (found by the art afore mentioned) 77, which added make 154, the quadrangle hath 84, as the rules of Quadrangles declareth, that ioyned with 154, bringeth 238, the whole ſumme of that Figure.





The other are but two ſuch ſegmentes as G I B, which meaſured as is before ſhewed and ioyned together bringeth 54. the Area of that figure. Likewiſe the laſt figure called a Lunula, A B C D is meaſured by deducting the ſegment A D C (founde by the former rules) 27 from A B C knowen in like maner 56 ½ there remaineth 29 ½ the Area or Superficies of the Lunula A B C D. For aſmuch as it is neceſſarie in meaſuring of theſe portions or ſegmentes of circles to knowe the diameter of the whole circle, wherof they are the fragmentes. I thinke it not amiſſe hereunto to adioyne a rule for the ſame purpoſe.

Example.

Admit A B C D *a portion or fragment of a circle. I deſire to know the longitude of his circles diameter, firſt I meaſure* A C *which I finde* 12. *likewiſe the length of the line* D B *from* D *the middle of the ſtraight line to* B *the middle or higheſt point of the arke which I ſuppoſe* 4. *Now diuide the ſquare of* A D *that is* 36 *by* 4. *ſo haue ye* 9, *which added to* 4. *bringeth* 13. *and that is the length of the Circles Dimetient, whoſe parte or fragment this figure* A B C D *is.*

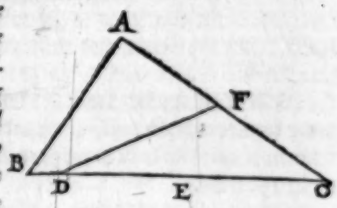
Theſe preceptes well noted there can no plaine Superficies offer it ſelfe whether it bee regular or irregular, framed of right lines enuironed with circular or mixte of both, but ye ſhall readily by diuiding it into triangles & circular fragments, finde out the Area and content ſuperficiall thereof. I will nowe therefore only, adioyne certaine queſtions for the partition and diuiſion of ground. And ſo paſſe on to the third kind of Geometrie, where you ſhall haue rules to meaſure, not onely the ſolide, but alſo the ſuperficiall contentes of all maner bodies, but to returne to theſe partitions my firſt queſtion ſhall be of triangles, and ſo on to figures of more difficultie.

The

The 15 Chapter.

There is a Trianguler field hauing on the one ſide a Well or Fountaine, this field muſt be equally diuided betweene two parties, and that in ſuch ſort that either of them may haue commoditie of that Fountaine, not comming on the others land. I demaund how that partition ſhall be made.

LEt *A B C* repreſent the Triangular field, hauing on the ſide *B C* a Well or Fountaine at *D*, from whence I would direct a hedge or ditche in ſuch ſort that it parteth the Triangle in two euen partes, firſt therefore I ſearche the middle of the ſide *B C*, which I ſuppoſe *E*, then my Inſtrument Topographicall at the Fountaine ſituate as he ought to bee, I turne the Alhidada, till I can eſpie thorough the ſightes the oppoſite Angle heere repreſented by *A*, then remooning mine Inſtrument to the foreſayd middle at *E*, the Alhidada remaining immooueable on the degrees tofore cutte, the Inſtrument againe duely ſituate, I view thorough the ſightes what parte of the ſide *A C* I can eſpie, Admit it *F*. Now ſay I that a Hedge, Dike, or other partition running from *D* the Fountaine to *F* the marke eſpied in the ſide *A C*, will diuide that Triangular figure exactly into two equall portions, and thus maye you without ſupputation onely by the aide of your Inſtrument Topographicall diuide any Triangle in two equall parts, and that from any part or pointe in any of his ſides aſſigned.



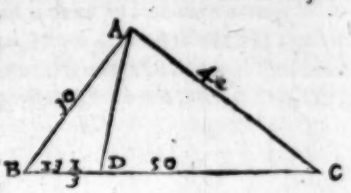
The 16 Chapter

To cut off from any Triangular fielde as many Acres, rodde, or other meaſures, as ſhall be required, and that by a line drawne from any Angle aſſigned,

YOu ſhall firſt meaſure the ſide ſubtending the Angle aſſigned, which here I will call the baſe: Secondly you ſhall meaſure the *Area* of the whole Triangle: Then multiplie the number of Acres or Rodds which you would cut off from the Triangle, by the length of the baſe, and the product diuide by the *Area* of that Triangle, the Quotient ſheweth how many Perches you ſhall meaſure in the baſe from one of the Angles to cut of your deſired number of Rodds.

Example.

Suppoſe A B C a Triangular peece of ground, hauing his three ſides *A B* 30, *A C* 40, *C B* 50 my deſire is with a right line from the Angle *A* to cut of one Acre of ground, to performe this by the arte before taught, I ſearche the Area of that figure which is 600 roddes. Now becauſe I would cut off from that Figure one Acre, and an Acre conteineth 160 rods: I multiply 160 in 50, the whole ſide ſubtending the aſſigned Angle, the product being 8000, I diuide by 600 the Triangles Area, my Quotient is 13 ⅓ ſo many Perches I reckon in the baſe *B C* from *B* to *D*, finally extending a line from *A* to *D*, I conclude a Superficies *A B D* one exact Acre, and this is there from that Triangular field one Acre cut of with the line *A D*.



The

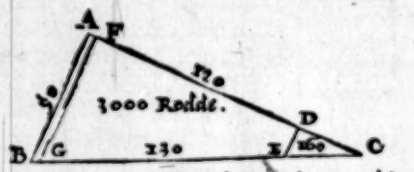
The 17 Chapter

To cut off from any Triangular piece of ground what quantitie of Perches ye liſt with a line Equidiſtant to one of the ſides.

TO performe this concluſion, it ſhall be requiſite to ſearch out what parts of the other two ſides this equidiſtant line ſhall cut, whereof I will giue you two rules, foraſmuch as this partition may two ſeuerall wayes be made: for the portion to be cut of eyther is adioyning to the parallele ſide, and then is it a quadrangular figure, or elſe to his ſubtending angle, and then is it a Triangle. For the Triangle ye ſhall thus doe, ſquare the ſides that the Parallele line ſhall cut, and the products ſeuerally multiplie in the number of Perches to be taken away, the ſurmounting ſummes, diuide by the Area of the whole Triangle, the Rootes quadrate of the quotients is the number of Perches to bee accompted in the correſpondent ſides, from the Angle that ſubtendeth the Parallele ſide. But if yee would haue the portion cut off, lye adioyning to the Parallele ſide, then ſhall you firſt deduct that portion from the whole Triangular Area, and with the remainder worke in like maner as ye were taught in the former rule, ſo will your quadrate rootes (accompted from the parallele ſide his ſubtendent Angle) ſhew to what part of either ſide the line ſhall bee drawne, to cut of the forenamed portion.

Example.

Suppoſe A B C the Triangle from which I would cut off an Acre, that is to ſay 160 rodde by a Parallele line to the ſide A B, firſt therefore I meaſure the Area of that whole Triangle as was before taught in this Booke, finding A B 50 Perche, A C 120, B C 130, and ſo conſequently the Area of that Triangle 3000 rodde, the Square of A C is 14400, the ſquare of B C is 16900, theſe augmented by 160, bring 2304000, and 2704000, and theſe diuided by 3000, produce in the Quotientes 768 and 901 1/9, the roote of 768 Perches (being 27 Perches 12 foote) I meaſure out in the ſide A C, beginning from C, admit it ende at D, likewiſe the roote of 901 1/9 Perches is 30 Pearches, and betweene 4 and 5 Inches, meaſuring therefore 30 Perches from C in the other ſide C B, I ſet vp a marke at E. Nowe if you drawe a ſtreight line from D to E, it ſhalbe a Parallele to A B,



and the peece of ground, repreſented by D E C, an exact Acre. But if you deſire to laye out this Acre at one of the ſides, as yee may ſee in the Figure ſignified by the Quadrilater Superficies A B G F. Then muſt ye deduct the foreſaid 160 roddes fro the Area of that Triangle, the remaine is 2840, which I multiplie as before in the ſquare of A C, ſo haue I 40896000. Likewiſe the ſame 2840, augmented by the ſquare of B C, produceth 48016000, theſe products ſeuerally deuided by 3000, the Area of the whole Triangle will yeelde in the Quotients 13632 and 16005 1/3 their Quadrate rootes are 116 Perches 12 1/2 foote, the length of the line C F, and 126 Perches 8 1/2 foote, the line G C, Or if yee deduct theſe rootes from the whole ſides A C, and B C there will remaine 3 rodde 4 foote from A to F, and 3 rodde 8 foote from B to G. And thus may you in all Triangular peeces of ground, exactly lay forth an Acre, or any other quantitie of ground yee will require, and that either againſt the ſide or Angle, euen as you will deſire.

The

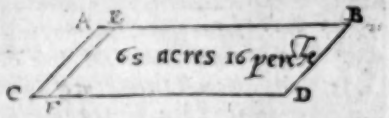
The 18 Chapter.

For partition of Parallelogrammes what kind ſo euer they be of, note theſe Rules enſuing.

Becauſe partition may ſundrie wayes be made according to the ſeuerall ſituation of the line wherewith it is deuided, I will firſt intreate of that diuiſion that is made by a Paralele vnto two of the Parallelogrammes ſides.

Example.

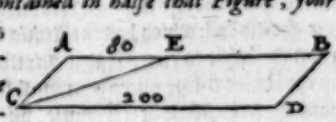
Admit therefore A B C D the peece of ground which by menſuration I finde to containe 65 Acres 16 Perches, and that I would cut off with a Parallel to A C 5 Acres, as ye may ſee done by the line E F, which were eaſily brought to paſſe if I knew the quantitie of the lines A E or C F, for knowledge of them therefore I worke thus. Firſt with my Square Geometricall or otherwiſe, as hath beene declared in Longimetra, I finde the length of A B 308 rodde, I multiplie therefore 800, for ſo many rodde are there in 5 Acres, by 308, there amounteth 246400 which diuided by 10416 the number of Perches contained in the whole Quadrangle, yeeldeth in the Quotient 23 Perches 11 foote, the length of the two lines A E & C F. Thus by making the partition E F, the portion A E C F, ſhall conteine exactly 5 Acres. This rule is generall for all Quadrangular peeces of ground, whoſe ſides be parallele, whether it be Square Rectangle, Rhombus or Rhomboides.



But if you deſire to make like partition by a right line iſſuing out of one Angle you ſhall thus worke: firſt conſider whether the portion yee would cut of be greater or leſſe than halfe the Parallelogramme: if leſſe multiplie as before the number of Perches, that yee would ſeparate from the Quadrangle in one of the oppoſitie ſides to that Angle whence your diuiding line iſſueth, and the product diuide by the medietie of the Area, the Quotient will ſhewe on what parte of that oppoſite ſide your diuiding line will fall. But if the aforeſaide portion to be ſeperate be greater than the medietie of the Parallelogramme ye ſhall deduct it from the whole Area thereof, and with the reſidue proceede in like manner as was before taught.

Example.

Admit A B C D a Quadrangular peece of ground, which being meaſured, is found to containe 50 Acres, and that my deſire is to cut off 10 Acres from that Figure, with a right line paſſing foorth from the Angle C, firſt therefore I meaſure the length of the ſide A B, finding it 200 rodde, 200 therefore I augment by 1600 the number of Perches in 10 Acres, there ariſeth 320000, which diuided by 4000 the number of rodde contained in halfe that Figure, your Quotient will be 80, the number of Perches from A to E, ſo that by the Partition C E, yee ſhall ſeperate the Triangle A C E containing exactly 10 Acres, but if the ſumme of Acres had beene greater than halfe, ye ſhould haue de-



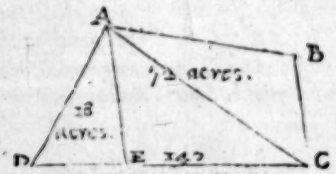
ducted it from the whole Area, and with the residue woorke in all respects as yee haue done with this, the onely difference is, that at the ende of your operation, whereas heere the Triangle A C E is the portion seperate, there it should be the residue or Quadrangle E C D B, for that there is no difference in the working, it were superfluous to vse mo Examples.

The 19 Chapter.

To cut off from any *Trapezium* or Quadrangular peece of grounde, what parte thereof yee list.

Example.

Admit A B C D a Trapezium, or Irregular Quadrangle, and that I would cut of a quarter of his Area, with a line issuing from the Angle A, first by the rules tofore giuen, I measure his content Superficiall, which I suppose 72 Acres, secondly I meete the Area of the Triangle A D C, finding it 50 Acres. Nowe bicause I would cut of a quarter of that Trapezium, I diuide his content that is 72 by 4, my Quotient is 18, and that multiplied in D C, (which heere I suppose founde by mensuration 146 Perches,) produceth 2520, and this diuided by 50 the number of acres in the Triangle A D C, yeeldeth in the Quotient 50 Perches 6 ½ foote, which measured out from D in the line D C, sheweth where the line of partition issuing fourth of the Angle A shall fall: and thus may you conclude that A D E, is the foresaid quarter of the whole Trapezium, and containeth 18 Acres.



The 20 Chapter

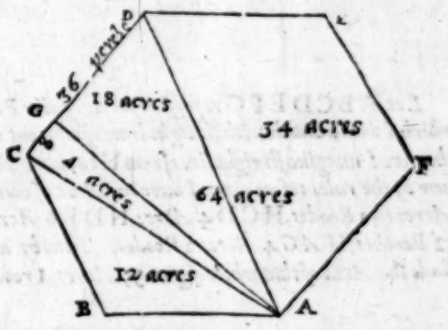
To diuide the Superficies of any irregular *Pollygonium*, with a straight line proceeding from any one of the Angles assigned in such sorte, that the partes shall retaine any proportion appointed.

When any proportion is giuen, there are two Numbers wherewithall it is expressed, and they are called Termini, those two you shall adde together, reseruing the Product for a Diuisor, then measure the Area of that whole Irregulare Pollygonium, which you shall multiplye in the lesser of those *Termini*, the Product shall you diuide by your reserued Diuisor, the Quotient is the content of the lesser Portion, which Deducted from the whole, leaueth the Superficies of the greater part. But to drawe the line of Partition that shall diuide this Pollygonium, it were necessary first to learne on what side and parte therof this line should fall, which you shall thus doe: Drawe or imagine right lines extended from the assigned Angle to euery Angle on either side, so shall ye make seuerall Triangles, whereof by the Rules tofore giuen, you must search the content superficiall. This done, ye shall compare the Area of the first triangle, with the Superficiall content of the lesser portion founde as is before declared.

If the Triangle be greater, then may you by the precepts before geuen, cut off from that Triangle so many Acres or Perches as that lesser Portion should containe, but if the Triangle be not greater, yee shall deduct it from the foresaid lesser portion, and the Remaine compare with the nexte Triangle, which if it be greater than that Triangle also, subtract from it the Superficies of the Triangle, and so proceede on till ye finde a Triangle greater than your Remaine, then may you say, that your partition or diuiding right line proceeding from the assigned Angle shall fall on the base or side subtendent of the assigned Angle in that last Triangle, but to know on what parte thereof, you must worke with your last Remaine and the Area of your last Triangle, as you were before taught in the diuision of Triangles. And in like manner drawe your diuiding line, which shall exactlye seperate that Irregular Polligonum, in two Superficies retaining the prescribed proportion: for more plainnesse perule the Example ensuing.

Example.

Admit A B C D E F, an irregular Hexagonum which I would diuide with a right line issuing foorth of the Angle A in such sorte that the greater parte might bee triple to the lesser: this proportion triple may be expressed with these two numbers 3 and 1, and they are called Termini or boundes of that proportion, these two added make 4, wherewith diuide the whole Superficies of that Polygonum, which I suppose by mensuration found 64 Acres, my Quotient will be 16. Admit also I finde the Area of my first Triangle A B C 12 acres, & the Area of the second A C D, 18 Acres. Now comparing 16 with the Triangle A B C, I finde it greater, deducting therefore the one from the other, there resteth 4 Acres, which forasmuch as it is lesse than the Area of the Triangle A C D, I conclude the partition or diuiding line shall fall on the side C D, but to learne on what part thereof, I worke as I was taught in the partition of Triangles thus, first with my Square Geometricall or otherwise, I measure the side C D.



Admit in 36 Perches, which multiplied in 4 the last Remaine bringeth 144, and that diuided by 18 the Acres contained in the Triangle A C D, produceth in the Quotient 8, so many rodde length shall you meete from C to G, in the side C D, finally extending a right line from the assigned Angle A, to the pointe G, you may conclude A F E D C Triple to A G C E.

The 21 Chapter

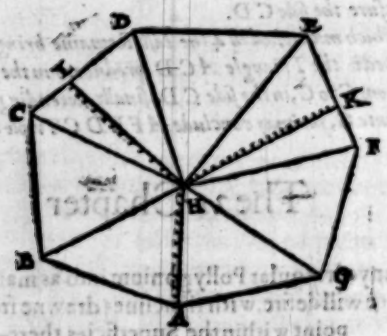
To diuide anye irregular Pollygonium into as many equall parts as yee will desire, with right lines drawne from anye point within the Superficies thereof assigned.

FIrst get the Area of the whole irregulare Pollygonium, which you shall diuide by the number of partes, wherevnto yee would diffeuer it, and the Quotient reserue, then from

your point assigned deducte or imagine lines to euery Angle of that figure, so shall yee parte it into sundrie Triangles, then compare your reserued quotient with some one Triangle beginning where yee list, and if yee finde the Triangle greater, then cut of from it a portion equall to your quotient, with a straight line proceeding from the angle adiacente to the assigned point, but if your quotient be greater than the first Triangle, deducte the one from the other, and compare the remaine with the seconde Triangle, which Triangle if yee finde greater, cut of from it a portion equall to your remaine, with a line issuing out of the Angle ioyninge to your assigned poynte, and then compare againe your reserued quotiente with the remaining Triangle, if your quotient be greater, seperate from the Triangle next ensuing a portion equall to the excesse or ouerplus, and that alwaies with a line issuing out of the Angle adiacent to the pointe first giuen. Thus proceeding till ye haue circulate the figure, alwaies drawing straight lines from the pointe assigned, yee shall in the ende departe the whole figure into as many equall portions as ye determined. This one thinge is to bee noted, that so ofte as your remaine or reserued quotient falleth out equall to anie of the Triangles, then shall you drawe any line from the assigned point, for the side of that Triangle serueth for the partition or dividing line.

Example.

Let ABCDEFG *represent an irregular Polygonium, or seuen sided peece of grounde, which I would diuide with streight lines issuing out of the pointe* H *into three equall partes, first therfore I imagine streight lines from* H *to euerie Angle, so is it diuided into 7 seuerall triangles: now by the rules tofore giuen I meete the Area of euerie triangle, finding* HAB 5 *Acres,* HBC 5 *Acres two Roodes,* HCD 4 *Acres,* HDE 6 *Acres 1 Rode,* HEF 5 *Acres,* HFG 5 *Acres 12 Perches,* HAG 4 *Acres 2 Roodes. Thus by adding all these triangles together you shall finde the Area of that whole figure 35 Acres, 1 roode, 12 Perches, which beeing reduced into Perches*



yeldeth this number 5652, and that diuided by 3 bringeth 1884 Perches, and that is the contents of euerie portion that this figure shoulde bee parted into.

First o

Firſte therefore comparinge it with the Triangle A B H, *I finde the Triangle the leſſer, the one dededucted from the other, there remaineth* 1084. *Perches, and this Remaine I finde likewiſe greater then the Triangle* C H B, *detracting therefore the one from the other, there remaineth* 204, *which compared with the thirde Triangle* C H D, *forasmuch as it is leſſe than the Area thereof. I ſearche by the Rules tofore giuen in diuiſion of Triangles, on what part of the Baſe,* C D *the diuiding line ſhall fall that proceedeth from the Angle* H: *admit it cut* C D *in the poynte* I. *from* I *to* H *extend a right line, & that ſhalbe the firſt partition. Now ſeeinge* H I D *Ioined to* D E H *the fourth Triangle, is leſſe than* 1884 *Perches my reſerued quotient, I deduct the one from the other, ſo there remaineth* 448 *Perches, I ſearch therefore by the Rules tofore giuen on what parte of* E F *the line ſhall fall, proceeding from* H *to cut off* 448 *Perches from* H E F: *but heere you muſt conſider that this portion of* 448 *roddes or Perches muſt be adiacent to the ſide* H E, *bicauſe the Quotient dooth exceede the Triangles by ſo much. This conſidered admit the partition* H K. *Now bicauſe my deſire was to deuide it onelye into three partes, I neede proceede no further, for thoſe two lines doe exactlye departe that whole irregular Pollygonium into three equall partes, whereof the firſt is that rightlined figure* I H A B C, *the ſecond is* I H K E D, *the thirde is* K H A G F, *and this partition is made with right lines iſſuing from the pointe* H *aſſigned, which was required. In this manner you may diuide anye Superficies into as many equall or vnequall partes as you liſt, or into what proportion yee will deſire, as by theſe fewe Examples the ingenious will readily conceane of any other: manifolde Rules might I giue you in theſe and like partitions, but knowing the premiſſes to the wittie are ſufficient, I will onely adioyne certaine Queſtions of ſome more difficultie, with the reſolution of them, and ſo finiſhe the ſecond kinde of menſuration.*

The 22 Chapter.

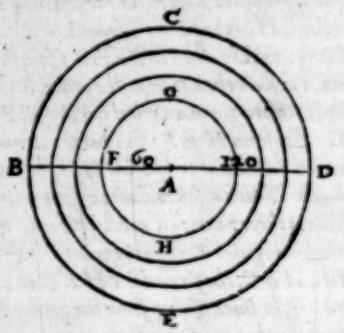
To diuide any Circle whoſe Semidiameter is knowne, with an other Circumference concentricall, in two ſuch partes that the one portion to the other ſhall retaine any proportion aſſigned.

WHen anye proportion is giuen, it conſiſteth of two numbers, as I haue before ſaide, that are called *Termini rationis*, thoſe numbers yee ſhall adde together, reſeruing the producte for a diuiſor, then multiplie the ſquare of the Semidiameter knowen by the leſſer of thoſe *Termini*, and the offcome diuide by the reſerued Diuiſor, from the Quotient thereof, reſulting, extract the Quadrate roote, for that is the Semidiameter of the concentricall Circle, whoſe Circumference ſhall diuide the former giuen Circle in two partes, retayning the proportion aſſigned.

Example.

Example.

Admit 120 *the Semidiameter of the Circle, B C D E, whose Area I would diuide with a concentricall Circumference in such sorte that one parte might be triple to the other, this proportion consisteth of* 3 *and* 1, *which added together make* 4, *now the Square of the Semidiameter being* 14400, *augmented by* 1 *the lesser of those* Termine, *produceth the same summe againe which diuided by* 4 *tofore reserued for that purpose yeeldeth* 3600, *whose Quadrate roote is* 60, *so much is A F the Semidiameter of that inwarde Circle, whose Circumference hath diuided the Circle B C D E in two partes, the lesser in the Circle F G H, and the greater is the annular Superficies conteined betweene those circumferences, the one being Triple to the other. In like manner may you diuide that annular Superficies into three other, euery one of them equall to the same inwarde Circle, whereof it were superfluous to adioyne any farther Example, forasmuch as the proportion of the partes once knowne, the operation is in all points agreeable with the former.*



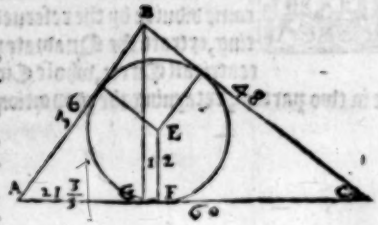
The 23 Chapter.

The three sides of a Triangle knowne, by supputation to get the greatest Circles Semidimetient that may bee described within that Circle.

YE shall first adde the two shorter sides together, reseruing the product, then subtract those sides the one from the other, the remaine multiplie in the former product, and the amounting summe diuide by the thirde or longest side, the Quotient detract from that longest side that was your last Diuisor, the Medietie of this remaine you shall multiplie in it selfe, and deduct the offrome from the Square of the shortest side, the root quadrate of the Remainder is the Perpendicular falling from the greatest Angle to the greatest side, which retaineth the same proportion to the Semidiameter of the inscribed Circle, that the Perimetrie of the Triangle dooth to the base or greatest side, working therefore by the rule of Proportion by three knowne, ye may readily finde the fourth.

Example.

Let A B C bee the Triangle, A B 36, *B C* 48, *A C*, 60, *nowe to get the greatest Circles Semidiameter that may be described within that Triangle, I woorke thus, first* 36 *added to* 48, *bringeth* 84, *then* 36 *detracted from* 48 *leueth* 12, *which multiplyed in* 84 *bringeth* 1008, *which diuided by* 60, *the*



the longeſt ſide yeeldeth in the Quotient 16 $\frac{4}{7}$. *This deducted from* 60 *the Diuiſor, the remaine will be* 43 $\frac{3}{7}$, *the medietie thereof is* 21 $\frac{1}{7}$: *whoſe Square deducted from the Square of* 36 *leaueth* 829 $\frac{11}{14}$. *Therefore doe I conclude* $\sqrt{}$ 829 $\frac{11}{14}$, *the Perpendicular B G, for the Semidiameter E F, I vſe the rule of proportion ſaying* 144 *the Permetrie, giueth* 60 *the baſe, what ſhall* $\sqrt{}$ 829 $\frac{11}{14}$ *by multiplication of the latter and diuiſion with the firſt, yee ſhall finde the fourth proportionall number* 12, *and that is the line E F or Semidimetient of the greateſt Circle that may be deſcribed within that Triangle.*

This is alſo to be noted, that the Medietie whoſe Square ye Subtract from the ſquare of the left ſide to get the Perpendiculare) is the line A G: which deducted from the whole baſe, leaueth the other line G C, and thoſe two parts of the baſes are called *Caſus*.

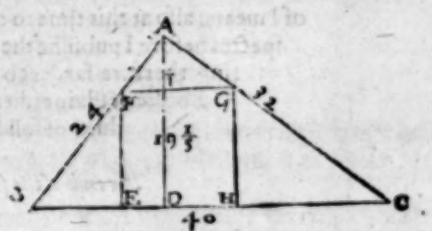
The 24 Chapter

To finde the greateſt Squares ſide that may be deſcribed within any Triangle whoſe ſides are knowne.

SVndrie rules might be giuen to reſolue this queſtion, but to auoide confuſion, I will onely declare the eaſieſt. yee ſhall therefore (as is before in this booke taught) ſearche the Perpendicular line falling from the greateſt angle to the longeſt ſide: this Perpendicular it behoueth you to diuide in ſuch ſorte, that the partes retaine ſuch proportion as booth the baſe or longeſt ſide to the ſaid Perpendiculare, ſo doeing the greater portion is the Squares ſide. But Arithmetically to attaine the quantitie of this longer portion, ye ſhall thus worke: Multiplie the Perpendicular in it ſelfe, and diuide the producte by the Baſe or longeſt ſide of the Triangle and the Perpendicular ioyned togither, the Quotient detracted from the whole Perpendiculare leaueth your deſire. Or multiplie the Baſe in the Perpendicular, and diuide by them bothe ioyned togither as before, ſo ſhall your Quotient produce the Squares ſide alſo.

Example.

A B C is the Triangle whoſe ſides are knowne, A B 24, *B C* 40, *A C* 32, *the Perpendicular founde, as is tofore declared, is* 19 $\frac{1}{7}$, *which diuided at I, in ſuch ſorte that D I to I A retaine the proportion of B C to D A, D I ſhall bee the ſide of the greateſt Square, that may be made within that Triangle: but to finde the length of D I, you muſt multiplie* 40 *the baſe B C in* 19 $\frac{1}{7}$ *the Perpendiculare, ſo haue you* $\frac{1}{7}$ *which diuided by* $\frac{136}{7}$ *the Perpendiculare and Baſe ioyned together yeeldeth* 12 $\frac{6}{11}$, *the line I D.*



Likewiſe

Likewise, if ye square the perpendicular, the mounting summe will be $\frac{9216}{25}$ which diuided by $\frac{296}{5}$ the former diuisor bringeth in the quotient $6\frac{92}{185}$, which deducted fro $19\frac{1}{5}$ the perpendicular, there remaineth $12\frac{180}{185}$, the side of the greatest Square agreeing with the former operation. In like manner if ye Deuide B D, whose longitude you were taught by the last Chapter to finde, and also A B. the left side of the Triangle in such sorte, that the greater sections to the lesser retaine the same proportion, that the base dooth to the Perpendicular, the squares of the two greater sections one deducted from the other leueth the Area of the greatest square that may be described within that Triangle. Also, if ye deuide any of the two lesser sides in two partes retaining the foresaid proportion of the Perpendiculare to the Base, the lesser of those Portions augmented by the base and the product deuided by the side bringeth in the quotient that greatest squares side. Thus also an other way, you may attaine the same: Diuide both the Casus, that is to saye, B D, and D C the distance of either Angle from the Perpendicular, in like manner as hath beene said of the Perpendicular, then adde both the smaller sections together, the resulting summe is the squares side: Euery of these wayes working, ye maie resolue this question, great pleasure shall the Arithmetrician (especially if he be seene in surde and radicall numbers) receiue, when hee shall perceiue so diuerse intricate and different operations alway in fine to produce the selfe same certaintie,

I Might here adioyne manie Propositions, concerning the deuiding of all Triangles, Quadrangles and other Polygona whatsoeuer, into equal or vnequal parts retaining any proportion prescribed: & likewise to finde out the Center of Grauitie, or Counterpeize Center of all Plaines, right lined Geometricalie without aide of Arithmeticall Calculations: the meanes also readilie to square all Triangles, Quadrangles and right lined Polygona, Geometricallie: to reduce the irreguler to reguler, Equilater and Equiangle, and to transforme the one into the other, to giue their proportions in lines, or to frame superficies of al kinds, to retaine any proportiō in right lines prescribed. But as my Father who laid the first foundation of this worke, sought to deliuer such conclusions as might be profitable and seruiceable to his Countrie, rather then by difficult & curious Demonstrations to stirre vp an admiration of his knowledge, so haue I forborne to adde or innouate any thing in this place. But for the satisfaction of such serchinge wittes as cannot contente themselues with matters necessarie, but will more curiouslie surueie what Arte is able to perfourme in matters difficulte, I meane at the ende of this treatise to adioyne some Problemes, that happelie maie somewhat better cōtent such aspiring Spirites, and yet, all such as haue most necessarie and commodious vses, aswell in the arte of Nauigation & Architecture Nauticall, as also in the arte of Fortification and Architecture Militare, and likewise in the newe science of great Artillerie, a tast wherof I meane also at this time to empart, for sundrie dewe respectes before I publishe the Treatise it selfe, forbearing therfore farder to enlarge this seconde Booke of Plaines, heere shall ensue the third of solide Bodyes.

The ende of the second booke.

The thirde kinde of Geometrie named Stereometra.

IN this thirde Booke yee ſhall receiue ſundrie Rules to meaſure the Superficies and Craſſitude of Solide bodies, whereof, although an infinite ſorte of different kindes might be imagined, yet ſhall I onely entreate of ſuch as are both vſually requiſite to be moten, and alſo may ſufficiently induce the ingenious to the menſuration of all other Solides, what forme or figure ſo euer they beare. And foraſmuch as in ſetting foorth their ſeuerall kindes, I haue choſen to vſe the accuſtomable and auncient names well knowne to any trauelled in Geometrie, rather than to forge new Engliſh wordes which can neither ſo breefely nor ſo aptly expreſſe the like effect, leaſt to the common ſorte anye obſcuritie might growe, I thinke good to adioine euery of their Diffinitions.

DIFFINITIONS.

1 A Solide bodye is that hath lengthe, breadth and thickneſſe, bounded or limited with Superficies.

2 Like Solides are ſuch as are encompaſſed with Superficies that are like and of equall number.

3 A Priſma is a Solide Figure comprehended of plaine Superficies, whereof two are equall, like, and Parallele, the reſt Parallelogrammes.

4 Pyramis is a Solide Figure incloſed with many plaine Superficies riſing from one, and concurring or meeting in a pointe.

5 A Sphere is a Groſſe or Solide body comprehended of one Conuex Superficies. In the middes whereof there is a pointe from whence all right lines to the ſame Superficies extended, are equall.

6 That pointe is called his Centre, and a ſtraight line by that Centre paſſing thorough this Solide bounded on eyther ſide with the Conuexe Superficies is called the Diameter of that Sphere.

7 Alſo intellectually yee may thus conceiue a Sphere to be made. Suppoſe a Semicircle (his Diameter remaining immoueably fixed) to be turned rounde about till it returne to the place whence it firſt began to moue, the figure ſo deſcribed is a *Sphere*.

8 Likewiſe, if a Rightangled Triangle (one of the containing ſides remaining fixed) be turned circularly about the Figure ſo deſcribed, it is called a *Cone*.

9 The right line that remaineth fixed is the *Axis*.

10 The Circle deſcribed by the other containing ſide is the *Baſe*.

11 The third line or *Hypothenuſa*, is the ſide of the Cone.

12 If a right angled Parallelogramme (the one of the ſides conteyning a Right angle remaining immoueable) be Circularly turned, the Figure ſo deſcribed, is a *Cylinder*, and the immoueable ſide is his *Axis*.

13 The Baſes are the Circles by reuolution of the two oppoſite Parallele ſides deſcribed.

14 The Altitude or heigth of any Solide bodye, is a line Perpendicularly falling from or highest part of the Solide vpon the plaine whereon the body lieth.

15 This Perpendicular or line of Altitude in direct solides falleth within the body, and vpon the Base, but in declining Solides, it falleth without the bodies and Bases.

16 As the Concourse of lines maketh a plaine angle, so the concurring or meeting of many Superficies in a point, maketh a Solide angle.

17 In euery Solide body a right line drawne from one Solide Angle to an other is called a line Diagonall. But if it passe betweene opposite Angles, it is named the Diameter.

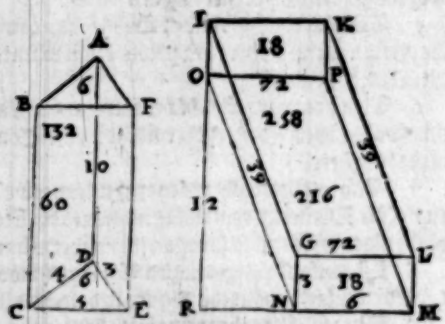
The fyrst Chapter.

To measure the contents Superficiall and Solide of any Prisma.

Or the Superficies, yee shall adde all the Parallelogrammes and Bases their Area togither, the resulting summe is your desire, but for the Crassitude, yee shall augment the Altitude of the solide in the Area of the Base, the product is the Grosse capacitie.

Example.

Considering of Solides called Prismata, there are two kindes, the one direct or vpright, whose Paralelogrammes are Perpendiculare to their Base, the other oblique or declining, whose Paralelogrammes are obliquely situate on their bases. Of either I minde to propound an Example, although one rule suffice them bothe. Admit therefore A B C D E F a direct or vpright Triangular Prisma, hauing the three sides of his Base E D 3, D C 4, E C 5, his altitude A D 10, so that by the rules giuen in Planimetra I finde the Area of either Triangle or base 6, the Paralelogrammes A F E D 30, A B C D 40 F B C E 50, all these ioyned together, make 132, the Superficies of that Prisma. Likewise for the Superficies of the declining Solide I K L M N O, by the Precepts giuen in Planimetra, I finde the Area of the right angled Paralelogrammes or bases either of them 18, the Area of the other Paralelogrammes I G N O 39, I G L K 72, K L M P 39, O P N M 72, and these ioyned together yeeldeth 258, that is the content Superficiall thereof.



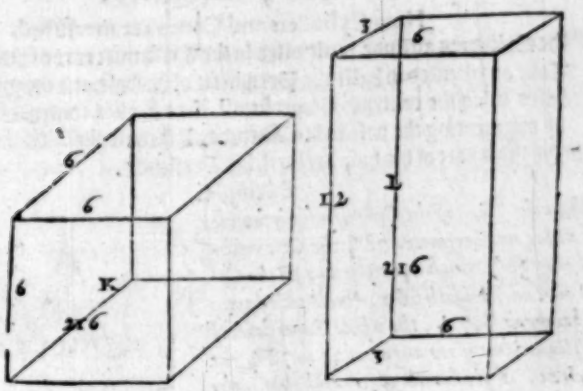
For the Crassitude I search the Altitude of either Solide, which in the vpright Prisma is, the ereared side of any of his Paralelogrammes, as A D, B C, or F E. For they are all equall, euery one of them being 10, but the Altitude of the declining Quadrangular Prisma is the Perpendicular O R, falling from O the top of the Prisma perpendicularly on M R a line drawne in the plaine whereon the body resteth, 10 therefore multiplyed in 6, produceth 60, the Solide quantitie of that Prisma: Also the Altitude O C being founde by mensuration 12 multiplied in 18 the Base, bringeth 216, and that is the Solide capacitie of the declining Prisma I K L M N O.

Although

Although these common peeces *K L* are moten as is tofore taught, yet yee may readily thus measure them, multiplie the length with the breadth, and the Product in the thicknesse, so haue ye the content or Crassitude.

Beholde the Figure.

The content of K 216 *Cubicall foote, the Crassitude of L* 216 *Square foote.*



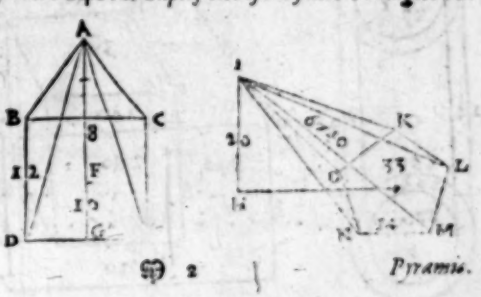
The 2 Chapter

How the contents Superficiall and Solide of a *Pyramis* may be measured whether it be direct or declining.

Or as much as euery *Pyramis* is enuironed with Triangular plaines rising fro the base, and concurring or meeting at the toppe, yee may by the rules giuen in *Planimetra*, measure the Area of euery Triangle, whose contents ioined together, and the Product to the base, yeeldeth the Superficies of that whole *Pyramis*. But for the content & olide whether the *Pyramis* be direct or inclinate, ye shall first measure the length of the line Perpendicularly falling fro the top to the plaine whereon the base resteth, which multiplied in the third part of the bases Area will produce the Crassitude of that *Pyramis*.

Example.

Suppose A B C D E an vpright Quadrangular Pyramis, whose base being a Square, I measure onely the side B C finding it 12, *which multiplyed in it selfe, bringeth* 144 *the Area of the base, then doe I measure the length of the line A G* 10, *for so am I taught in Planimetra, that multiplied in* 6, *halfe D E bringeth* 60, *the Area of the Triangle A D E, but seeing the other Triangles are equall to the same, considering they haue equall bases and altitudes, I needs make no more adoe, but multiply* 60 *by* 4 *so resulteth* 240 *the Superficies of the foure Triangles concurring at A, and this added to* 144, *the base bringeth* 384 *the whole content superficiall of that Pyramis. But for the Crassitude I measure the Altitude or line A F falling from A Perpendicularly vpon the base, this Perpendiculars length I suppose* 8, *which multiplyed in* 144 *produceth* 1152, *and that is the Solide content of that Quadrangular*



Pyramis. In like manner to attaine the Craſſitude of the declining Pentagonall Pyramis I K L M N O, I meaſure the Altitude or Perpendicular I H 20. Likewiſe the Area of the Pentagonall baſis, whoſe ſide is 14, which by the precepts giuen in Planimetra I finde 337, theſe multiplied together yeeld 6740, the Solide content of the declining Pyramis.

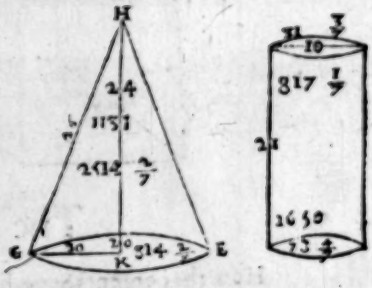
The 3 Chapter.

How Cylinders and Cones are meaſured.

The Cylinders altitude multiplied in the Circumference of his baſe, or the ſide of the Cone augmented in halfe the Peripherie of his baſe, and the products added to their baſes, bring the contents Superficiall. The Solide content of a Cylinder is gotten by augmenting the baſe in his Altitude. Likewiſe the Altitude of the Cone multiplied in the third part of his baſe yeeldeth his Craſſitude.

Example.

Admit the Area of the Cylinders baſe founde by the Rules in Planimetra 78 4/7 the Circumference being 31 1/7, which multiplyed in 21 the Cylinders altitude yeeldeth 660, which added vnto 157 1/7 bringeth 817 1/7, the whole Superficies of that Cylinder, but if yee augment 21 in 78 4/7 the baſes Area, it produceth 1650, and that is the Craſſitude or Solide capacitie of that Cylinder. Likewiſe for the Superficies of the Cone G H E, I augment 26 the ſide G H in 31 1/7 halfe the Circumference G F E, there reſulteth 817 1/7, whervnto if yee adioyne 314 2/7 that Baſis Area there amounteth 1131 3/7, the Cones content Superficiall, and by multiplying 24 the Altitude in 104 4/21, the third part of the Circle G F E, there ariſeth 2514 2/7, the Solide content of the Cone G H E.



A note to finde the Altitude of a Solide Cone.

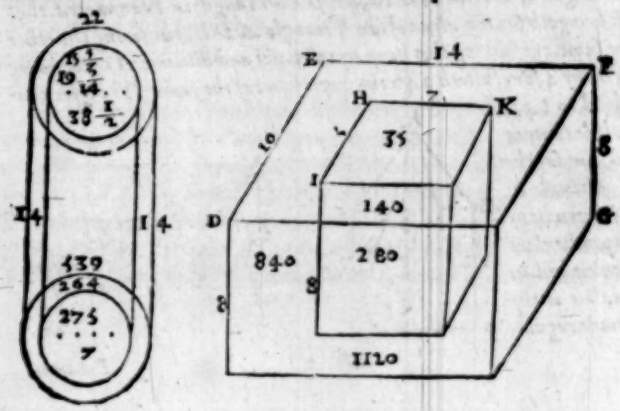
Multiplie the ſide of the Cone in it ſelfe, and likewiſe the Semidiameter of the Circular baſe, theſe deducted the one from the other the Quadrate roote of the Remainder is *Axis* or Altitude of the Cone.

The 4 Chapter

How excauate or holow timber, &c. is meaſured.

You ſhall by the Rules afore, ſearche the content or Craſſitude, as though it were not hollow, then meaſure the capacitie of that hollowe, the one ſubtracted from the other, the Remaine vttereth the magnitude of that excauate body.

Example.



Admit this rounde holowe Timber logge A B C were to bee meaſured, the heigth being 14 foote, the Diameter of the outwarde Circle 7 foote, the Dimetient of the inwarde Circle 5 foote, the Circumference of the bigger 22 foote, the content Super-

Superficiall or plaine of it 38 $\frac{1}{2}$ *the Circumference of the leſſe* 15 *foote and* $\frac{9}{14}$, *his plaine* 19 $\frac{9}{14}$. *Now multiply* 38 $\frac{1}{2}$ *in* 14. *ſo haue ye the craſſitude of the whole rounde peece of Timber which is* 539 *foote. Then augment* 19 $\frac{9}{14}$ *into* 14 *riſeth* 275 *foote, which ſubſtracted from* 539. *remaineth* 264 *ſo many foote is in the craſſitude about that hollowe piece of timber. Or thus ye may bring it to paſſe: Put* 19 $\frac{9}{14}$ *from* 38 $\frac{1}{2}$, *that is the one ſuperficiall content from the other and the remaine multiplie in the heigth* 14. *So haue ye as afore* 264 *foote. An other example of timber hollowed and ſomewhat faſhioned like vnto a troughe: Suppoſe it to be D E F G, whoſe outward bredth D E is* 10 *foote, the length E F* 14 *foote, the heigth F G* 8 *foote, the inward bredth of the hollow H I* 5 *foote, the length H K* 7 *foote, the heigth as before. Now multiplie* 10 *in* 14. *ſo haue ye* 140. *This augmented in* 8 *riſeth* 1120. *Then multiplie* 7 *in* 5, *that maketh* 35, *which augmented in* 8 *ſurmounteth* 280, *that ſumme by ſubtraction taken from* 1120 *leaueth* 840. *ſo many foote is the craſſitude of this hollowe troughe D E F G, euen thus of all other. By that which is ſpoken ye may gather howe to get the capacitie of all maner regular veſſels, and that is performed in multiplying the plaine or content ſuperficiall of the inwarde baſe or bottome in the profunditie or heigth.* Example 35 *the inward baſes plaine augmented in* 8, *the heigth produceth* 280. *So many cubicall feete is in the hollowe veſſell. Theſe* 280 *augmented in* 51, *bringeth foorth* 14280 *pints of water, farther diuided by* 8 *ſheweth* 1785 *gallons. Thus of all other, and that for liquour conteined in any maner veſſell.*

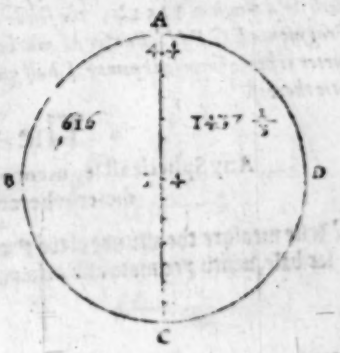
The 5 Chapter.

Howe the craſſitude and Superficies of a ſphere is to be meaſured.

FOr the Superficies yee ſhall multiplie the Diameter in his Circumference or gette the plaine of that Circle, as before in Planimetra I haue taught, and encreaſe it by 4. ſo haue yee the quantitie ſuperficiall, and that multiplied in the firt part of the diameter produceth the craſſitude. Likewiſe the diameter multiplied in his ſquare, and the offrome in 11, this product deuided by 21 produceth in the quotient alſo the ſolide content of the Sphere. Manie moe rules could I giue you for the like effect. But for better vnderſtanding of theſe, beholde the example enſuing.

Example.

A B C D the Globe whoſe Diameter A C is 14. *his Circumference being* 44. *the one multiplied by the other, bringeth* 616, *and that is the Superficies of the Globe which encreaſed by* 2 $\frac{1}{7}$ *bringeth* 1375 $\frac{1}{7}$ *and that is the ſolide content whiche is likewiſe thus produced* 14. *multiplied in his ſquare yeeldeth* 2744, *this againe in* 11, *maketh* 30184, *which diuided by* 21 *giueth in the Quotient* 1437 $\frac{1}{7}$, *the Craſſitude agreeable to the former worke.*



The

The 6 Chapter.

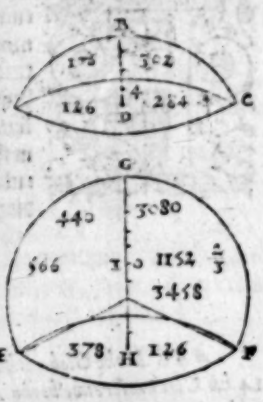
Howe fragmentes or partes of a Globe are measured.

YE shall augment the whole superficies of the Globe by the altitude or thicknesse of the Fragment, and the offcome deuide by the dimetient, your quotient is the Conuex Spherical superficies of the Fragmente, whereunto adding his Base or Circle ye produce the whole superficies.

For the crassitude if it be lesse than halfe the Globe ye shall first detracte the Altitude of the fragment from the Semidiameter of the Sphere, the remaine ye shall augmente by the Circular Base. This producte shall ye substracte from a number amounting by the multiplication of the Semidiameter in the conuex Spherical superficies of the fragmente, the thirde parte of the remaine is the crassitude or contente solide of that Fragment. But if the fragment be greater than halfe the Sphere, then shall ye deducte the Spheres semidimetient from the altitude of the Fragment and the remainder multiplied in the circular Base the producte must be added to the number produced by multiplication of the Semidiameter, in the conuex Spherical superficies of the Fragment, the thirde part of this resulting summe is the desired crassitude.

Example.

Admit A B C D a fragment of the former globe, whose diameter is 14. and that B D the Altitude thereof be 4, the spheres whole Superfices being 616, multipled by 4, bringeth 2464, and that deuided by 14, yeeldeth in the quotient 176, the conuex Superficies, whereunto if yee adioyne 126, the Area of the Base or circle, whose dimetient is A C, there amounteth 302, the whole Superficies of that portion A B C D. By the like working yee shall find the Superficies of E F G H 566. But for their crassitude I worke thus, In the lesser portion I finde the altitude 4, which deducted from 7, the spheres Semidimetient, leaueth 3, which augmented by 126, the bases Area bringeth 378, and this deducted from 1232 (a number produced by multiplication of the Spheres semidimetient in the conuex Superficies of the segment) leaueth 854, whose third parte is 284 1/3, so much is the solide contente of A B C D. But for the crassitude of the other portion, I augment 440 his conuex superficies in 7 there ariseth 3080. Likewise I multiplie 3. the difference betweene the semidiameter and the fragmentes altitude in 126, thereof commeth 378, these added together, make 3458, which deuided by 3, yeldeth for a quotient 1152 1/3, the solide crassitude of the other Fragmente E G F H. But for as muche as the Spheres Diameter is heere supposed knowne, I shall giue you a rule readily by supputation to learne the quantitie thereof.



The 7 Chapter.

Any Spherical segment propounded to atteyne the spheres diameter whereof it is the fragment.

FIrste measure the altitude of the Fragment: Secondlie, the Semidiameter of the circular base, which yee maie also attaine by knowledge of his circumference as was taught in

in *Planimetra*, then ſquare the Semidiameter, and diuide the product by the fragments altitude, the Quotient added to your Diuiſor yeelds the ſpheres Diameter. Or thus, adde the ſquare of the Altitude to the ſquare of the Semidimetient, and diuide that product by the Altitude, ſo will your quotient expreſſe the Diameters quantitie.

Example.

A B C D the Spherical Segment his *Altitude B D* 4, the *Semidiater* of his circular *Baſes A D* nigh $6\frac{1}{2}$, the Square thereof is 40, which diuided by 4, yeelds in the *Quotient* 10, which added to 4, bringeth 14 the *Diameter* of the Sphere. Likewiſe the Square of 4 is 16, added to 40 the Square of *A D* bringeth 56, which diuided by 4, giueth for the *Quotient* 14, the Spheres, Dimetient agreeing with the former working.

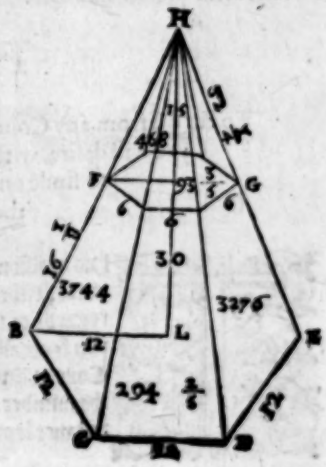
The 8 Chapter,

Howe a Pyramis cutte off, may bee meaſured.

IF the *Pyramis* be vnperfect, yea cutte off in the toppe, continue it by rule and line wittily layde to the two contrarie ſides, and where the ioyning and common meeting is there that *Pyramis* is whole and perfect, then meaſure that whole by the Arte afore, and alſo the *Pyramis* going from the toppe of the vnperfect to the common meeting. This doone, yee ſhall ſubtract the Craſſitude of the leſſe from the whole continued *Pyramis*, ſo the Remaine without doubt is the magnitude of the vnperfect *Pyramis*.

Example.

Let A B C D E repreſent an Hexagonall imperfect Pyramis encompaſſed with two Equiangle Hexagonall Parallele plaines and 6 Iſoſcheles Trapezia, euery of their Baſes being the ſides of the greater Hexagonum 12, *euery ſide of the leſſer Hexagonum* 6. *Now, applying rules or lines vnto the ſides B F, G E, I finde them to concurre at H. Thus haue you two pyramides, the greater hauing his Baſe an Equiangle Hexagonum, whoſe ſide is* 12: *the other is the pyramis H F G, whoſe baſe is the Hexagonum F G, euery of his ſides being* 6, *the Altitude of the leſſer pyramis is* 15 *the Altitude of the greater* 30. *Alſo by the Rules giuen in* Planimetra, *yee ſhall find the Area of the leſſer Hexagonum* $93\frac{3}{4}$, *the Area of the greater* $294\frac{3}{4}$, *and ſo conſequently by multiplication of them in the third part of their altitudes as was taught before in the menſuration of whole pyramides, the Solide content of the leſſer will be* 468, *the Craſſitude of the greater is* 3744, *the one deducted from the other, leueth* 3276, *the magnitude of that vnperfect pyramis F B C D E.*



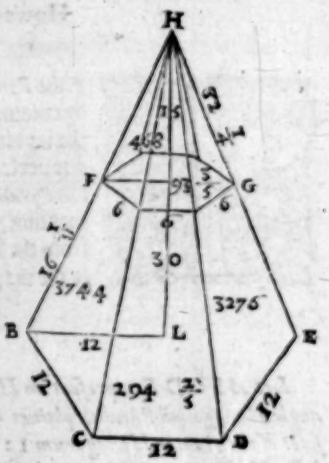
A note

A note howe by supputation to get the Perpendiculares or Altitudes of those Pyramides.

FOr that it were tedious and painefull in a Solide of great quantitie, by rule or line to searche out these Perpendiculares, yee shall by Arithmetike attaine them thus: multiplie the greater Hexagonall side by the side of the vnperfect Pyramis, and the product diuide by the difference or ouerplus of the sides Hexagonall, the Quotient sheweth the side of the whole great Pyramis, from whose Square (if yee deduct the Square of the Circles Semidiameter that encloseth the greater Hexagonum which yee were taught in *Planimetra* how to finde) the roote Quadrate of the Remainder is the Altitude of the greater Pyramis, which augmented againe by the lesser Hexagonall side, and the offcome diuided by the greater, bringeth in the Quotient, the altitude of the lesser Pyramis.

Example.

The vnperfect Pyramis side F B being nighe 16 $\frac{1}{4}$, *I multiplie by* 12 *the greater Hexagonall side, there ariseth* 193 $\frac{1}{2}$, *and that diuided by* 6, *the lesser side Hexagonall produceth in the Quotient* 32 $\frac{1}{2}$, *the Pyramidall side H B, from the Square thereof, if yee deduct the Square of B L, the circles Semidiameter that encludeth the greater Hexagonum being* 144, *there will remaine* 900, *whose roote is* 30, *the altitude of the greater Pyramis, which augmented by* 6, *the lesser Hexagonall side, and diuided by* 12, *produceth in the Quotient* 15 *the Altitude of the lesser Pyramis. These lines for that they are some of them incommensurable, cannot exactly bee expressed, saue onely in surde numbers: but so nighe as is requisite for any Mechanicall mensuration, this operation declareth them: such as are expert in Algebra by the former rules with irrationall numbers may precisely shew their quantitie. The very like operation is required in perfect and vnperfect Cones, for measuring their Perpendiculars and Crassitude. Whereof to the ingenious there needs no other Example.*



The 9 Chapter.

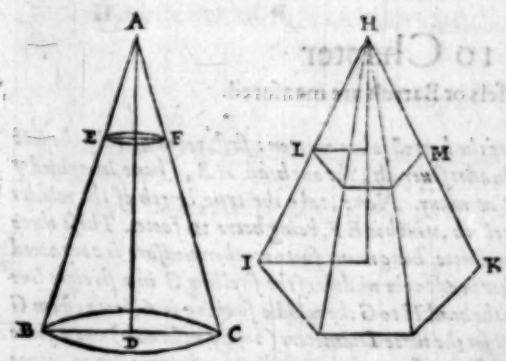
To cut off from any Cone or Pyramis what parte or portion thereof yee will desire, with a plaine Equedistant to the base, and to finde on what parte of the Solides side the Section shall fall.

THis diuision of Cones and Pyramides is in two respects to bee made, either in consideration of their contents Superficiall, or in regarde of their Solide quantities, for either of them I shall giue you seuerall rules. First, it behoueth yee to measure the side of the Cone or Pyramis, that side shall yee diuide by the quadrate roote of the number expressing the parte assigned, your quotient is the line or distance from the toppe of the Cone or Pyramis, to the Section or

place

place where the plaine Parallele to the baſe ſhall paſſe, which will diuide or cut off from the aſſigned Solides Superficies the deſired parte. The ſame line or Diſtance is alſo found with diuiſion of the Cones ſide ſquared by the number expreſſing the part to be cut off, for the roote quadrate of the Quotient ſhall be your deſire. But if yee would from any Cone or Pyramis in like manner make ſeparation, in ſuch ſorte that their Solide Contentes might retaine like proportion, then ſhall yee multiplie the ſide of the Cone or Pyramis, firſt in himſelfe, and then againe in the offcome, and this laſt product ſhall yee diuide by the number expreſſing the parte ye would ſeperate, the roote Cubicall of your Quotient is the ſide of the leſſer Cone or Pyramis to be cut off from the toppe of the greater.

Example.

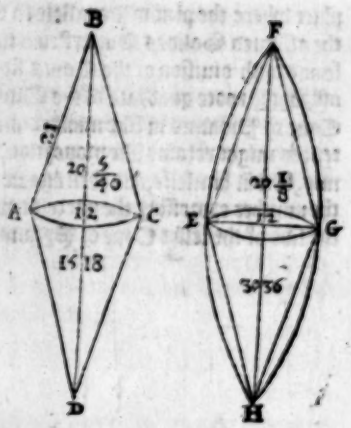


Admit A B C the Cone, from whoſe ſummitie or top, I would cut off a portion, for example, the Cone A E F whoſe Superficies ſhould be the 9 parte of the great Cone A B C, his Superficies. I take therefore the roote Quadrate of 9, that is 3, wherewith I diuide 100 *the ſide of the great Cone there ariſeth in the Quotient* $33\frac{1}{3}$ *the line A F or ſide of the leſſer Cone, the ſame number I produce alſo by the ſecond rule thus, I diuide* 10000 *the Square of A C, the Cones ſide by* 9, *the Quotient yeeldeth* $1111\frac{1}{9}$, *whoſe roote is likewiſe* $33\frac{1}{3}$, *the ſide A F. But for operation in Solides, I ſhall giue you an other Example: Suppoſe H K the ſide of the Pyramis H I K* 120, *from the which Pyramis I would cut off an other portion, for example, H L M that might be an eight part of the great Pyramis H I K, the number of the ſide* 120, *I multiplie in it ſelfe thereof ariſeth* 14400, *which augmented againe by* 120, *maketh* 1728000, *and this diuided by* 8 (*for that is the number wherewith the part is expreſſed*) *I finde in the Quotient* 216000, *whoſe roote Cubike is* 60, *and that is the length of H M or ſide of the Pyramis H L M, whoſe Solide content is the eight part exactly of the great Pyramis H I K. Euen ſo ſhall yee doe in all other regulare Pyramides, how many equall ſides ſoeuer his baſe haue: but if the ſides of his baſe be vnequall, and ſo conſequently the ſides of the Pyramis alſo vnequall then ſhall yee worke with euery ſide ſeuerally as I haue by example ſhewed ye in one: or elſe worke by the rule of three, for all the ſides are proportionally diuided: this to the wittie will ſuffice.*

Heere followeth other faſhioned bodies ſomewhat ſtrange in figure, but in effect thoſe that to fore are meaſured.

THe firſt A B C D appeareth to be two round Pyramides ioyned togither, whoſe Diameter A C is as the round Pyramis afore meaſured 12 *foote, the height or perpendiculare line* 20 *foote and* $\frac{1}{20}$, *the ſide* 21, *wherefore I conclude the Craſſitude of the one parte or halfe of it to be euen as the round Pyramis before meaſured, that is* 759 *foote, which doubled, make* 1518, *the*

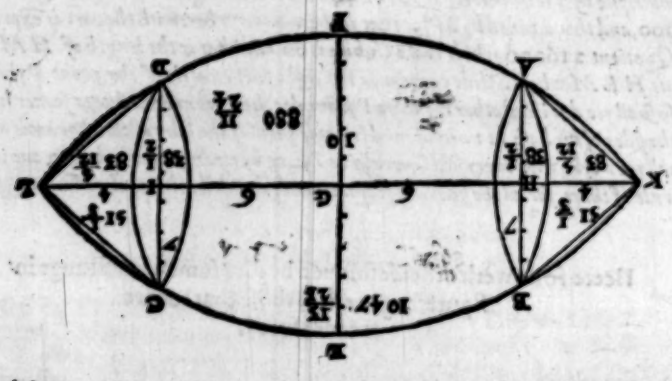
whole content of this Figure, the other Figure enſuing is like to an Egge, and not very vnlike to the Figure aboue. Wherefore to meaſure him, yee ſhall worke as yee did in the other, the Craſſitude of that ſo had, and then doubled, bringeth the whole content of him. Alſo this Figure E F G H is like in content to the other A B C D, according to that forme whoſe Craſſitude is 1518, this doubled maketh 3036 foote, ſo much ye may affirme the Craſſitude or quantitie of that figure like vnto an Egge. Now to get the quantitie ſuperficiall of him, yee ſhall multiplie the Arke F E H in the halfe Circumference of the Circle whoſe Diameter is E G, or the whole Circumference in halfe the arke, ſo haue yee the Superficies.



The 10 Chapter

How Wine Veſſels or Barrels are meaſured.

Vppoſe A B C D were the barrell to bee motten: firſt, yee muſt take the iuſt meaſure of both the heades ſeuerally, the one head A B, I haue imagined 7 foote, the other C D as many. Nowe, take the true heigth of the middes where the barrell ſwelleth, which is E F, being heere 10 foote. Theſe three Diameters keepe, then note howe many foote or other meaſure is contained from the mids of either head to the middle of the ſwelling G in a ſtreight line moten within the Barrell. Behold from the head H to G the middle ſwelling is 6. foote, from G to this letter I as manye: nowe yee muſt ſet the three Diameters (tofore reſerued) vpon ſome carde, paper, or other plaine, the one differing from other according to their meaſures, as yee may ſee in the Figure, then encloſe them with an Arke on both ſides, cutting the middle line, which line croſſeth the Diameter: Squirewiſe in the middle, the one Arke is K B E C L, the other is K A F D L, the arkes drawne as yee ſee, lo a figure is made like vnto an Egge:



Truely all ſuch faſhioned figures I haue taught you to meaſure immediatly afore. In him ye may find 1047 11/11, now if ye pull away by any art from this whole ſum the number of feet conteyned in the peeces of the figure ſuperfluous & more than the barrell at either end, then the magnitude or capacitie of that veſſell muſt needs remaine, as by the Example is plainely perceiued, although it would ſeeme I had entreated ſufficiently, yet I thinke it worthie of remembrance to tell you how to ſearch the content of theſe peeces at the ends of the figure, I muſt ſuppoſe that ye know the meaſuring of a round Pyramis which is entreted before, imagine the diameter B A which is 7 foot to be the baſe, K H

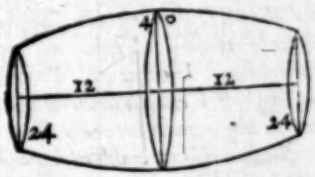
the heigth of the peece heere imagined a Pyramis: *now if yee (as is before mentioned) multiply the plaine of the base in the third parte of the heigth, the content of that figure will come according to that fashioned meeting, which is* $51\frac{1}{7}$ *with this number by the precepts of proportion yee shall worke as followeth: first know the length of H L that is* 16 *foote, againe G L being* 10 *foot, both these added make* 26. *Now say* 16 *giueth* 26, *what shall* $51\frac{1}{7}$ *being, so haue yee* $83\frac{1}{13}$. *so many foote is the peece K B A, and the other C D L as many, which added make* $166\frac{2}{13}$ *pull this out of the number conteyned in the whole Figure afore found, which is* 1047 *so haue yee left* $880\frac{11}{14}$ *the capacitie of your Barrell. Thus the learned and famous* Archimedes *hath taught in the* 31 *proposition of his booke* de Sphæroibus & Conoidibus.

An other mensuration of Vessels more common.

SOme vse when the base plaine of the extreames and middes of any Vessell are not one, to subtract the lesse from the bigger Superficies, the medietie that remaineth deduct from the bigge plaine: then whatsoeuer is left adde to the lesser Superfitices, the halfe of the product sheweth the true base to be multiplyed in the heigth, of the which commeth the contente.

Example.

THe Plaine of the bigger Superficies conteyneth 40 *foote, the lesser* 24, *the diffrence is* 16. *the halfe* 8 *subtracted from* 40, *leaueth* 32, *this added to the lesse Superficies* 24 *make* 56. *the halfe is* 28 *which multiplyed in* 24 *bringeth* 672, *the whole content. Lo the figure.*



Another note of measuring.

IF the Diameter of the Vessell bee in the middes 10 foote, but at the ends eight, adde them, and take halfe, so haue yee nine, thus nine foote shall be thy Diameter, whose plaine or base (as before) multiplied in the Longitude or heigth, bringeth a true contente. Note well this kinde for pillers and other bodies, when they haue Diameters or Bases not agreeable.

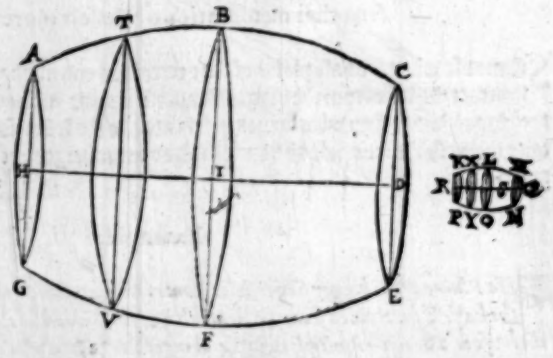
The 11 Chapter,

One rule generall exactly to measure all kinde of wine Vessels.

FOrasmuch as there are sundrie kindes of wine Vessels, as the tun, the Pipe, the Punchion, Hogsheads, Buttes, Barrels, &c. euery of them differing from other aswell in quantitie as in fashion, to teache seuerall rules for euery sorte it were ouer tedious, leauing therefore manifolde precepts and examples that I might in this case prescribe, I shall for breuitie sake set foorth one onelye rule generall, whereby yee shall be able with the aide of a small proportionall Vessell, not onely to measure the whole capacitie of any manner Wine

Veſſell, but alſo the defaulte and quantitie of licquoure therein conteyned, when it is partly emptie. And firſt I will beginne with this ſmall proportionall veſſell, which it behoueth you to procure by ſome ſkilfull Artificer, ſo made that their lengths retaine the ſame proportion that their Circumferences or Circles, aſwell at the ends and middes, as alſo at other like and proportionall diſtances taken in the length of either Veſſell: as by the example following ſhall more plainelie appeare.

Example.



Admit S and I the two Veſſels, I the greater, S the leſſer: now to examine whether theſe 2 be proportional I ſearche firſt their lengths, finding H D 50 inches. and R Q 10 inches, likewiſe I ſearche the longitude of euery depth or diameter, that is to ſay, B F and L O the diameter or depths of the middell, and A G, C E, K P, M N, the Circles of the endes, now if B F, A G and C E retaine the ſame proportion to K P, L O, M N, (euery one particularly compared to his like) that H D the length of the greater doth to R Q the longitude of the leſſer, then are thoſe Veſſels likely proportionall: but for greater certeintie (becauſe errour maye growe vpon ſmall difference) it were requiſite alſo to meaſure in either of them one other Circumference or hoope, as for example in the greater, the Circle paſſing by T V exactly in the middle betweene A G and B F, and in the leſſer the Circle X Y ſituate in like manner betweene K P and L O. Now if yee finde the proportion of theſe two Circumferences alſo agreeable with the former lengthes and Circumferences, ye may aſſuredly gather the ſe veſſels to be proportionall. But if yee finde anye diſcrepance or variance betweene them, yee ſhall by the aide of ſome skilfull Artificer, reforme it in the leſſer, till yee haue brought it fully agreeing with the proportion of the greater, which is readily prooued by the rule of proportion. Whereof I minde not heere to vſe mo wordes, leaſt in ſeeking nee deleſſe plaineſſe, I become ouer tedious: your ſmall veſſell thus prepared, ye ſhall in the next chapter be taught how to vſe it,

The 12 Chapter,

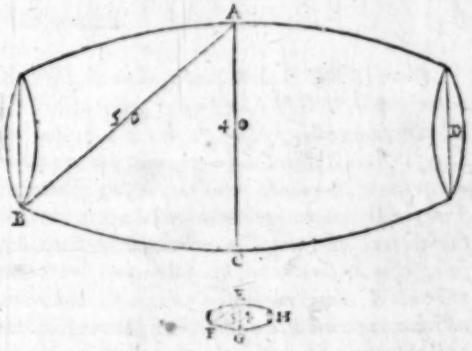
How by this ſmall prepared Veſſell to meaſure the quantie of the greater.

Henſoeuer yee will meaſure anye manner of Wine Veſſels, it behoueth you to conſider what ſorte they are of, and accordingly to prepare your leſſer proportionall veſſell, then ſhall yee in either of them meaſure the profunditie or greateſt Diameter, and alſo the Diagonall or croſſe lines from the bung holes to the oppoſite or loweſt part of either baſe, theſe diagonall lines yee ſhall ſquare, and the products ſeuerally multiply in his correſpondent Diameter, of theſe ſurmounting ſummes yee ſhall diuide the greater by the leſſer, your Quotient openeth how many times the leſſer Veſſell is conteyned in the greater, or if yee augment the greater of thoſe laſt ſurmounting ſummes by the Craſſitude of the leſſer veſſell, and diuide by the leſſer ſumme, your Quotient will declare the Solide content of the greater Veſſell.

Example

Example.

Suppose A B C D the Wine vessell, whose capacitie I would knowe E F G H my lesser proportionall vessell, first I measure the Diameters E G and A C. Admit the first 4, the other 40, againe I mete the Diagonall lines. Admit I finde A B 50 E F 5, the Square of 4 is 16, which augmented by 5, bringeth 80, the Square of 40 is 1600, & that augmented by 50, yeeldeth 8000, which diuided by 80, bringeth in the Quotient 1000, so many times is the lesser conteyned in the greater. Now to learne how many pottles or gallons is conteined in that great Vessell, I trie first how much liquour my little prepared Vessell will holde: Admit it conteine ¼ of a pinte, this augmented by 4000 produceth 60000, which diuided by 80, bringeth 750, so many pintes yee may conclude in that great Wine vessell which reduced to Gallons diuiding by 8 yeeldeth 93 Gallons 3 quartes, the exact quantitie of liquour that such a Vessell will conteine, but if it happen that your Vessell be not throughly filled, and that ye desire as well to know how much liquor would suffise to fill it, as also what quantitie is therein conteyned, ye shall in the next Chapiter receiue therein perfect instruction.



The 13 Chapter.

How both the liquour and default or emptinesse in Wine Vessels partelye filled is to bee moten.

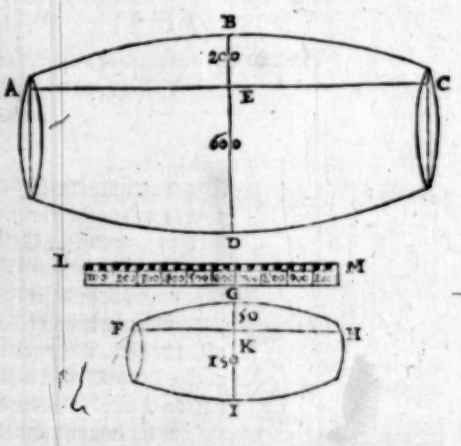
FOr greater exactnesse yee shall prouide a fine streight rodde of 4 or 5 foote in lengthe, exactly diuided into 1000 equall portions. Then shall yee moue the Vessell whose liquoure or default yee woulde measure, till it lie leuell the bung hole directly vpwarde, this doone take your rodde diuided as is before declared, and let it descende perpendicularly down thorough the bung, till it come to the bottom of the vessell. This done, note what part the bung or vpmost heigth of the Wine Vessell will touch of the rod, and likewise howe many partes thereof is wette with the liquour, and laste of all howe many partes are conteyned betweene the liquour and the bung hole. The first I call the Diameter or profundite of the Vessell, the second the parts of liquour, and the third shall bee named (for distinction) the partes of default or emptinesse. This doone yee shall also measure with your diuided rodde how many partes the Diameter of your lesser vessell conteyneth, these partes augmented by the partes of liquour before founde, and the product diuided by the diameter of the Wine Vessell, yeeldeth in the Quotient the partes of liquour, for your little prepared Vessell, which it behooueth yee so to situate that it maye also lye leuell as did the greater vessell, then powre in so muche Water, that by prooving with your diuided rodde yee maye finde the partes of liquour exactlye agreeing with your former quotient:

This

This doone augment the Diameter of the Wine veſſell Cubically, that is to ſay, by his owne Square, and the product in the Quantitie of liquor now being in your little prepared Veſſell, the product diuided by the Cube of your leſſer Veſſels Dimetient yeeldeth in the Quotient the true quantitie of liquor contained in the Wine Veſſell, and that againe deducted from the whole content of the Veſſell found, as was in the former Chapters declared, leaueth the default or emptineſſe, that is, how many gallons or other meaſures is requiſite to fill vp the ſaid Veſſell.

Example.

Admit A B C D the Wine Veſſell partlie filled, A D C E the liquor, A B C E the defaulte or emptineſſe, F G H I my ſmaller proportionall Veſſell prepared as I haue tofore in the laſt Chapter declared, L M the ſtraight rodde diuided into 1000 parts, which being let perpendicularly fall from B till it touch the oppoſite ſide at D, I finde B D 800 partes, E D that is ſo much as is wette of the rodde 600 partes, likewiſe placing the rodde in my leſſer veſſell at G, admit I finde the Diameter or profunditie G I 200 partes, this number multiplyed by 600, the partes of fulneſſe yeeldeth 120000, which diuided by 800, bringeth in the Quotient 150, the partes of fulneſſe for my ſmaller veſſell: filling therefore the proportionall ſmall Veſſell with liquor till it riſe vnto K, that is to the 150 parte in my diuided rodde L M, finally by meaſure I make triall what quantitie of liquour is in my little veſſell, Admit it 3 pottles: Nowe to learne the quantitie of liquor conteyned in the great, firſt I multiplie B D the greater Diameter being 800 in in his Square, thereof ariſeth 512000000, and that againe by 3, ſo haue I 1536000000, which diuided by 8000000 (the Cube of G I the leſſe Dimetient) produceth 192. ſo many pottles is there in that wine veſſell. Nowe if yee deſire to knowe howe manye pottles or Gallons more will fill it full, yee may by the laſt Chapter ſearche how many Gallons the whole veſſell will conteyne, and from that withdrawe 96 Gallons, the quantitie of liquor alreadie therein conteyned, the remaine is your deſire. I thinke it not neceſſarie in this caſe to adioyne anye farther Example, for that this was in the former Chapter plainelye both by rule and example already declared. This kinde of menſuration ſerueth not onely for wine veſſels, whereof there are ſundrie formes, but alſo for all manner bodies, what kinde of faſhion ſoeuer they be of, and likewiſe for their fragments or partes, and it is grounded vpon this Theoreme. All like Solides retaine among themſelues triple proportion of their like or correſpondent ſides, that is to ſay, looke what proportion the Cube of one ſide in the leſſer Solide, retaineth to the Cube of his correſpondent ſide in the greater Solide, the ſame proportion doth the leſſer body retaine to the greater: ſo that with the ayde of the golden rule the firſte three being knowne, the fourth is readily founde: And as this rule ſerueth for the whole bodies, ſo is it alſo to be applyed to all fragments or partes: ſo that the Superficies wherwith the Solides are diuided be like, and make equall Angles with the ſides and Superficies of eyther Solides, for thereupon it muſt conſequently enſue, that theſe fragments correſpondentlye compared are alſo proportionall.



Thus much I thought to adioyne touching the reaſon and demonſtration of this rule, whereby

whereby the ingenious practizer might the better retaine in memorie the operation, and also be able to applie the same to sundry other vses not heere mentioned if occasion be offered. And for as much as this kinde of measuring dependeth wholy vpon the small proportionall Vessell, whose quantitie is supposed knowne, I thinke it not amisse to giue one rule generall for all small vessels how irregular soeuer they be (their fashion not regarded) practize to finde their Solide quantities.

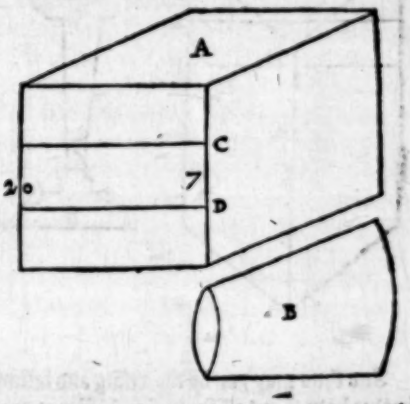
The 14 Chapter.

To measure exactly the Solide content of any small body, how disordred or irregular soeuer it be, the forme or fashion not regarded.

WE shall prepare a hollow vessell of Cubicall forme so large, that it may contein the small irregular body, which being placed therein, yee shall poure in so much water that it couer altogither the body, then make a marke where the *Superficies* or vpmost part of the water toucheth. This done take out the same irregular body, and marke againe directly vnder the former where the brimme of the water now toucheth, for the distance of these two markes multiplied by the Square of the Cubes side, produceth the Crassitude of that irregular body.

Example.

Admit A the Cubicall hollowe vessell, whose inward side I suppose 20 inches, B is the irregular body, whose Crassitude I desire, first therefore I put the Solide into my hollowe Cube, and powring in water till it be thoroughly couered. Admit the brim of the water reach vnto C, then taking out that irregular bodye againe: admit the Superficies of the water fall to D, I measure the distance betweene C and D, suppose it 7 inches, which multiplied in 400, the Square of the Cubes side produceth 2800, so many cubicall inches are conteined in that irregular body B. Neither is it of great importance whether your vessell A be an exact Cube. For what kind of Prisma soeuer it be, alwayes rising vniformely, and his Paralelogrammes being Perpendicular vpon their base, ye may (changing the forme of your Calculation) bring the same to passe, onely whereas in the Cube ye did multiply his sides Square in the distance betweene the water markes, yee shall now multiply the same distance or difference of water markes in the base of the Prisma, the resulting summe is the irregular Solides Crassitude. And thus may you alwayes frame your containing vessell, according to the forme and quantitie of the irregular body that ye desire to measure. Meruellous is the appliance of this kinde of mensurations, and strange conclusions may be performed thereby, wherein although I meane not in this treatise to reueale any secrets, reseruing them for an other place, yet to giue some light to the ingenious to proceede in applying them farder, I shall not thinke it tedious to shewe how it may be vsed to discouer the waight of such things, as no way possibly by ballance may be found.



The 15 Chapter.

How the waighte of any parte or portion of a Solide body may be knowne without seperation thereof from the bodye, whereby it moughte be paised or waighed in Ballance.

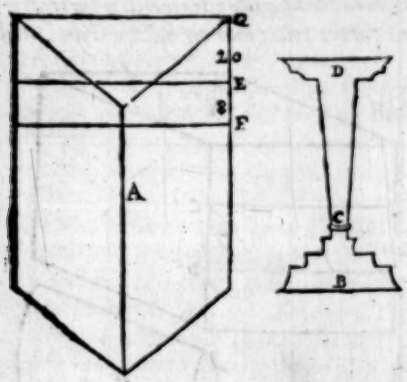
FOr as much as neither by common Ballance, neither by that kinde of Ballance which the *Italians* vse, called Statarea, nor anye other hitherto inuented, the waighte of anie Fragmente or parte of Solide bodie maie bee knowen without Seperatinge

or

or cutting that Fragment off from the whole bodie, whereby it may by it selfe alone bee paised in the Ballance, I thought it not amisse aswell for the rarenesse of the matter, as for the necessarie vses thereof, to set foorthe this meane of searching waighte by water, with the ayde of Arithmetike: your Vessell therefore being prepared as is toforе declared, whether it be Cube or Rectangular Prisma it forceth not, yee shall first fill it full with water, and throwe the Solide body thereinto, then softly lift that body out of the water till such time as there remaine no more in the water than that portion, whose waight yee desire to knowe, at that instant make a marke on one side of the Vessell where the Superficies of the water then toucheth, then take out the bodie altogether. This doone, measure the Distance from your marke to the Superficies of the water as it is now after the body is taken quite out. Likewise measure the distance of the waters Superficies from the toppe of the vessell. This doone, augment the waight of the whole body by the lesser distance, and diuide by the greater, your Quotient will shewe the true waight of the Fragment or portion.

Example.

Admit B C D a piller of an hundred pound in waight, being of Bras, Iron, Siluer, or anye other Metall, my desire is to knowe the waight of the foote or portion from B to C, first therefore



putting the hole piller into the vessell A, I fill it full of water, then lifting it softly vp till all the piller be out of the water, saue onely the foote or Fragment B C, I finde the Superficies of the water fallen to E, then doe I lift out the whole piller, leauing no parte thereof within the Vessell, admit now the water salne vnto F, and that by measuring I finde E F 8 inches, and G F 20 inches, 8 multiplyed in 100 the whole pillers waighte yeeldeth 800, which diuided by 20 (the greater distance) from the top of the Vessell to the lowest water marke, bringeth in the Quotient 40 so many pound waight, I conclude the pillers foote or portion B C.

And thus may you by the rising and falling of the water with the aide of Arithmetike, knowe how to cut off from this Piller or any other bodye what portion, quantitie or waight yee will prescribe, alway it is supposed that it is *Corpus Homogeneum*, that is to saye of matter and kinde of substance equally dispersed through out: otherwise if it be *Corpus Heterogeneum*, that is to say of partes vnlike in substance, it requireth more curious calculation, which I reserue for an other place, minding heere onelye to make an Introduction for the wittie to proceede farther, and not at large to disclose such secrets as may be shewed heerin. By this meanes did *Archimedes* finde the exact quantitie of euery seuerall mettall that was in the Kings Crowne at *Syracusa*, without opening or breaking any parte thereof: and many more and no lesse straunge conclusions may be done thereby.

The ende of the third Booke called
Stereometria.

A Mathematicall Diſcourſe of the 5 Platonicall Regular Solides, and their Metamorphoſis into 5 other compound rare Geometricall bodies: With an hundred Theoremes at leaſt of the Authors inuention, neuer mentioned before by anye other Geometrician.

The Preface.

ALthough the Rules and Preceptes already giuen abundantlie maie ſuffice for the exact menſuration of any manner Solides or veſſelles that are vſually occupied, or maie be proponed to be moten, yet for the ſatisfaction alſo of ſuch as deliting in matters onlie new, rare & difficile, ſeeke to reache aboue the common ſorte, I haue thought good to adioyne this Treatiſe of the 5 *Platonicall* bodies, meaninge not to diſcourſe of their ſecrete or myſticall appliances to the Elementall regions and frame of Cœleſtiall Spheres, as thinges remote and farre diſtant from the Methode, nature and certaintie of Geometricall demonſtration, only heere I intende Mathematically to conferre the Superficiall and Solide capacities of theſe Regular bodies with their Circumſcribing or inſcribed ſpheres or Solides, and Geometrically by *Algebraycal* Calculations to ſearchout the ſides, Diameters, *Axes*, Altitudes and lines *Diagonal*, together with the Semidimetients of the Equiangle Baſes, containing or contained Circles, and as in the Problemes ye ſhall perceiue by Precepts and Examples, the quantities and Proportions of all theſe lines, Superficies and Craſſitudes, with numbers Rationall and Radicall expreſſed, ſo haue I by *Theoremes* ſowne the ſeede of rules innumerable, wherein the ſtudious may delite himſelfe with infinite varietie: Finally I ſhall in the laſte Chapiter by Diffinitions and *Theoremes* partly ſet forth the forme, nature and proportion of other fiue vniforme Geometricall Solides, created by the transformation of the fiue bodies Regulare or Platonicall: but not in ſo ample manner, as the noueltie or difficultie of the matter requireth, meaning as I ſee theſe the firſt frutes of my ſtudies liked and accepted, to beſtowe time and trauaile vpon an other peculiar Volume, which ſhall containe, not onely the demonſtrations of theſe Theorems of Spherall ſolides, but alſo of *Conoydall*, *Parabolicall*, *Hyperbolicall* and *Ellepleyſall* circumſcribed and inſcribed bodies, and manie moe lines and ſolides produced by the ſections of theſe, and reuolution of their ſections. But to returne to this preſent Treatiſe, let no man muſe that writing in the Engliſh toung, I haue retained the Latin or Greeke names of ſundry lines and figures, as cordes Pentagonall, lines Diagonall, Icoſaedron, Dodecaedron or ſuch like, for as the Romanes and other Latin writers, notwithſtandinge the copiouſe and abundant eloquence of their tongue, haue not ſhamed to borrow of the Grecians theſe and many other termes of arte: ſo ſurely do I thinke it no reproche, either to the Engliſh tongue, or any Engliſh writer, where fit words faile to borowe of them both, but rather ſhould we ſeeme thereby to do them great iniurie, theſe beeing in deede certaine teſtimonies and memorials where ſuch ſciences firſt tooke

their originall, and in what languages and countries they chiefllie floriſhed, which names or wordes how ſtraunge ſoeuer they ſeeme at the firſte acquaintaunce, by vſe wyll growe as familiar as theſe, a Triangle, a circle, or ſuch like, which by cuſtome and continuance ſeeme meere Engliſh, yet to auoide all obſcuritie that maie growe by the noueltie of them, I haue adioined euery of their diffinitions, and ſo proceeded to Problems and Theoremes with ſuch methode, as howe obſcure or harde ſoeuer they appeare at the firſt, through the rareneſſe of the matter: I doubt not but by orderly reading, the ingenious Student, hauing any meane taſte of coſſicall numbers, ſhall find them plaine and eaſie.

Diffinitions.

PReportion is a mutuall or enterchangeable relation of two magnitudes, being of one kind, compared together in reſpect of their quantities.

The ſecond diffinition.

When the preportion of two magnitudes is ſuch as maie be expreſſed with numbers, then it it certaine and apparant and here is called rationall: But when the proportion is ſuch as cannot be expreſſed with numbers, but with their rootes onelie, then is that proportion certaine alſo, but not apparante, and therefore here I name it ſurde or irrationall.

The third diffinition.

When there be three ſuch magnituds or quantities that the firſt to the ſecond retaine the ſame proportion that the ſecond doth to the third, thoſe quantities are ſaid to be proportional, and the firſt to the third retaineth double the proportion of the firſt to the ſecond, and the ſecond is named meane proportionall betweene the firſt and the laſt.

The fourth diffinition.

When foure magnitudes are likewiſe in continuall proportion, the firſt and the fourth are the extremes, and the ſecond and third the meanes, and the extreames are ſaid to haue triple the proportion of the meanes.

The fifte diffinition.

Anie line or number is ſaide to be diuided by extreame and meane proportion, when the diuiſion or ſection is ſuch or ſo placed, that the whole line or number retaine the ſame porportion to the greater parte, that the greater doth to the leſſer.

The ſixth diffinition.

A line is ſaide to be equall in power with two or moe lines, when his ſquare is equall to all their ſquares.

The ſeuenth diffinition.

A line is ſaid to matche a ſuperficies in power, when the ſquare of that line is equall to the ſuperficies.

The eight diffinition.

When anie equiangle triangle, ſquare, or Pentagonum is in ſuch ſorte deſcribed within a circle, that euerie of their angles touche the circumference, their ſides are called the trigonall, Tetragonall and pentagonall Cordes of that circle.

The ninth diffinition.

About euerie equilater Triangle, Square, or Pentagonum, a circle maie be deſcribed, preciſelie touching euerie of thoſe figures, and that circle ſhall bee called the circumſcribinge or containing circle.

The

The tenth diffinition.

Also within euery of these Equiangle Figures a Circle may be drawne, not cutting but only touching euery of their sides, this is called the inscribed Circle.

The eleuenth diffinition.

Any right line drawne from Angle to Angle in those Equiangle figures passing through the Superficies, I name the line Diagonall.

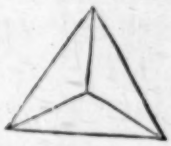
The twelfth diffinition.

A right line falling from any Angle of these Superficies Perpendicularly to the opposite side shall be named that plaines Perpendiculare.

The 13 Diffinition.

Tetraedron *is a body Geometricall, encompassed with foure equall Equiangle Triangles.*

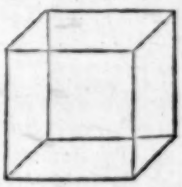
Tetraedrum.



The 14 Diffinition.

Hexaedron *or* Cubus *is a Solide Figure, enclosed with sixe equall Squares.*

Cubus.



The 15 Diffinition.

Octaedron *is a body comprehended of eight equall Equiangle Triangles.*

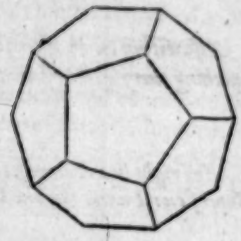
Octahedrum

The 16 Diffinition.

Icosaedron *is a Solide Figure, vnder twentie equall Equiangle Triangles conteyned.*

Icosaedrum

Dodecaedron.



The 17 Diffinition.

Dodecaedron *is a Solide comprehended of twelue equall Equiangle Pentagonall Superficies.*

The 18 diffinition.

These fiue bodies are called Regular, and about euery of them a Sphere may bee described, that shall with his concaue Peripherie exactly touch euery of their Solide Angles, and it shall be called their comprehending or Circumscribing Sphere or Globe, and these Solides shall be called the inscribed or conteyned bodyes of that Sphere.

The 19 diffinition.

Also within euery of these regulare bodyes a Sphere may be described that shall with his conuex Superficies precisely touche all the Centres of those Equiangle Figures wherewith these bodies are enuironed, and such a one I terme their inscribed or conteyned Sphere, and those bodyes shall be termed the circumscribing Solides of that Sphere.

The 20 diffinition.

The Semidiameter of this inscribed Sphere, forasmuch as it is the very Axis *or* Kathetus *of euery* Pyramis *hauing his base one of the Equiangle plaines, and concurring in the Centre, of which* Pyramides *(intellectually conceiued) these bodies seeme to be compounded, it shall be named the* Axis *or* Kathetus *of that body.*

The 21 diffinition.

Euery of these bodies side, I call any one of those equall right lines wherewith these Equiangle Figures are enuironed that comprehend and include these bodies.

The 22 diffinition.

Any one of the Figures wherewith these Solides bee enuironed is called the base of that Solide.

The 23 diffinition.

A line falling from any Solide Angle of these bodyes Perpendicularlye on the oppoſite plaine or base, shall be named that Solides Perpendiculare.

The 24 diffinition.

The power of any line giuen is sayd to be diuided into lines retaining extreame and meane proportion, when two such lines are found as both make their Squares ioyned together, equall to the Square of the line giuen, and also holde such proportion one to another as the two partes of a line diuided by extreame and meane proportion.

The 25 diffinition.

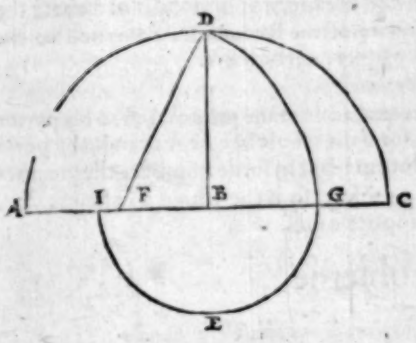
One of these Regulare Solides is said to be described within an other, when all the Angles of the internall or inscribed bodye at once touche the Superficies of the comprehending or circumscribing Regulare Solide.

The

The fyrst probleme

To diuide any line or number by extreame and meane Proportion.

ADmit the line Diuisible AB extend him foorth sufficiently, then erect vpon B a Perpendicular to A B as ye were taught in the first booke. Now, place the one foote of your compasse in B, and extending the other to A, describe the Semicircle A D C, then diuide A B in halfe at F, and placing the one foote of your compasse immoueable in F (making F D a Semidiameter) describe the Arke D G. This done, set the one foote of your Compasse in B and opening the other to G, drawe the Semicircle G E I, so is your line A B parted by extreame and meane proportion in I the greater portion B I the lesser I A. But Arithmeticallye to finde the same proportions yee must multiplie the Square of your geuen number in 5, and diuide by 4, from the Square roote of the Product, deduct the halfe of your giuen number, the Remainder is the greater portion, which subtracted from the whole leueth the lesser parte.



Example.

The number giuen 12, his Square 144, multiplyed by 5 and diuided by 4 yeeldeth 180, the halfe of the giuen number 6 deducted from the Square roote of 180, leaueth $\sqrt{\text{ʒ}}$ *180---6, which is the greater portion, and that againe deducted from 12, leaueth this Apotome 18—* $\sqrt{\text{ʒ}}$ *180 the lesser portion.*

Yee maye also by anye of the partes knowne, finde the quantitie of the whole thus: Augment the parte giuen by 12, and diuide by his correspondent Portion, the Quotient is your desire.

For Example, Suppose the lesser parte of a line diuided by extreame and meane Proportion, (whose quantitie I desire) 10, which augmented by 12, bringeth 120: this diuided by 18— $\sqrt{\text{ʒ}}$ *180 bringeth 15+* $\sqrt{\text{ʒ}}$ *125, so much is the whole line whereof 10 is the lesser parte. Againe, deducting 10 from 15+* $\sqrt{\text{ʒ}}$ *125, there remaineth 5+* $\sqrt{\text{ʒ}}$ *125, & that is the greater portion. But forasmuch as bothe this and the former conclusion may sundrie other wayes bee resolued, heere shall ensue Theoremes which the Geometer will applye aswell to this purpose as diuerslye to other necessarie conclusions, whereof some shall heereafter ensue.*

The

The fyrst Theoreme.

ANy number parted by extreame and meane proportion, the Square of the greater parte is equall to the Rectangle or number produced by multiplication of the lesse portion in the whole.

The 2 Theoreme.

If any line or number be parted by extreame and meane Proportion, and to the whole yee adioyne the greater portion, that surmounting number or line is also diuided by extreame and meane proportion, his greater portion being the firste giuen line, and the lesser the line or number adioyned.

The 3 Theoreme.

Any line or number parted by extreame and meane proportion, the Square of the whole added to the Square of the lesser parte, is triple to the Square of the greater portion.

The 4 Theoreme.

A line or number diuided by extreame and meane proportion, if yee abate the lesser portion out of the greater, the square of the Remainder adioyned to the square of the greater portion is triple to the square of the lesser.

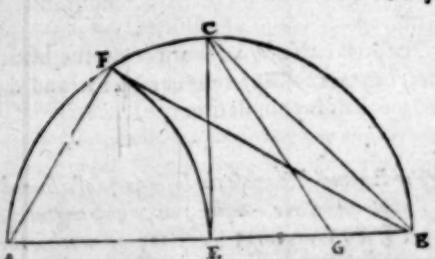
The 5 Theoreme.

The proportion of a line parted by extreame and meane proportion to his partes cannot be expressed in rationall numbers, for if the whole line be rationall, the parts are either of them an irrationall called Apotome, but in surde numbers the proportion may be thus declared, as $\sqrt{3}\,5$---1 to 3--$\sqrt{3}\,5$, so is the whole to the greater, and the greater to the lesse, for they are Proportionall.

The 2 Probleme

The Dimetient of any Circle giuen to searche out the sides or Cordes Trigonal, Tetragonal, Pentagonal and Decagonal.

SVppose the Dimetient A B of the Circle A B C D giuen, diuide this Diameter equally in the pointe E, whereupon erect the Perpendicular E C and from C to B drawe a right line, then fixe the one foot of your Compasse in A and opening the other to E, drawe the Arke E F, making intersection with the Circle in F, from thence extende right lines to A B, This doone, diuide the Semidiameter B E, as yee were taught by the first Probleme, by extreame and meane Proportion in the point 6: Finally, from G to C drawe a streight line, so haue ye F B the Corde Triagonall C B the Tetragnall, C G Pentagouall, and E G Decagonall, Nowe Arithmetically by the Diameter knowne to attaine these Cordes, yee must thus worke: triple the square of the Semidiameter the roote Quadrate of the product is the side Trigonall, double the Semidiameters square, and the roote Quadrate thereof is the tetragonall Corde: Diuide the Semidimetient by extreame and meane proportion the greater Portion is the side Decagonall, whose Square ioyned to the Semidimetients square produceth the square of the Corde Pentagonall.



Example

Example.

Admit the Diameter A B 12. the halfe is 6, whose Square tripled, yeeldeth 108. this ſurde number therefore $\sqrt{3}\,108$ *is the Trigonall Corde. The Square of 6 doubled is 72, whoſe roote is the line Tetragonall. The Semidiameter E B 6 diuided by extreame and meane proportion as was taught in the firſt Probleme, yeeldeth E G, the greater Portion* $\sqrt{3}\,45 - 3$, *and that is the line Decagonall: whoſe Square ioyned with the Square of 6 produceth* $90 - \sqrt{3}\,1620$, *whoſe roote vniuerſall is the line C G or Pentagonall Corde.*

The Diameter of the Circle 12
- *The Corde Trigonall* $\sqrt{3}\,108$
- *The Corde Tetragonall* $\sqrt{3}\,72$
- *The Corde Pentagonall* $\sqrt{3}$ vni. $90 - \sqrt{3}\,1620$
- *The Corde Decagonall* $\sqrt{3}\,45 - 3$

Now becauſe long working with irrationall numbers, may breede confuſion in ſuch as are not perfect in the rules of Algebra, yet may by the rule of proportion (ſuppoſing the Spheres Diameter 1000) reduce ſurd numbers to integers, although not ſo exactly as the ſubtiltie of Geometricall Demonſtration requireth: conſidering theſe Cordes cannot preciſely be expreſſed in rationall numbers) yet for any Mechanicall operation or manuall menſuration, the difference ſhall not be ſenſible.

Example.

12 $\mathcal{Z}\sqrt{3}\,108$ 1000 $\mathcal{Z}$ 866

12 $\mathcal{Z}\sqrt{3}\,72$ 1000 $\mathcal{Z}$ 707

12 $\mathcal{Z}\sqrt{3}$ vni. $90 - \sqrt{3}\,1620$ 1000 $\mathcal{Z}$ 588

12 $\mathcal{Z}\sqrt{3}\,45 - 3$ 1000 $\mathcal{Z}$ 309

Diameter 1000

Cordes.	
Trigonall	866
Tetragonall	707
Pentagonall	588
Hexagonall	500
Decagonall	309

As in theſe operations by the Diameter yee are brought in knowledge of the Cordes, ſo by the knowledge of any Corde, yee may alſo finde both the Diamster and all other Cordes, whereof to the writer there needs no farther Example.

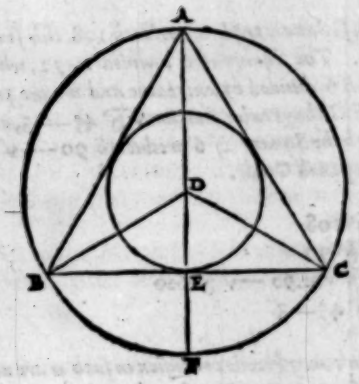
The 3 probleme

The ſide of any Equiangle Triangle geuen, to finde out the Semidiameters of his containing and contained circles, with the true quantitie of the Area.

THe Semidiameter of the containing Circle, it behoueth you to ſquare the ſide geuen of the Triangle, and diuide the Product by 3 the roote Quadrate of the Quotient is your deſire. The halfe of that roote is the Semidiameter of the contained Circle, which multiplyed in the Circuite or Perimetry of the Triangle yeeldeth a number, whoſe medietie is the content Superficiall or Area of that Figure.

Example.

Example.



The side giuen A B 6, the Square thereof diuided by 3 produceth 12, the roote thereof beeing $\sqrt{3}\ 12$ *is the Semidiameter D C, whose halfe* $\sqrt{3}$ *is the Semidimetient D E, which multiplyed in 18 the Triangles Perimetrie produceth* $\sqrt{3}\ 972$, *whose medietie being* $\sqrt{3}\ 243$ *is the quantitie Superficiall or Area of the Triangle A B C. The Semidiameter of the contained Circle is also thus founde. From the Semiperimetry of the Triangle deduct euery side, noting euery difference seuerally, then multiply these differences together, the Zenzicubike roote of the Product is your desire.*

The side geuen AB 6.

- *The containg Circle — Semidiameter B D* — $\sqrt{3}\ 12$
- *The contained Circles — Semidimetient D E* — $\sqrt{3}\ 3$
- *The Superficies or Area of the equilater Triangle* — $\sqrt{3}\ 243$

Any Equilater Triangles Area is also thus gotten, multiply the Square of his side by it selfe, and the product by 3, the offcome diuide by 16, the Quotients roote Quadrate is the Area.

Or multiplie the Square of the side giuen by 3, and diuide the offcome by 4, that Quotient, augment againe by the Square of halfe the side, the roote Quadrate of the Product is the Triangles content.

Or diuide the Triangles side in three equall partes the Square of one third parte multiplye by 3, and diuide the offcome by 4, the roote Quadrate of your Quotient encreased by the Triangles Semiperimetrie, produceth his Superficies also.

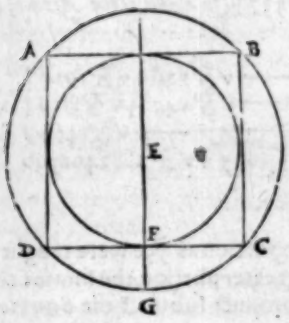
The 4. Probleme

The side of any Square knowne by Supputation to attaine the Semidiameters of his exteriour and interiour Circles, with the content of his plaine or Area.

THe halfe of your geuen side is the Semidiameter of the internall Circle, and if yee double the Square of this Semidiamiter, and from the Product extract a Quadrate roote, that roote is the Semidiameter of the externall Circle. Nowe for the Area either multiplye the side first geuen in it selfe, or else your lesser Semidiameter in halfe the Perimitrye of your Square.

Example.

Example.



Admit A B the Squares side 10, the halfe being 5 is the Semidimetient of the inward Circle, the Square thereof doubled is 50, whose Quadrate roote is E G, the containing Circles Semidimetient. Nowe for the Superficies I augment 10 by it selfe, there ariseth 100. Likewise 5 the internall Circles Semidiameter multiplyed by 20 the Squares Semiperimetry yeeldeth also 100, so muche is the Area of the Square A B C D.

A B the side of the square 10. { *E G the containing Circles Semidiameter* $\sqrt{\text{ʒ}}$ 50
E F the inscribed Circles Semidimetient 5
The Area of the Square A B C D 100.

The 5 Probleme

The side of any Equiangle Pentagonum giuen by Arithmeticke to learne his circumscribing and inscribed Circles Semidimetients with the exact quantitie of his plaine Superficies.

IT behooueth yee to knowe that all Equiangle Pentagonall figures, of what capacitie soeuer they bee, their sides to the Semidiameters of their containing Circles alway retaine one proportion, so that hauing already by the second Probleme (the semidiameter geuen) found the side, yee may contrariwise by the rule of proportion in any other, by the side giuen finde the semidiameter, from whose square if ye deduct halfe the sides square, the roote Quadrate of the Remainder is the Semidimetient of your Circle interior, the medietie whereof multiplyed by the Perimetrie or circuite of your figure yeeldeth the content superficiall or Area thereof.

Example.

Suppose the side of your Pentagonum A B C D E 14, for the Semidiameter A F, I worke by the rule of Proportion thus, $\sqrt{\text{ʒ}}$ *vniuers.* 90—$\sqrt{\text{ʒ}}$ 1620 *geueth* 6, *what yeeldeth* 14, *the fourthe proportionall is* $\sqrt{\text{ʒ}}$ *vniuers.* $\frac{635040 + \text{R.Q.}\,80655160320}{6480}$ *which by Reduction is brought to this number* $\sqrt{\text{ʒ}}$ *vniuers.* $98 + \sqrt{\text{ʒ}}\,1920\frac{4}{5}$, *and that is the containing Circles Semidimetient being very nigh* $11\frac{19}{21}$ *for exactly neither by integer nor fraction it can be expressed. Now for F G, I must deducte* 49 *the square of C G fro* $98 + \sqrt{\text{ʒ}}\,1920\frac{4}{5}$ *the Square of the Semidiameter tofore founde, so there romaineth* $49 + \sqrt{\text{ʒ}}\,1920\frac{4}{5}$, *whose roote vniuersall is the line F G or Semidimetient of the intrinsicall circle, which augmented by* 35 *the Semiperimetrie, produceth* $\sqrt{\text{ʒ}}$ *vniuers.* $60025 + \sqrt{\text{ʒ}}\,2882400500$, *which surde*

number reſteth betweene 337 and 338 being verye nighe 337 $\frac{1}{4}$, ſo much is the Area of the Pentagonum.

The ſide of the Pentagonum 14

The containing Circles Semidiameter —— $\sqrt{3}$ v. 98 + $\sqrt{3}$ 1920 $\frac{3}{4}$

The internall Circles Semidimetient —— $\sqrt{3}$ v45. + $\sqrt{3}$ 1920 $\frac{3}{4}$

The line Diagonall —— $\sqrt{3}$ 245 + 7

The Pentagonal Area —— $\sqrt{3}$ vni. 60025 + $\sqrt{3}$ 2882400500

Other Rules to performe the ſame.

Diuide the ſide giuen by extreame and meane proportion as yee were taught in the 2 Probleme, and to the whole ſide, adde his greater portion, the ſumme that thereof reſulteth multiply in it ſelfe, and from the product ſubtract the Square of halfe the ſide giuen, with the roote Quadrate of this remainder diuide the Square of halfe the ſide, the Quotient adde to your Diuiſor, the medietie of this reſulting ſumme is your greater Semidiameter, which ſubducted from your former Diuiſor, leaueth the Semidimetient of the intrinſicall Circle, and this augmented by the Pentagonall Semiperimetrie produceth his Area.

Or thus, your giuen ſide diuided as before by extreame and meane proportion, to the whole adde his greater portion, the product ſquare, and thereto adde the ſquare of the ſide Pentagonall, the offcome diuide by 5 your Quotientes Quadrate roote is the greater ſemidiameter, which augmented by the Pentagonal ſide, and the ſumme thereof amounting, diuided by double his greater portion parted as is before ſaide by extreame and meane proportion, your Quotient will expreſſe the quantitie of the leſſer ſemidimetient: for the Area yee may augment the Diameter of the contained Circle by $\frac{4}{5}$ of the Pentagonal ſide: Or halfe that leſſer ſemidimetient in the whole circuite of the figure, the ſurmounting ſummes declare his ſuperficiall quantitie.

Many moe rules might in this matter bee giuen, but I thinke it better to adioyne certaine Theoremes, whereby the ingenious ſhall be able not onely to conceiue the ground and reaſon of theſe already taught, but alſo of himſelfe inuent ſundrie other no leſſe certainely and perchance more ſpeedily performing the premiſſes.

Theoremes.

The firſt Theoreme.

THe ſquare of any Equiangle triangles ſide, is triple to the ſquare of his conteyning circles ſemidiameter.

The ſecond Theoreme.

The ſquare of an Equilater triangles ſide to the inſcribed Circles ſemidimetients ſquare is as 12 to 1.

The third Theoreme.

The ſemidiameter of a ſquares conteyning Circle, is meane proportionall betweene his ſide and the ſemidiameter of his conteyned Circle.

The fourth Theoreme.

In any Equiangle Pentagonum the ſquare of his ſide is equall to the ſquare of his containing Circles ſemidiameter, and the ſquare of the Corde Decagonall ioyned together.

The fifth Theoreme.

The Diameter of the Circle deſcribed within a Pentagonum is equall to the ſides Hexagonall and Decagonall of the comprehending Circle.

The 6 Theoreme.

The ſquare of the ſide Pentagonall togither with the ſquare of the line ſubtending the Pentagonall Angle, hath prooportion to the conteyning Circles Semidiameters ſquare, as 5 to 1.

The 7 Theoreme.

The ſquare of halfe the ſide Pentagonall being deducted fro the ſquare of the line Diagonall ſubtending one of the Pentagonall Angles, the roote Quadrate of the remainder is equall to both Semidiameters of the containing and conteyned Circles ioyned togither.

The 8 Theoreme.

In euery Equiangle pentagonum the conteyning Circles ſemidimetient retaineth the ſame proportion to the conteyned Circles Diameter that the pentagonall ſide dooth to his line Diagonall.

The ninth Theoreme.

The Diagonall line in euery Equiangle pentagonum dooth diuide the perpendicular (falling from his ſubtendent angle to the oppoſite ſide) by extreame and meane proportion.

The tenth Theoreme.

In euery equiangle pentagonum the lines Diagonall doe cut themſelues by extreame and meane proportion.

The 11 Theoreme.

If within one circle an equilater triangle and a ſquare be deſcribed, that ſquare retaineth the ſame proportion to the ſquare of the triangles inſcribed circles Diameter, that the Semidiameter of the triangles conteyning Circle dooth to the Semidiameter of the conteyned.

The 12 Theoreme.

Whereas with one Circle an Equiangle Pentagonum and a equilater triangle are deſcribed, the pentagonall inſcribed Circles Semidiameter parted by extreame and meane proportion, the greater portion is equall to the triangles conteyned Circles Semidimetient.

The 13 Theoreme.

An equilater triangle and an Equiangle pentagonum being within one Circle deſcribed, if two ſquares be made, the firſt equall to the ſquare of the pentagonall ſide, and the ſquare of the pentagonall line Diagonall added togither, the ſecond equall to the ſquare of the ſame Diagonall and the ſquare of the line, whereby the Diagonall exceedeth the ſide pentagonall, the ſide of the firſt ſquare to the ſide of the ſecond, retaineth the ſame proportion that the pentagonal diagonall doth to the ſide of the equilater triangle.

The 14 Theoreme.

The tetragonall Cordes ſquare exceedeth the ſquare of the Cordes pentagonall and Decagonall, by the ſquare of a line meane proportionall betweene the corde Decagonall and the exceſſe or difference of the cordes Hexagonall and Decagonal.

The 15 Theoreme.

An equiangle triangles ſide being rationall, his perpendicular and ſemidiaters as well of the conteyning as contained Circles, are all irrationall, but their ſquares may be expreſſed in number.

The 16 Theoreme.

The Area of an Equilater triangle beareth to the ſquare of his ſide proportion as his perpendicular to his baſe doubled.

The 17 Theoreme.

The Area of an Equiangle triangle beareth proportion to the ſquare of his ſide, as $\sqrt{3}$ 3 to 3.

The 18 Theoreme.

Double the Area of an equilater triangle is meane proportionall betweene the square of his side, and the square of the perpendicular falling from one of his angles to the opposite side.

The 19 Theoreme.

The Area of an equilater triangle to the square that is described within his conteyning Circle reteyneth the proportion of $\sqrt{3}\ 3\frac{3}{7}$ to $\frac{3}{7}$.

The 20 Theoreme.

The side of an Equiangle triangle being rationall, his Area is irrationall, but the q uare there of may with number be declared.

The 21 Theoreme.

The Area of an equiangle Pentagonum, to the triangles Area that is described within his containing Circle, retaineth such proportion as fiue times the line Diagonall of the Pentagonum to the perimetrie of the triangle.

The 22 Theoreme.

If an Equiangle Pentagonum and an equilater triangle be both described in one circle, the Rectangle contained of the line Diagonal, and the triangles perpendicular, retaineth proportion to the pentagonall Area, as 6 to 5.

The 23 Theoreme.

The pentagonall side being rationall, the Area is irrationall proportioned to the square of the side, as $\sqrt{3}$ vniu. $\frac{11}{14}+\sqrt{3}\ \frac{121}{44}$ to an vnitie.

The 24 Theoreme.

That square whose side is meane proportionall betweene the internal circles semidiameter, and the pentagonall semiperimetrie, is equall to the pentagonall Superficies.

The 25 Theoreme.

When an Equiangle pentagonum, and an Equilater triangle be both within one Circle described, a meane proportionall betweene $\frac{1}{5}$ of the triangles perpendicular, and the line Diagonal is equal in power to the pentagonal Superficies.

The 6 probleme

The side of any Tetraedron giuen, to searche out the Semidiameters of the circumscribing and inscribed Spheres.

Irst as is taught in the third probleme ye must get the circles semidiameter that conteineth the Equiangle triangle, whose side yee haue: Multiplie the square of this Semidiameter by 9, and diuide by 2, the roote Quadrate of the Quotient is the circumscribing spheres dimetient, the medietie whereof if yee augment by it self, and from the product subtract the square of the semidiameter tofore found, the roote Quadrate of the Remaine is the Semidiameter of the conteyned Sphere.

Or thus more easily and speedily. Multiply the side giuen in it selfe, the offcome diuide by 24 the roote Quadrate of the Quotient is the internall Spheres semidiameter, which tripledlyeeldeth the semidiameter of the comprehending Sphere.

Example.

The Trigonall side supposed 10, the containing Circles Semediameter is $\sqrt{3}\ 33\frac{1}{3}$, *whose square augmented by 9, and diuided by 2, yeeldeth in the quotient 150, whose quadrate root is the dimetient of the Sphere, the medietie here of squared, is* $37\frac{1}{2}$, *from which if ye withdraw* $33\frac{1}{3}$, *there remaineth* $4\frac{1}{6}$, *whose quadrate roote is the Axis of this Tetraedron.*

Likewise, if by the second rule ye diuide 100 with 24 the quotient is $4\frac{1}{6}$, *whose square roote being the Axis of this tetraedron if ye triple it, the resulting number wil be* $\sqrt{3}\ 37\frac{1}{2}$, *and that is the Semidiameter of the circumscribing Sphere, ye may also (by deducting 25 halfe the sides*

Square

Square from 100. *the square of the whole side*) *finde the Perpendiculare, for the roote of the remainder being* $\sqrt{3}\cdot 75$ *is the Perpendicular of the base. for the Solides perpendiculare I deduct* $33\frac{1}{3}$ *the containing circles Semidimetientes square, from* 100 *the square of the side giuen, there remaineth* $66\frac{2}{3}$, *whose roote is the solides perpendiculare.*

The side of Tetraedron 20

- *The containing circles semidiameter* — $\sqrt{3}\cdot 33\frac{1}{3}$
- *The containing circles semedimetient* — $\sqrt{3}\cdot 8\frac{1}{3}$.
- *The perpendicular of the Base* — $\sqrt{3}\cdot 75$.
- *The comprehending spheres semediameter* $\sqrt{3}\cdot 37\frac{1}{2}$.
- *The inscribed spheres semidimetient* $\sqrt{3}\cdot 4\frac{1}{2}$.
- *The Solides perpendiculare* — $\sqrt{3}\cdot 66\frac{2}{3}$.

Thus also an other way yee shall most speedily finde these Spherall Semidiameters, square the side giuen, that squares medietie ye shall triple the roote Quadrate of the product is the comprehending Spheres dimetient, which diuided by 3, *bringeth the inscribed spheres Diameter, the medieties of these are the Semidiameters, whereof the lesser augmented by* 4, *produceth the Solides perpendiculare.*

Theoremes of Tetraedron. 1.

THe square of Tetraedrons side, is equall to the square of his perpendiculare and the square of his contayning circles Semidiameter added togither.

The 2 *Theoreme.*

The square of Tetraedrons inscribed circles Semidiameter withdrawne from the square of his bases Perpendiculare, leaueth the square of the solides perpendiculare.

The 3 *Theoreme.*

This Solides perpendicular is equall to his Axis, and containing Spheres Semidimetient.

The 4 *Theoreme.*

The perpendiculare of tetraedrons base is equall to his containing and contained Circles Semidiameter.

The 5 *Theoreme.*

Tetraedrons containing spheres dimetient retaineth the same proportion to his Axis, that the square of his side dooth to double the square of his contained circles Semidimetient.

The 6 *Theoreme.*

Tetraedrons perpendicular is $\frac{1}{4}$ of his comprehending spheres dimetient.

The 7 *Theoreme.*

Tetraedrons Axis is a sixt part of his comprehending spheres Dimetient.

The 8 *Theoreme.*

Tetraedrons containing spheres Diameters square retaineth the same proportion to the square of his side, that the perpendiculare of his base dooth to the Semidiameter of his containing Circle.

The 9 *Theoreme.*

The square of tetraedrons comprehending spheres Dimetient contayneth the square of his inscribed circles Semidiameter 18 times.

The tenth Theoreme.

The square of tetraedrons conteyned circles Semidiameter is double to the Semidiameters square of his conteyned sphere.

The 11 *Theoreme.*

Tetraedrons perpendicular retaineth the same proportion to his basis containing Circles Semidiameter, that the contained Circles semidimetient dooth beare vnto his Axis.

The 12 *Theoreme.*

The square of the comprehending spheres dimetient to the containing Circles semidiameters square, hath proportion as 9 vnto 2.

The 13 *Theoreme.*

Tetraedrons side being rationall, his containing Spheres Diameter is irrationall, and their proportion is as $\sqrt{3}\cdot 2$ vnto $\sqrt{3}\cdot 3$

The

The 14 Theoreme.

Tetraedrons side being rationall, the Axis is surde, and it beareth proportion to the side as 1 to $\sqrt{3}\cdot 24$.

The 15 Theoreme.

The Tetraedrons comprehending Spheres dimetient, is equall in power to these fiue, the containing Circles Semidiameter, the containing Spheres Semidimetient, the inscribed circles Semidiameter, the Axis, and the solides Perpendicular.

The 7 Probleme

The side of any Hexaedron giuen, to finde the Semidimetients of the containing and contayned Spheres.

INcrease the side by it selfe, the offcome triple from the product extract the roote Quadrate the halfe thereof is the semidimetient of the circumscribing sphere, from whose square if yee detract the square of that Circles semidimetient that comprehendeth one of the Cubes quadrate plaines the root square of the Remainder is the semidimetient of the inscribed sphere: the Circles semidiameter is found by the fourth Probleme, admitting the line giuen the side of a square.

Example.

The Cubes side 6, the square thereof tripled 108, halfe the roote thereof is $\sqrt{3}\cdot 27$, and that is the Semidiameter of the containing sphere, his containing circles Semidiameter is $\sqrt{3}\cdot 18$, whose square deducted from 27, leueth 9, his roote quadrate being 3 is the Axis or contained speres Semidimetient.

The Cubes side 6

The line Diagonall	$\sqrt{3}\cdot 72$
The containing Circles Semid.	$\sqrt{3}\cdot 18$
The contained circles Semid.	3
The containing Spheres Semid.	$\sqrt{3}\cdot 27$
The Axis or inscribed Spheres Semidimetient	3

Theoremes of Hexaedron. 1

THe square of the Cubes comprehending spheres dimetient is equal to the square of his side, and the square of his basis line Diagonall.

The second Theoreme.

The square of his line Diagonall contayneth the square of his Axis 8 times.

The third Theoreme.

The square of the Cubes comprehending spheres semidiameter is equall to the square of halfe the line Diagonall of his base, and the square of his Axis added together.

The fourth Theoreme.

The semidimetient of the Cubes contained circle is equall to his Axis.

The 5 Theoreme.

The Hexaedron side is meane proportionall betweene his containing Circles semidiameter and his bases line Diagonall.

The 6 Theoreme.

The Cubes line Diagonall to his Axis retaineth triple the proportion of his side to his containing Circles semidiameter.

The

The 7 Theoreme.

The comprehending ſpheres Dimetient to the Cubes ſide, obſerueth the proportion of $\sqrt{3}$ 3 to 1.

The 8 Theoreme.

The Cubes ſide being a line rationall, his Axis is alſo rationall.

The 9 Theoreme.

The Hexaedrons comprehending Spheres Dimetient being rationall, his Axis is a ſurde and bereth proportion to the Dimetient, as 1 to $\sqrt{3}$ 12.

The 10 Theoreme.

The comprehending ſpheres Diameter hath the ſame proportion to the line Diagonalll of his baſe that the Cubes ſide hath to the Semidimetient of his contayning Circle.

The 8 probleme

The ſide of Octaedron giuen, to ſearche out Arithmetically the contayning Spheres Diameter and the Axis thereof.

DOuble the Square of the ſide, and from the offcome extract the roote Quadrate, ſo haue yee the comprehending Spheres Dimetient the halfe (being the Semidiameter) if from the Square thereof, you abate the ſquare of that circles Semidiameter which contayneth one of the Equilater triangles whereof the body is framed, the roote Quadrate of the Remainder is the Axis.

Or thus more briefly, ſquare the ſide giuen, the product diuide firſt by 2 then by 3, the roote ſquare of the firſte Quotient is the containing ſpheres ſemidimetient, the ſecond Quotient deducted fro the firſt, leaueth a remainder, whoſe roote quadrate is the Axis.

Or thus more readily, from the ſixte parte of the giuen ſides ſquare, extract the Quadrate roote, ſo haue yee the Axis, whoſe ſquare augmented by 12, produceth the ſquare of the comprehending ſpheres Diameter.

Example of the firſte.

THe ſide giuen 10 *his ſquare* 100, $\sqrt{3}$ 50 *being the Quadrate roote of halfe that ſquare, is the comprehending ſpheres Semidimetient. from the ſquare thereof abating* $33\frac{1}{3}$ *the Square of the containing Circles Semidiameter, the Remainder is* $16\frac{2}{3}$, *whoſe Quadrate roote is the Octaedrons Axis.*

Example of the ſecond rule.

100 *the Square of the ſide diuided firſt by* 2 *yeeldeth* 50, *then by* 3 *bringeth in the Quotient* $33\frac{1}{3}$ $\sqrt{3}$ 50 *is the containing ſpheres Semidiameter, the one Quotient deducted from the other leueth* $16\frac{2}{3}$, *his roote being* $\sqrt{3}$ $16\frac{2}{3}$ *is the Axis.*

Example of the third precept.

The ſides ſquare giuen 100 *diuided by* 6 *produceth in the Quotient* $16\frac{2}{3}$, *the roote Quadrate thereof is the Axis, the ſame number augmented by* 12 *maketh* 200, *wherefore I conclude* $\sqrt{3}$ 200 *the comprehending ſpheres Diameter.*

The ſide of Octaedron 10.	*The containing Circles Semidiameter.*	$\sqrt{3}$ $33\frac{1}{3}$
	The contained Circles Semidimetient.	$\sqrt{3}$ 8 $\frac{1}{3}$
	The containing Spheres Semidiameter.	$\sqrt{3}$ 50
	The Axis.	$\sqrt{3}$ $16\frac{2}{3}$

Theoreme

Theoremes of Octaedron. 1.

THe Axis of Octaedron is meane proportionall betweene the conteyning and conteyned Circles Semidiameters.

The ſecond Theoreme.

The Octaedrons ſide is meane proportionall betweene the Diameter and Semidiameter of the circumſcribing ſphere.

The third Theoreme.

The Axis Square increaſed by 12 yeeldeth the comprehending Spheres Diameters ſquare.

The 4 Theoreme.

The conteyning ſpheres Diameter hath the ſame proportion to his Axis, that the Octaedrons ſide hath to the conteyned Circles Semidiameter.

The 5 Theoreme.

The ſide of Octaedron being rationall, the comprehending ſpheres Dimetient cannot be expreſſed with number, but his ſquare to the ſides ſquare is double.

The 6 Theoreme.

The ſide of Octaedron being rationall, his Axis is not in number to be expreſſed, but the ſquare thereof to the ſquare of the ſide is as 1 to 6,

The 7 Theoreme.

The Octaedrons containing Spheres Semidiameter is equall in power with his Axis and contayning circles Semidiameter.

The 8 Theoreme.

The ſide of Octaedron is equall in power with the Axis, the comprehending ſpheres Semidiametient, and the comprehending Circles Semidiameter.

The 9 Theoreme.

The Diameter of Octaedrons comprehending ſphere being rationall, his ſide is a ſurde, and retaineth ſuch proportion to the Diameter as $\sqrt{3}\cdot\frac{1}{2}$ vnto 1.

The 10 Theoreme.

The Diameter of Octaedron being rationall, his containing Circles Semidiameter is irrationall, and hath proportion to his Diameter, as $\sqrt{3}\cdot\frac{1}{2}$ vnto 1.

The 11 Theoreme.

Octaedrons ſide being rationall his Axis is ſurde, and beareth proportion to the ſide, as $\sqrt{3}\cdot\frac{1}{2}$ vnto 1.

The 12 Theoreme.

Octaedrons ſide being rationall his contained Circles Semidiameter is irrationall, hauing proportion to the ſide as $\sqrt{3}\cdot\frac{1}{12}$ to 1.

The 13 Theoreme.

The Axis of Octaedron being rationall, his comprehending ſpheres Semidimetient is irrationall and their proportion is as an vnitie vnto $\sqrt{3}\cdot 3$.

The 14 Theoreme.

The ſide of Octaedron being rationall, that line which is equall in power to the Semidiameters of his contayned ſphere and Circle, is alſo rationall, and the comprehending ſpheres diameter to that line hath triple the proportion of the contayning Circles Semidiameter to the Axis.

The 15 Theoreme.

Octaedrons contayned Circles Semidimetient, his Axis, the containing Circles Semidiameter, and the inſcribed ſpheres Diameter, are 4 lines in continuall geometricall proportion.

The

The 9 Probleme

The side of an Icosaedron measured, by supputation to finde his Axis and containing spheres dimetient.

ADmitting the side measured a Corde Pentagonall, you shall by the fourth probleme searche out the containing circles Semidiameter, and multiplie the square thereof by 5, the Quadrate roote of the product is the Dimetient of the comprehending sphere, and so consequently the halfe thereof shall be the semidiameter, from whose square if ye subtract his containing circles Semidimetients square, the roote Quadrate of the Remainder is the *Axis*.

Example.

Admit the side of Icosaedron 12, which supposed a Corde pentagonall, the side Hexagonall or Semidiameter found (as was taught in the fifth Probleme) is $\sqrt{\mathfrak{z}}\ v.\frac{576}{10-R20}$ *the square of this semidiameter increased by 5 maketh* $\frac{3880}{10-R20}$ *whose Quadrate roote is the containing Spheres diameter, his medietie being* $\sqrt{\mathfrak{z}}\ v.\ \frac{739}{10-R20}$ *is the Semidiameter. Nowe to attaine the Axis I search first (as was taught in the third Probleme) the Circles Semidiameter that contayneth the Icosaedrons Triangulare base, finding it* $\sqrt{\mathfrak{z}}\ 48$, *whose square deducted from the square of the containing spheres semidiameter leaueth* $\frac{730}{10-R20}-48$, *whose quadrate roote is the Axis of this Icosaedron.*

The side of Icosaedron 12	*The Circles semidiameter, whereon Icosaedron is framed.*	$\sqrt{\mathfrak{z}}\ \frac{576}{10-R20}$
	The Semidiameter of the contayning Circle.	$\sqrt{\mathfrak{z}}\ 48.$
	The Semidimetient of the contained circle.	$\sqrt{\mathfrak{z}}\ 12.$
	The comprehending spheres semid,	$\sqrt{\mathfrak{z}}\ \frac{730}{10-R20}$
	The Axis or Cathetus	$\sqrt{\mathfrak{z}}\ v.\frac{730}{10-R20}-48.$

Theoremes of Icosaedron and his partes. 1.

THe Diameter of Icosaedrons comprehending sphere retayneth such proportion to the Diameter of the circle whereon the Icosaedron is framed, as $\sqrt{\mathfrak{z}}\ 5$ to 2.

The second Theoreme.

The semidiameter of this Circle whereon the Icosaedron is framed with two cordes Decagonal of the same circle doe make the comprehending spheres Dimetient.

The third Theoreme.

The comprehending spheres Semidiameter, dooth match in power the Axis and containing circles Semidimetient.

The 4 Theoreme.

If the semidiameter of that circle wherevpon the Icosaedron is framed, be diuided by extreame and meane proportion, the square of Icosaedrons side is equall in power to the foresaid semidiameter, and his greater portion.

The 5 Theoreme.

The Diameter of the comprehending sphere being a line Rationall, the Icosaedrons side is a line Irrationall, called of *Euclide* Minor.

The 6 Theoreme.

If the ſide of Icoſaedron be a line rationall, the Dimetient of the comprehending ſphere ſhalbe an irrationall line called Maior.

The 7 Theoreme.

The Diameter of the comprehending ſphere being a rationall, the ſemidiameter of the Circle wherevpon the Icoſaedron is made, ſhall be a line irrationall, but his ſquare is rationall, and the comprehending ſpheres diameters ſquare containeth it 5 times.

The 8 Theoreme.

The Icoſaedrons ſide being a line rationall, the Semidimetient of that circle whereon the body is framed will be an irrationall, called of Euclide *Maior*, and it retaineth vnto this Solids ſide the proportion of $\sqrt{3}$ v. $\frac{1}{2}+\sqrt{3}\,\frac{1}{12}$ vnto an vnitie.

The 9 Theoreme.

The Icoſaedrons ſide being rationall, his conteyning Circles ſemidiameter is Irrationall, and beareth proportion to that ſide, as 1 to $\sqrt{3}$ 3.

The 10 Theoreme.

The comprehending ſpheres diameter being Rationall, his conteyning Circles Semidiameter is an irrationall of that kinde, which Euclide calleth *Minor*, and it beareth proportion to the Diameter as $\sqrt{3}$ vniuerſ. $\frac{1}{6}-\sqrt{3}\,\frac{1}{180}$ vnto an vnitie.

The 11 Theoreme.

The Icoſaedrons ſide admitted Rational, the contained Circles Semidiameter is a ſurde, and reteyneth ſuch proportion to the ſide, as $\sqrt{3}\,\frac{1}{12}$ vnto 1.

The 12 Theoreme.

Icoſaedrons conteyning ſpheres dimetient being rationall his contayned Circles Semidimetient is an Irrationall called *Minor*, and the proportion betweene them is as 1 vnto $\sqrt{3}$ vniuerſalis $\frac{1}{12}-\sqrt{3}\,\frac{1}{1440}$.

The 13 Theoreme.

Icoſaedrons ſide ſuppoſed rationall, his axis is an Irrational called *Binomium*, and beareth proportion to the ſide, as $\sqrt{3}$ vni. $\frac{7}{12}+\sqrt{3}\,\frac{1}{12}$ vnto 1.

The 14 Theoreme.

The Diameter of Icoſaedrons containing ſphere admitted Rationall, his axis is Irrationall, and reteyneth ſuch proportion to his dimetient as $\sqrt{3}$ vni. $\frac{1}{12}$, $+\sqrt{3}$ $\frac{1}{720}$ vnto 1.

The 15 Theoreme.

The ſemidiameter of the conteining and conteined circles, although the proportion of eyther to the ſpheres dimetient be ſurde, yet the proportion between themſelues is rationall, and as 2 to 1.

The 16 Theoreme.

Icoſaedrons ſide being parted by extreame and meane proportion, if ye adioyne to the ſide his greater portion, the ſquare of that whole line added to the ſquare of the ſide, produceth the ſquare of the comprehending ſpheres diameter.

The 17 Theoreme.

The diameter of that circle whereon Icoſaedron is framed, retaineth ſuch proportion to the contayning Circles Semidimetient of the body, that the Icoſaedrons ſide doth to the Corde Pentagonall of the conteyned Circle.

The 18 Theoreme.

If from the ſquare of the comprehending ſpheres dimetient ye abate the Icoſaedrons ſides ſquare, that line which matcheth in power the Remaine, being deuided by extreame and meane proportion, will make his greater Segment the Icoſaedrons ſide.

The 19 Theoreme.

The Pentagonal line Diagonal of that Circle, whereon Icoſaedron is framed, exceedeth in power the contayned ſpheres Diameter, by a line matching in power the exceſſe of Icoſaedrons baſis conteining circles Diameters ſquare, aboue the ſquare of his ſide.

The 20 Theoreme.

Icoſaedrons ſide is meane proportionall betweene his Diameter and the Decagonall Corde of that circle whereon Icoſaedron is framed.

The

The 10 Probleme

The side of Dodecaedron giuen, by calculation to finde his Axis and conteyning spheres diameter.

VSing the aide of the fift Probleme, if you search out the triangular or trigonall corde of that circle that containeth this Dodecaedrons Pentagonall base, admitting the same an Icosaedrons side, yee may by the last chapter search out the Axis and containing spheres dimetient, which are in all points equal and agreeable in proportion with the Axis and sphericall Diameter of this Dodecaedron.

Or thus more speedily and with lesse confusion in working, by the first Probleme diuide this Dodecaedrons side giuen by extreame and meane proportion, adding thereto his greater portion, then triple the square of this whole line, the roote quadrate of the product is the conteyning spheres Dimetient. Againe it behooueth you by the fift Probleme, to learne the Semidiameter of the Circle that contayneth one of the Dodecaedrons Pentagonall Superficies, and deduct the square thereof from the square of the comprehending spheres semidimetient, the roote Quadrate of the Remainder is the Axis of the body.

Example.

The side of Dodecaedron 10 diuided by extreme and meane proportion by the first probleme maketh his greater portion $\sqrt{\text{ʒ}}\,125-5$, *which added to 10, produceth* $\sqrt{\text{ʒ}}\,125+5$, *the square hereof tripled is* $450+\sqrt{\text{ʒ}}\,112500$ *the roote square heereof is the comprehending Spheres dimetient, whose medietie being* $\sqrt{\text{ʒ}}\,\text{v}.112\frac{1}{2}+\sqrt{\text{ʒ}}\,\frac{112500}{16}$ *is that speeres Semidiameter. Againe the Semidiameter of this Dodecaedrons containing circle I find by the 5 probleme* $\sqrt{\text{ʒ}}\,\text{v}.50+\sqrt{\text{ʒ}}\,500$, *whose square taken from the square of the conteyning spheres semidiameter leaueth* $62\frac{1}{2}+\sqrt{\text{ʒ}}\,\frac{112500}{16}-\sqrt{\text{ʒ}}\,500$, *whose roote Quadrate is the Axis or inscribed spheres Semidiametient.*

The side of Dodecaedron 10
- *The conteyning circles Semid.* $\sqrt{\text{ʒ}}\,\text{vni}.50+\sqrt{\text{ʒ}}\,500$.
- *The conteyned circles Semidimetient* $\sqrt{\text{ʒ}}\,\text{vni}.25+\sqrt{\text{ʒ}}\,500$.
- *The diameter of the conteining sphere* $\sqrt{\text{ʒ}}\,\text{v}.450+\sqrt{\text{ʒ}}\,112500$
- *The Axis* $\sqrt{\text{ʒ}}\,\text{vni}.62\frac{1}{2}+\sqrt{\text{ʒ}}\,\frac{112500}{16}-\sqrt{\text{ʒ}}\,500$
- *The Pentagonall diagonal line* $\sqrt{\text{ʒ}}\,125+5$.

The Axis of this body is also thus founde, I adde the line Diagonall being $\sqrt{\text{ʒ}}\,125+5$ *to 10 the Dodecaedrons side, thereof ariseth* $\sqrt{\text{ʒ}}\,125+15$, *this medietie squared maketh* $\frac{35}{4}^{\circ}+\sqrt{\text{ʒ}}\,\frac{28125}{4}$ *from whence I deduct* $25+\sqrt{\text{ʒ}}\,500$ *the square of the conteyned Circles Semidimetient, there remaineth* $\frac{25}{4}^{\circ}+\sqrt{\text{ʒ}}\,\frac{28125}{4}-\sqrt{\text{ʒ}}\,500$, *whose roote Quadrate is the Dodecaedrons Axis exactly agreeing with the foomer operation.*

Theoremes of Dodecaedron. 1

THe square of the Diameter of Dodecaedrons comprehending sphere, is triple to the square of his basis line Diagonall.

The second Theoreme.

The Dodecaedrons side being added to the Diagonall line of his basis maketh a line, whose square ioyned to the square of the side, produceth the square of the conteyning Globes dimetient.

The 3 Theoreme.

The Diameter of Dodecaedrons comprehending sphere being Rationall, the side of that body shall be an Irrationall called *Apotome*, bearing proportion to the Diameter as $\sqrt{3}$ vni. $\frac{3}{4}$ — $\sqrt{3}$ $\frac{1}{12}$ vnto 1.

The 4 Theoreme.

The side of Dodecaedron being rationall, his comprehending spheres dimetient is an irrationall *Binomium*, hauing the squares of his names or compounding lines in proportion as 5 to 1, and this irrationall diameter to the Dodecaedrons side, retaineth proportion, as $\sqrt{3}$ vniuers. $\frac{3}{4}$ + $\sqrt{3}$ $\frac{4}{3}$ vnto an vnitie.

The 5 Theoreme.

If Dodecaedrons side bee adioyned to his Diagonall line the miedietie thereof squared is eqnall to the square of the Axis and the square of the contained circles semidiameter.

The 6 Theoreme.

Dodecaedrons side being rational, the conteyning circles Semidimetient is an irrationall called *Maior*, and beareth such proportion to the side, as $\sqrt{3}$ vni. $\frac{1}{3}$ + $\sqrt{3}$ $\frac{1}{12}$ vnto 1.

The 7 Theoreme.

The square of Dodecaedrons side added to the square of his Basis Diagonall, produceth a quantitie, whose fifth parte is the conteyning circles Semidiameters square.

The 8 Theoreme.

Dodecaedrons side being Rational, the Diagonall of his basis shal be an Irrationall called *Binomium*, retaining proportion to the side, as $\sqrt{3}$ $\frac{1}{4}$ + $\frac{1}{4}$ vnto 1.

The 9 Theoreme.

Dodecaedrons Axis being deuided in extreame and meane proportion, maketh his greater part the lesse Semidimetient of his basis.

The tenth Theoreme.

The side of Dodecaedron being rationall, his basis conteyned circles Diameter is an irrationall *Maior*, proportioned to the side, as $\sqrt{3}$ v. 1 + $\sqrt{3}$ $\frac{4}{1}$ to an vnitie.

The 11 probleme

The side of Tetraedron knowne, to finde his superficiall and solide content.

FOr the superficies yee shall augment the squared square of the side by 3, the products Quadrate roote is your desire, or multiplie the containing circles Semidimetient in the Perimetrie of the triangle, or augment the contayned Circles Semidiameter in the perimetrie of the triangle and double the product so haue you by euery of these operations his content superficiall. For his Crassitude thus, Multiply the Semidimetient of the containing Circle in the Axis of the body, and the product in the side. Or the semidimetient of the contained Circle in the Diameter of the conteyned sphere, and the offcome in the side: Or the Axis in the perimetrie of the triangle, and the product in the semidiameter of the contayning circle, the offcome diuided by 3, yeeldeth in the quotient the solide quantitie.

Examples of the superficiall quantitie.

The side of Tetraedron 10, *his squared square* 10000 *multiplied in* 3, *bringeth* 30000, *conclude* $\sqrt{3}$ 30000 *the superficiall content. Likewise the side being* 10. *I finde by the third*

Probleme

Probleme his containing circles Semidiameter $\sqrt{3}\ 33\frac{1}{7}$ *which increased by* 30 *the triangular Perimetrie yeeldeth* $\sqrt{3}\ \frac{9000}{7}$. *Also the semidiameter of the intrinsecal circle being* $\sqrt{3}\ 8\frac{1}{7}$ *multiplyed in* 30 *bringeth* $\sqrt{3}\ \frac{9000}{77}$ *which doubled amounteth to* $\sqrt{3}\ 30000$ *the whole Superficies of that body agreeing with both the former operations.*

Examples of the solide capacitie.

The side admitted as before 10, *by the thirde Probleme the Semidiameter of the containing circle is* $\sqrt{3}\ 33\frac{1}{7}$, *the Axis yee shall by the sixth Probleme finde* $\sqrt{3}\ 4\frac{1}{2}$ *these multiplyed together make* $\sqrt{3}\ \frac{1000}{14}$. *and this againe in* 10, *bringeth* $\sqrt{3}\ \frac{10000}{14}$. *Likewise the Semidiameter of the contained Circle founde by the thirde Probleme* $\sqrt{3}\ 8\frac{1}{7}$, *augmented by* $\sqrt{3}\ \frac{10}{7}$, *the contayned Spheres Dimetient, there ariseth* $\sqrt{3}\ \frac{1210}{7}$. *and that againe in* 10 *the side triangular bringeth* $\sqrt{3}\ \frac{12100}{7}$. *Also the Axis* $\sqrt{3}\ 4\frac{1}{2}$ *multiplyed by* 30 *the Triangles Perimetrie, produceth* $\sqrt{3}\ \frac{1210}{2}$, *and this againe in* $\sqrt{3}\ 33\frac{1}{7}$ *the containing Circles Semidiameter maketh* $\sqrt{3}\ \frac{121000}{14}$, *whose third part is* $\sqrt{3}\ \frac{40333\frac{1}{3}}{14}$ *the Solide capacitie of that body exactly agreeing with the former workings.*

The side of Tetraedron 10

- *The containing circles Semidiameter* $\sqrt{3}\ 33\frac{1}{7}$
- *The contained circles Semidimetient* $\sqrt{3}\ 8\frac{1}{7}$
- *The containing Spheres Semidimetient* $\sqrt{3}\ 37\frac{1}{2}$
- *The Axis or Kathetus* $\sqrt{3}\ 4\frac{1}{2}$
- *The content Superficiall* $\sqrt{3}\ 30000$
- *The content Solide* $\sqrt{3}\ 1382\frac{8}{9}$

Theoremes of Tetraedrons Superficiall and Solide Contents.

THe side of Tetraedron being Rationall, the line that matcheth in power his superficies is an Irrational, retaining proportion to the side as $\sqrt{3}\ 3$ vnto 1.

The 2 Theoreme.

The Tetraedrons Superficies to the square of his side beareth proportion as $\sqrt{3}\ 3$ vnto 1.

The 3 Theoreme.

The Triangle whose basis is equall to the Perimetrie of this solides basis, and his altitude or Perpendicular to the Diameter of the containing Circle is equall to the Superficiall quantitie of this body.

The 4 Theoreme.

The right angled Paralelogramme, whose altitude is the contained circles diameter, and his base the triangular perimetrie, is equall to the Tetraedrons superficies.

The 5 Theoreme.

That equilater triangle is equall to the Tetraedrons superficies, whose side beareth proportion to the solides side as 2 to 1.

The 6 Theoreme.

A Quadrangular Prisma having his basis a square, whose side is meane proportionall betweene the Axis and the containing Circles Semidimetient, and his altitude the Tetraedrons side, is equall to the crassitude of Tetraedron.

The 7 Theoreme.

The solide of Tetraedron may by imagination be parted into 4 equall Trigonall Pyramides, having for their bases the triangles of that body, and meeting with their toppes or Vertices in the center of the Sphere.

The

The eight Theoreme.

A triangular Prisma, whose altitude is equall to the third part of the Axis, and his base one of the equilater triangles wherewith the body is encompassed, is equall to one of those Pyramides that meete with their toppes or Vertices at the Center of the body.

The ninth Theoreme.

A right angled quadrangular direct Prisma, hauing for his longitude, latitude and profunditie these three lines, Tetraedrons side, his Axis and containing Circles Semidimetient, is equall to the Tetraedron.

The tenth Theoreme.

If foure right lines be found in continuall proportion, so as the first being equall to the Tetraedrons side, beare proportion to the last, as $\sqrt[3]{72}$ vnto 1, the Cube of the second is equall to the Tetraedron in Crassitude.

The 12 Probleme

The side of a Cube measured, to finde his Superficiall and Solide quantitie.

MVltiplie the side first by 2, then by 3, these Products multiplyed together, declare the superficiall content. Or double the comprehending Spheres Diameters square. Or triple the Diagonal lines square. Or multiply the Cubes base by 6, euery of these wayes yee haue the Superficies of this body.

For the Crassitude thus, Augment the side in the base, or the Axis in the square of the line Diagonall, or the superficies in the sixth part of his altitude, any of the products is your desire.

Example.

Admit Hexaedrons side 10, *by the seuenth Probleme yee shall finde the Diameter of his containing Sphere* $\sqrt[3]{300}$, *his Axis* 5, *his line Diagonall* $\sqrt[3]{200}$. *The side first increased by* 2 *maketh* 20, *then by* 3 *bringeth* 30, *these together multiplyed make* 600. *Likewise the Square of the comprehending Spheres Diameter* 300 *doubled maketh* 600, *and the square of the line Diagonall being* 200 *tripled maketh also* 600, *I conclude therefore* 600 *the content Superficiall.*

Example of the Solide Content

10 *the side augmented in* 100, *the base maketh* 1000. *Also* 5 *the Axis in* 200 *the square of the line Diagonall produceth* 1000. *Likewise* 600 *the Superficies in* $\frac{1}{6}$ *the sixth part of the side, yeeldeth also* 1000, *thus I finde by all these rules* 1000 *the Cubes solide quantitie.*

The side of the Cube 10

The containing circles semidiameter	$\sqrt[3]{50}$
The contained circles Semidiameter	5
The containing Spheres Dimetient	$\sqrt[3]{300}$
The Axis or Kathetus	5
The line Diagonal	$\sqrt[3]{200}$
The Cubes superficiall quantitie	600
The Solide Capicitie	1000

Theoremes

Theoremes of Hexaedrons superficies and Crassitude. *The first Theoreme.*

THe comprehending spheres Dimetients square doubled is equall to the Cubes superficiall content.

The second Theoreme.

A Paralelogramme whose Altitude is double to the Cubes side, and his base triple, is equall to the superficies of that Solide.

The third Theoreme.

That square is equall to the Cubes superficies, whose side is meane proportional betweene the Cubes side and a line 6 times his length.

The 4 Theoreme.

Any Prisma whose base is equall to the square of the Cubes basis line Diagonall, and his altitude the Cubes Axis, is equall to his solide quantitie.

The fift Theoreme.

The Cube may intellectually bee deuided into 6 Quadrate Pyramides, euerye of them hauing to his base one of the Cubes square basis, and concurring or meeting at the centre of the sphere, which is a common Vertex to them all.

The sixt Theoreme.

Any Pyramis hauing his Base equall to the square of the comprehending spheres Diameter, and his altitude the side of the Cube, is equall to his Crassitude.

The seuenth Theoreme.

A Tetraedron whose side is proportioned to the Cubes side, as $\sqrt{3\,3}$ 12. vnto an vnitie, haue equall superficiall contents.

The eight Theoreme.

An Octaedron whose side is equall to a line meane proportionall betweene the Cubes side and his Diameter, hath his superficies equal to the Cubes.

The ninth Theoreme.

If 7 lines be continually proportional, the last retaining proportion to the first as 72 vnto 1, the Tetraedron of the second is equal to the Cube of the first.

The tenth Theoreme.

If 4 lines be continually proportionall the last bearing proportion to the first, as 1 to $\sqrt{3\,\frac{1}{2}}$ the cube of the first is equal to the Octaedron of the second.

The 13 probleme

Octaedrons side giuen to searche his Superficiall and Solide content.

MVltiplye the contayning Circles diameter in the circuite or perimetrie of the Triangle, or increase the squared square of the side giuen by 12, the quadrate roote of the product is the Superficies: for the Solide content worke thus, multiplie the Diameter of the interior circle in the Diameter of the internal sphere, and the product in the side. Or the Dimetient of the contayning Circle in the Axis of the body, and the product in the side. Or the Axis in the perimetrie of the triangle, and the resulting summe in the contayning Circles Dimetient, the product diuided by 3 yeeldeth the Solide content.

Example.

The side of Octaedron supposed 10, the contained circles Diameter by the thirde probleme I finde $\sqrt{3\,\frac{40}{7}}$, *which augmented by 30, the triangles perimetrie produceth* $\sqrt{3\,\frac{360}{7}}$.

The

The squared square of the triangulare side is 10000 *which augmented by* 12 *bringeth* 120000, *whose Quadrate roote is this bodyes superficiall quantitie.*

The Example of the Solide content.

The side being 10 *by the third Probleme I find the containing circles Semidimetient* $\sqrt{\text{ʒ}}\ 33\frac{1}{3}$ *the contayned Spheres Diameter by the eight probleme is* $\sqrt{\text{ʒ}}\ 66\frac{2}{3}$ *these multiplyed togither make* $\frac{20000}{9}$, *which increased by* 10 *the triangulare side produceth* $\sqrt{\text{ʒ}}\ \frac{2000000}{9}$.

The Diameter of the containing circle $\sqrt{\text{ʒ}}\ \frac{400}{3}$ *augmented by* $\sqrt{\text{ʒ}}\ \frac{50}{3}$ *the Axis, bringeth* $\sqrt{\text{ʒ}}\ \frac{20000}{9}$ *and this againe in* 10 *maketh as before* $\sqrt{\text{ʒ}}\ \frac{2000000}{6}$.

The Axis $\sqrt{\text{ʒ}}\ \frac{50}{3}$ *multiplyed by* 30 *the triangulare perimetrie, bringeth* $\sqrt{\text{ʒ}}\ \frac{45000}{3}$ *and this againe in* $\sqrt{\text{ʒ}}\ \frac{400}{3}$ *the contayning circles dimetient, produceth* $\sqrt{\text{ʒ}}\ \frac{18000000}{9}$ *whose third parte is* $\sqrt{\text{ʒ}}\ 222222\frac{2}{9}$ *the Solide capacitie of that Octaedron.*

The side of Octaedron 10	*The contayning Spheres dimetient* $\sqrt{\text{ʒ}}\ 200$
	The contayning circles Diameter $\sqrt{\text{ʒ}}\ \frac{400}{3}$
	The Axis or Cathetus $\sqrt{\text{ʒ}}\ 16\frac{2}{3}$
	The internall circles Diameter $\sqrt{\text{ʒ}}\ 33\frac{1}{3}$
	The content superficiall $\sqrt{\text{ʒ}}\ 120000$.
	The solid capacitie $\sqrt{\text{ʒ}}\ 222222\frac{2}{9}$

Theoremes of Octaedrons content superficiall and Solide. 1.

THe side of Octaedron admitted Rationall, his content superficiall is Irrationall, and the line that matcheth it in power is a surde called of Euclide a line Mediall, retaining such proportion to the side, as $\sqrt{\text{ʒ}}\ 12$ to 1.

The 2 Theoreme.

The superficiall quantitie of Octaedron beareth proportion to the square of his side as $\sqrt{\text{ʒ}}\ 12$ to an vnitie.

The 3 Theoreme.

A rectangulare parallelogramme hauing the one of his containing sides the perimetrie of Octaedrons triangulare basis, and the other equall to the dimetient of his conteyning circle, is equall to the whole superficies of this body.

The 4 Theoreme.

An Equilater triangle, whose side beareth proportion to the side of Octaedron, as $\sqrt{\text{ʒ}}\ 8$ to 1 is equall to the capacitie superficiall of that body.

The 5 Theoreme.

The square whose side is meane proportionall betweene the containing circles diameter, and the perimetrie of this bodies basis, is equall to his superficies.

The 6 Theoreme.

A Quadrangulare Prisma hauing to his base a square (whose side is meane proportionall betweene the conteyned Circles Diameter and the contayned spheres diameter, and his altitude the solides side is equall to this Octaedrons Crassitude.

The 7 Theoreme.

This body may be diuided into 8 equal trigonal Pyramides, whose bases are the equilater triangles wherewith this solide is enuironed, euerye of these Pyramides meete at the center of the contayning and contayned sphere, which is the common vertex to them all.

The 8 Theoreme.

To euery of these Pyramides that Prisma is equall, whose altitude is the contayned

The eight Theoreme.

To euery of theſe Pyramides that Priſma is equall, whoſe altitude is the contayned ſpheres Semidiameter, and his baſe $\frac{3}{4}$ of one of thoſe Equilater Triangles, area, wherewith this body is enuironed.

The 9 Theoreme.

A Quadrangulare direct Pyramis hauing his baſe equall to the ſquare of a line in meane proportion betweene the Axes and the perimetrie of the Triangle, and his altitude the contayning circles Dimetient, is equall to this regulare ſolide.

The tenth Theoreme.

If foure lines in continuall proportion haue the firſt and greateſt equall to Octaedrons ſide, retaining proportion to the laſte, as 1 to $\sqrt{3}\ \frac{2}{3}$ the Cube of the ſecond is equall to this Solide.

The 14 probleme

The ſide of Icoſaedron knowne, by ſupputation to learne the contents ſuperficiall and ſolide of that bodie.

Conſidering this body is enuironed with 20 equilater triangles, yee ſhall by the thirde probleme get one of thoſe Triangles Area, which increaſed by 20 yeeldeth your deſire. Or augment the ſquared ſquare of the giuen ſide by 75, and from the product extract the roote Quadrate. Likewiſe if you adde the Semidiameter of the inſcribed Circle to the Diameter of the circumſcribed circle, the offcome multiplyed by double the Triangles perimetrie, produceth the Icoſaedrons content Superficiall. For the ſolide capacitie adde the contayning and contayned circles ſemidimetients together, the product increaſe by double the ſide knowne, the reſulting ſumme augmented by the Icoſaedrons Axis (which yee were in the ninth probleme taughte to finde) yeeldeth the groſſe capacitie. Alſo if yee augment the ſquared ſquare of the ſide by $8\ \frac{1}{4}$ the Quadrate roote of the product increaſed by the Axis of this body bringeth the deſired Craſſitude. Or if yee augment the ſides ſquared ſquare by $2\ \frac{1}{12}$ and the reſulting ſumme againe in the ſquare of the inſcribed ſpheres Dimetient, the roote Quadrate of the product is likewiſe the Icoſaedrons ſolide capacitie.

Examples of the former Rules.

The Icoſaedrons ſide ſuppoſed 12, by the third Probleme yee ſhall finde the Area of the Equilater Triangle $\sqrt{3}$ 3888, *which increaſed by 20 produceth* $\sqrt{3}$ 1555200. *Likewiſe the ſquared ſquare of 12 being 20736 augmented by 75, yeeldeth 1555200, whoſe quadrate roote is your deſired ſuperficies. In like manner if yee adde* $\sqrt{3}$ 12, *the Semidimetient of the inſcribed circle, to* $\sqrt{3}$ 192 *the Dimetient of the circumſcribed Circle (for ſo ſhall yee finde them by the thirde Probleme the Triangles ſide being 12) the product will bee* $\sqrt{3}$ 300, *which multiplyed by 72 double the Triangles Perimetrie, yeeldeth againe* $\sqrt{3}$ 1555200 *the Superficies of that whole Icoſaedron.*

Nowe for the solide content if you adde $\sqrt{3}\cdot 192$ *(for so is the Dimetient of the containing circle) to* $\sqrt{3}\cdot 12$ *the Semidimetient of the contained circle, there will amount* $\sqrt{3}\cdot 300$, *which augmented by 24 the triangles side doubled bringeth* $\sqrt{3}\cdot 17280$, *and this by* $\sqrt{3}\cdot v.\frac{720}{10-R\,20}$ -48, *(for so is the Axis as yee may by the ninth Probleme perceine) multiplied, produceth* $\sqrt{3}\cdot vm.\ 7257600+\sqrt{3}\cdot 48372940800000$, *and that is the Solide capacitie of this body. The same summe is also produced by multiplication of* 20736, *the squared square of* 12, *in* $8\frac{1}{7}$, *and the product thereof in* $\sqrt{3}\cdot v.42+\sqrt{3}\cdot 1620$, *the Axis. Also if yee multiplye* 20736 *the sides squared square by* $2\frac{1}{11}$ *the offcome will bee* 43200, *which augmented againe by* 168 $+\sqrt{3}\cdot 25920$ *the square of the inscribed spheres Dimetient produceth* $7257600+\sqrt{3}\cdot 48372940800000$, *whose vniuersall Quadrate roote is the Solide capacitie of this Icosaedron, and by Reduction to numbers rationall it falleth out betwe ene* 3769 *and* 3770.

The side of Icosaedron 12.

The containing circles diameter. $\sqrt{3}\cdot 192$.
The conteyned circles semid. $\sqrt{3}\cdot 12$.
The comprehending spheres Semid $\sqrt{3}\cdot v.\frac{720}{10-R\,20}$.
The Axis, $\sqrt{3}\cdot v.\frac{720}{10-R\,20}-48$.
The superficiall content $\sqrt{3}\cdot 1555200$.
The solide capa. $\sqrt{3}\cdot v.7257600+\sqrt{3}\cdot 48372940800000$.

Theoremes of Icosaedrons contents

Superficiall and Solide.

The first Theoreme.

THe superficies of Icosaedron to the square of his side, beareth the proportion of $\sqrt{3}\cdot 75$ to 1.

The 2 Theoreme.

Anye Parallelogramme whose base is equall to double the Perimetrie of the Icosaedrons triangulare base, and his altitude to the circumscribing Circles Diameter and the inscribed Circles Semidiameter added togither, is equall to the Icosaedrons 20 triangles.

The 3 Theoreme.

When Tetraedrons side retayneth such proportion to Icosaedrons side, as Icosaedrons Diameter dooth to the Semidiameter of the Circle whereon it is framed, that Tetraedrons Superficies is equall to Icosaedrons.

The 4 Theoreme.

When the square of Octaedrons side contayneth the square of Icosaedrons sides medietie 10 times, then is that Octaedrons superficiall quantitie equal to Icosaedrons.

The fift Theoreme.

A direct Quadrangulare rightangled Prisma that hath for his three Dimensions, those three right lines, double the side, the Axis, and a line componed of the Diameter of the contayning Circle, and the Semidiameter of the contayned Circle, is equall to the solide content of Icosaedron.

The

The 15 Probleme

The side of Dodecaedron giuen, to searche out by Arithmeticall calculation the Superficiall and solide content.

THis bodye as was before declared among the Diffinitions is encompasſed with 12 equall Euiangle Pentagonal Superficies, ſo that, if yee ſearche the Area of any one (as was taught in the fift Probleme) and increaſe the ſame by 12, the Dodecaedrons ſuperficies reſulteth. Or by the forenamed fift Probleme yee ſhall learne the quantitie of the contayning Circles ſemidiameter, and alſo of the line Diagonall, the ſemidiameter yee ſhall augment by 3, the line Diagonall by 5, theſe products multiplyed together, bring the deſired Superficies alſo.

In like manner for the Craſſitude if yee multiply the ſemidiameter of the contayning Circle in the line Diagonall and the offcome in the Axis of this body) which yee were in the tenth Probleme taught to finde) the product augmented by 5, yeeldeth your deſire. Or the Semidiameter of the contayned circle in the Dimetient of the contayned ſphere, and the product in the Pentagonall Perimetrie bringeth the ſolide content alſo.

Example.

Admit the Dodecaedrons ſide 1 *by the* 5 *and* 10 *Probleme. I finde his contayning ſpheres ſemidimetient,* $\sqrt{3}$ *vni.* $\frac{9}{8}+\sqrt{3}\,\frac{81}{320}$, *the Axis* $\sqrt{3}$ *v.* $\frac{9}{8}+\sqrt{3}\,\frac{1215}{3136}-\sqrt{3}\,\frac{1}{20}$, *the containing Circles Semidimetient* $\sqrt{3}$ *v.* $\frac{1}{2}+\sqrt{3}\,\frac{1}{20}$, *the line Diagonall of the baſe* $\sqrt{3}$ *vniuerſ* $\frac{3}{2}+\sqrt{3}\,\frac{9}{4}$. *Nowe, for the Dodecaedrons ſuperficies, I ſearche firſt the Area of one Pentagonum, which I finde by the fifth Probleme* $\sqrt{3}$ *vni.* $\frac{25}{16}+\sqrt{3}\,\frac{625}{3136}$, *this augmented by* 12, *produceth* $\sqrt{3}$ *v.* $\frac{900}{4}+\sqrt{3}\,\frac{810000}{20}$. *Likewiſe the contayning Circles ſemidimetient tripled, maketh* $\sqrt{3}$ *vni.* $\frac{9}{8}+\sqrt{3}\,\frac{81}{320}$, *and the line Diagonall of the baſe increaſed by* 5, *yeeldeth* $\sqrt{3}$ *vni.* $\frac{25}{2}+\sqrt{3}\,\frac{81250}{16}$, *theſe multiplyed together, produce* $\sqrt{3}$ *vni.* $\sqrt{3}\,\frac{2531250}{16}+\sqrt{3}\,\frac{253125}{80}+\sqrt{3}\,\frac{4556250}{16}+\sqrt{3}\,\frac{4556250}{80}$, *which contracted, maketh* $\sqrt{3}$ *v.*225.$+\sqrt{3}$ 40500, *ſo much is the Dodecaedrons ſuperficiall quantitie, and being reduced to rationall numbers, it falleth out betweene* 20 *and* 21 *very nighe* $20\frac{13}{20}$.

Nowe, to attaine the ſolide quantitie I augment $\sqrt{3}$ *vni.* $\frac{9}{8}+\sqrt{3}\,\frac{81}{320}$ *the contayning Circles ſemidimetient, in* $\sqrt{3}$ *v.* $\frac{3}{4}+\sqrt{3}\,\frac{9}{64}$ *the line Diagonall, there ariſeth* $\sqrt{3}$ *vni.* $1+\sqrt{3}\,\frac{9}{20}+\sqrt{3}\,\frac{81}{320}$ *and this againe multiplied in* $\sqrt{3}$ *vni* $\frac{9}{8}+\sqrt{3}\,\frac{1215}{64}-\sqrt{3}\,\frac{1}{20}$ *the Dodecaedrons Axis the product augmented by* 5, *maketh* $\sqrt{3}$ *vni.* $10\frac{9}{8}+\sqrt{3}\,\frac{281250}{1024}+\sqrt{3}\,\frac{7593750}{1024}+\sqrt{3}\,\frac{2278125}{64}+\sqrt{3}\,\frac{759375}{1024}+\sqrt{3}\,\frac{14062500}{1024}-\sqrt{3}\,\frac{625}{20}$, *ſo much is the Dodecaedrons craſſitude.*

Againe, by the other rule I augment $\sqrt{3}$ *v.* $\frac{9}{4}+\sqrt{3}\,\frac{81}{320}$ *the ſemidimetient of the contayned circle, in* $\sqrt{3}$ *v.* $\frac{9}{4}+\sqrt{3}\,\frac{405}{64}-\sqrt{3}\,\frac{405}{20}$ *the Diameter of the inſcribed ſphere, there of reſulteth* $\sqrt{3}$ *v.* $1\frac{7}{40}+\sqrt{3}\,\frac{405}{64}+\sqrt{3}\,\frac{9}{16}-\sqrt{3}\,\frac{1}{20}$, *which multiplyed againe in* 5, *the Pentagonall perimetrie produceth* $\sqrt{3}$ *vni.*$29\frac{1}{8}+\sqrt{3}\,\frac{2278125}{64}+\sqrt{3}\,\frac{759375}{16}\sqrt{3}\,\frac{455625}{20}$ *this is alſo the Dodecaedrons ſolide capacitie exactly agreeing with that former operation, and by reduction is founde to reſt betweene* $\sqrt{3}$ 58 *and* $\sqrt{3}$ 59, *being very nighe* $7\frac{9}{16}$.

The side of Dodecaedron 1.
- The containing circles semidimetient $\sqrt{3}$ v. $\frac{1}{2}+\sqrt{3}\ \frac{1}{10}$.
- The conteyned circles semidiameter $\sqrt{3}$ v. $\frac{1}{4}+\sqrt{3}\ \frac{1}{6}$
- The line Diagonall $\sqrt{3}$ vni. $\frac{1}{4}+\sqrt{3}\ \frac{1}{4}$
- The containing spheres Semidimetient $\sqrt{3}$ vni $\frac{3}{4}+\sqrt{3}\ \frac{135}{216}$
- The Axis or Kathetus $\sqrt{3}$ v $\frac{5}{4}+\sqrt{3}\ \frac{125}{216}-\sqrt{3}\ \frac{1}{10}$
- The content superficiall $\sqrt{3}$ vni 225 $+\sqrt{3}$ 40500
- The Crassitude or solide capacitie $\sqrt{3}$ v. 29 $\frac{1}{4}+\sqrt{3}\ \frac{15125}{64}$

Theoremes of Dodecaedrons Superficiall and Solide quantities.

The first Theoreme.

A Pentagonall Equiangle superficies (whose side to the side of Dodecaedrons retayneth the proportion of the comprehending Spheres Diameter to the medietie of the line Diagonall) is equall to the Dodecaedrons content Superficiall.

The 2 Theoreme.

A rightangled Parallelogramme contayned of the Pentagonall perimetrie, and triple the Diameter of the contayned Circle, is equall to Dodecaedrons Superficies.

The 3 Theoreme.

That Cube whose side is meane proportional betweene the Semidiameter of the internal Circle, and the perimetrie of the Pentagonal base, hath a superficial content equal to Dodecaedrons.

The 4 Theoreme.

When an Icosaedrons side is meane proportional betweene the Pentagonal bases Diagonal and his circumscribing Circles Trigonal Corde, the Superficies of that Icosaedron is equal to the Superficies of Dodecaedron.

The 5 Theoreme.

If two meane proportional lines be founde betweene the Pentagonal Diagonal and his circumscribing Circles Corde Trigonal, that Icosaedron whose side is the meane proportional nigheſt to the Corde Trigonal, is equal to the Dodecaedron.

The 16 Probleme

The Diameter of anye Sphere knowne, to searche out the sides, Axes and containing or contayned Circles Semidiameters of all such bodyes Regulare as are therein to be described, both Arithmetically and Geometrically.

COnsidering that in anye Sphere all the Regulare bodyes maye bee described, as it is by *Euclide* sufficientlye Demonstrated, and that both the proportion of their sides, Axes, and Perpendiculares to the Spheres Dimetient, and also the maner of working in euery of them is differente, I thinke it best to adioyne seuerall Rules for euerye of their

their operations, and firſt for the inuention of their ſides peruſe the precepts following: ſquare the Spheres Diameter, and from the product deduct $\frac{1}{7}$, the roote Quadrate of the Remainder is the Tetraedrons ſide: Double the ſquare of the Spheres Semidiameter, ſo haue yee the ſquare of Octaedrons ſide, diuide the Diameters ſquare by 3, the roote Quadrate of your Quotient is the Cubes ſide. For the Icoſaedrons ſide ye ſhall diuide the Diameters ſquare by 5, the roote of the Quotient note, and ſuppoſing the ſame a Circles ſemidiameter, by the ſecond probleme ſearche out the Circles Pentagonall Corde, for that is the Icoſaedrons ſide. Finally, for the ſide of Dodecaedron, yee ſhall diuide the ſquare of the ſpheres Diameter by 3, and from the product extract the Quadrate roote, this roote diuided by extreame and meane proportion, (as yee were taught in the firſte Probleme) yeeldeth for his greater parte the Dodecaedrons ſide. Nowe the ſides of euerye Regular bodye thus founde, for their perpendiculares, Axes, and Semidiameters of their containing and contayned Circles, yee ſhall reſotte for Tetraedron to the ſixte Probleme, for Octaedron to the ſeauenth, for Hexaedron to the eight, and ſo in order for the reſte, where yee are taught by the ſide to ſearche out all thoſe forenamed lines, and ſo ſhall yee finde the exact quantitie of all perpendiculares Axes and Semidiameters of any regulare body in that ſphere contayned.

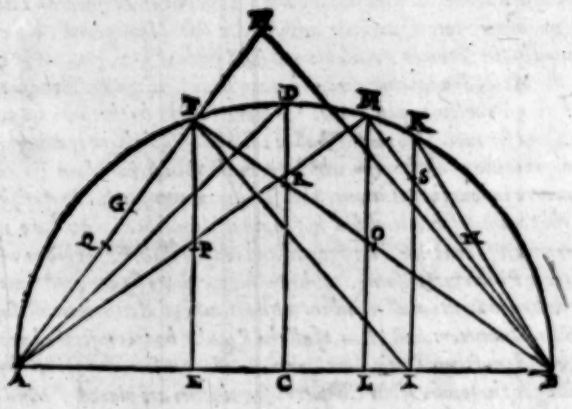
Example.

It would bee ouer tedious to ſhewe the Calculation for finding of euerye particular line, wherefore I ſhall onely giue Examples of the ſides, for that the practiſe of their Rules hath not beene yet ſet foorth in anye former Probleme. Admit the Spheres Diameter that ſhall comprehend theſe Regulare bodyes 10, the ſquare thereof diuided by 3, yeeldeth $33\frac{1}{3}$, which deducted from 100, leaueth $66\frac{2}{3}$, the Quadrate roote thereof is the inſcribed Tetraedrons ſide. The ſquare of the Spheres ſemid. is 25, which doubled maketh 50, the Zenzike *roote thereof, is the Octaedrons ſide, 100 diuided againe by 3, yeeldeth as before $33\frac{1}{3}$, the roote ſquare thereof is the Cubes ſide, which diuided by extreame and meane proportion (as was taught in the firſte Probleme) maketh the greater Segment $\sqrt{3}\,41\frac{2}{3} - \sqrt{3}\,8\frac{1}{3}$ ſo much I affirme the contayned Dodecaedrons ſide. Nowe, for the ſide of Icoſaedron I diuide 100 by 5, thereof ariſeth 20, whoſe roote Quadrate admitted a ſide Hexagonall, his correſpondente Corde Pentagonall by the ſeconde Probleme yee ſhall finde $\sqrt{3}\,v.50 - \sqrt{3}\,500$, the ſides thus knowne for the Axis, Semidiameters, and other lines, yee muſt vſe the ſame ſupputation that you did in thoſe 5 Problemes paſt, where yee were taught by the ſides knowne to attaine all the other lines, and for more plainneſſe I ſhall at the end of this chapiter adioyne a Table containing the true quantitie of all the reſt, which yee may vſe in ſteede of an Example to direct you, if happely you erre in your ſupputations, and for the further ſatiſfaction of ſuch as ſeeke to reache beyonde the common ſorte, and will not content themſelues with bare rules and precepts, vnleſſe they may alſo conceiue ſome ground and reaſon of their workings, I haue thought good to euery of theſe Problemes enſuing, to adioyne his peculiare figure, with meanes Geometricall (no regarde had to Irrationall numbers without aide of Arithmeticall ſupputation) to ſearche out the ſides, Diameters, and Axis, of all the Regular bodyes inſcribed or circumſcribed of Spheres, by knowledge of their Diameters, or mutuallye conferred together by knowledge of ſome ſide, according to the tenure of the Chapiter wherein they are placed. And although breuitie (which in this trifeling treatiſe I haue cheefely affected) compell me not to ſtay in making demonſtration of euery rule and Theoreme, yet the very conſtruction of the figures well wayed and conferred with Euclides 5 laſt bookes of Solides, will geue ſufficient light to the ingenious both to vnderſtand the cauſe of theſe, and to inuent many moe whereof there is no end.*

Geometri-

Geometrically without aide of Arithmeticall calculation, to attaine the quantitie of all these forenamed lines yee shall thus worke.

Admit the Diameter of the comprehending sphere geuen A B, which yee shall as was taught in the first booke diuide in two equall partes at C, and in three at E, A E being a third part, vpon either of those sections errear Perpendiculars, and (describing a Semicircle vpon the Diameter) note their intersections with F D, drawing lines from either of them to A B, so is A F the Cubes side, A D Octaedrons side, F B the side of Tetraedron, A F diuided by extreame and meane proportion (as yee were taughte in the firste Probleme at G) maketh A G the Dodecaedrons side, which extended out to H, yee shall make F H equall to F G drawing the righte line H B, and from F extende a Paralele to H B, till it crosse the Diameter in I, erecting thereupon the Perpendiculare I K, so is the Corde K B the inscribed Icosaedrons side, I L is a third parte of I B, M L a Paralele to I K, M B is the Diameter of Icosaedrons basis containing circle, whose medietie M N is the Diameter of the contained circle, the halfe thereof M S parted by extreame and meane proportion in V, so as S V be the greater segment. V N will be the Semidiameter of Dodecaedrons contained circle, and N B the Semidimetient of his containing circle. N C the Axis both of Icosaedron and Dodecaedron, V B is the Perpendicular of Dodecaedrons basis, and M A his Solides altitude, B S the perpendicular of Icosaedrons basis, and M A likewise his Solides altitude, A F (the inscribed cubes side) is also Dodecaedrons Basis line Diagonall, E F is the greater Semidiameter of Tetraedrons base, and E P his medietie the lesser Semidiameter, E C Tetraedrons Axis, E B his Perpendicular or altitude, F B the cubes line Diagonall, O B his medietie the greater Semidiameter of the cubes base, O C the lesse Semidiameter, and Hexaedrons axis Octaedrons containing circles Semidimetient, O F the Sediameter of his contained circle R O, his Axis C O, and A F his altitude. Thus haue yee Geometrically in one Figure, the exact quantities, and proportions of all the regular bodyes sides, Diameters, Axes, Perpendiculars, and lines Diagonall, whereby yee may also be able both to conceiue some reason of such rules as are past, or Theoremes that shall ensue, And also inuent diuers meanes to abreuiate such painfull calculation as by the former rules yee shall be forced to enter into, while yee laboure with Irrationall number to searche out the hidden proportions of these vnknowne lines, as by proofe the industrious will soone perceiue.



The com-

The comprehending Spheres Dimetient ginen A B 10

- Tetraedrons
  - Syde, B F √3' 66 2/7
  - Basis greater semid, F E √3' 22 2/7
  - Basis lesse semid. P E, √3' 5 1/7
  - Axis, E C, 1 1/7
  - Altitude, E B, 6 2/7
- Hexaedrons
  - Syde, F A, √3' 33 1/7
  - Basis greater Semid. O F, √3' 16 2/7
  - Basis lesse semid. O C, √3' 8 1/7
  - Axis C O, √3' 8 1/7
  - Altitude A F, √3' 33 1/7
- Octaedrons
  - Syde A D, √3' 50
  - Basis greater semid, O F √3' 16 1/7
  - Basis lesse semid. O R √3' 4 2/7
  - Axis Q F, √3' 8 1/7
  - Altitude A F √3' 33 1/7
- Icosaedrons
  - Syde K B, √3' v. 50 — √3' 500
  - Basis greater sem. M N, √3' v. 16 2/7 — √3' 55 5/9
  - Basis lesse semid. N. S √3' v 4 1/2 — √3' 3 17/28
  - Axis, C N, √3' v. 8 1/7 + √3' 55 5/9
  - Altitude M A, √3' v. 33 1/7 + √3' 888 2/9
- dodecaedrons
  - Syde A G, √3' 41 2/7 + √3' 8 1/7
  - Basis greater sems. N B, √3' v. 16 2/7 — √3' 55 5/7
  - Basis lesse semid. N V, √3' v. 4 1/2 + √3' 3 17/28
  - Axis C N, √3' v. 1/7 + √3' 55 5/9
  - Altitude M A, √3' v. 33 1/7 + √3' 888 2/9.

Theoremes of the Regular bodyes in one containing Sphere described.

The first Theoreme.

THe containing Spheres Diameters square to the square of the inscribed Tetraedrons side is as 3 to 2.

The 2 Theoreme.

The Spheres Dimetient is in power double to Octaedrons side.

The 3 Theoreme.

The Spheres Diameter is in power triple to the Cubes side.

The 4 Theoreme.

The Spheres Dimetient being Rationall, Icosaedrons side is such an Irrationall as *Euclide* calleth *Minor*, and beareth proportion to the Diameter, as √3' vni. 1/2 — √3' 1/28 to 1.

The 5 Theoreme.

The Spheres Dimetient Rationall, Dodecaedrons side is an Irrationall Apotome, retayning proportion to the dimetient, as √3' v. 1/15 — √3' 1/11 vnto 1.

The sixt Theoreme.

Tetraedrons Axis, is a sixth part of his Spheres dimetient.

The 7 Theoreme.

Octaedron and the Cubes Axis are equall, and triple in power to the Axis of tetraedron.

The

The 8 Theoreme.

The Axis of Icosaedron and Dodecaedron are equall, either of them being such an Irrationall as Euclide calleth Maior, the Spheres Dimetient admitted Rationall.

The 9 Theoreme.

Tetraedrons side, is the meane proportionall betweene his Perpendicular and his Diameter.

The tenth Theoreme.

Octaedrons containing Circles Semidimetient is meane in proportion betweene his inscribed Circles Semidiameter and Tetraedrons side.

The 11 Theoreme.

The Hexaedrons side is a meane proportionall betweene Tetraedrons side, and Octaedrons basis containing Circles Semidiameter.

The 12 Theoreme.

The Cubes side being Rationall, the Dodecaedrons side is an Irrational Apotome bearing proportion to the side of the Cube as $\sqrt{3}\cdot 1\frac{1}{4}-\frac{1}{2}$ vnto 1.

The 13 Theoreme.

Octaedrons side being Rationall, the side of Icosaedron is such an Irrationall as Euclide nameth Minor, and retaineth proportion to Octaedrons side as $\sqrt{3}\cdot$ vn. $1-\sqrt{3}\cdot\frac{1}{3}$ to an vnitie.

The 14 Theoreme.

Octaedrons side being Rationall, the Dodecaedrons side is an Irrationall Apotome, retaining proportion therevnto as $\sqrt{3}\cdot$ v.1.$-\sqrt{3}\cdot\frac{1}{5}$ vnto an vnitie.

The 15 Theoreme.

The Semidiameter of Dodecaedrons basis containing circle, hath the same proportion of the Cubes side, that Icosaedrons side hath to the Diameter of his containing sphere.

The 16 Theoreme.

Hexaedrons side being Rationall, the contayned Dodecaedrons circles semidiameter of Icosaedron is an Irrationall named by Euclide *Maior*, hauing proportion to the Cubes side, as $\sqrt{3}\cdot$v.$\frac{1}{4}+\sqrt{3}\cdot\frac{1}{12}$ vnto 1.

The 17 Theoreme.

Tetraedrons side admitted Rationall, the conteyned Circles semidiameter of Icosaedrons is an Irrationall *Minor*, bearing proportion to the Tetraedrons side as $\sqrt{3}\cdot$ vniuers.$\frac{1}{4}-\sqrt{3}\cdot\frac{1}{12}$ vnto $\sqrt{3}\cdot 2$.

The 18 Theoreme.

Octaedrons conteyned circles Diameter to the dimetient of Dodecaedrons conteyned circle, hath proportion as 1 to $\sqrt{3}$v.1.$+\sqrt{3}\cdot 2$.

The 19 Theoreme.

Dodecaedrons contayned Circles semidiameter being an Irrationall Maior, Icosaedrons conteyned circles Semidimetient shall be an Irrationall *Minor*, bearing proportion as $\sqrt{3}$v.1.$+\sqrt{3}\cdot\frac{1}{3}$ to $\sqrt{3}\cdot$vni.1.$-\sqrt{3}\cdot\frac{1}{3}$.

The 20 Theoreme.

Dodecaedrons side, to Octaedrons Axis, retayneth such proportion, as the greater

ter parte of a line diuided by extreame and meane proportion, to the medietie of the whole.

he 21 Theoreme.

If from the ſquare of Dodecaedrons Dimetient, yee ſubtract the ſquare of his ſide, the roote of the Remainder (diuided by extreame and meane proportion) maketh his greater part the Cubes ſide, and his leſſe the Dodecaedrons ſide.

he 22 Theoreme.

A right line equall in power to the Diameter and ſemidiameter of Dodecaedrons baſis conteyning circle, retaineth the ſame proportion to their conteyning ſpheres Diameter, that Dodecaedrons ſide dooth to the ſide of Icoſaedron.

he 23 Theoreme.

Dodecaedrons ſide retaineth the ſame proportion to the ſide of the cube, that the Semidiameter of Dodecaedrons conteyning Circle dooth to the Diameter of his conteyned circle.

he 24 Theoreme.

Dodecaedrons baſis internall Circles Semidiameter diuided by extreame and meane proportion, maketh his greater parte the Icoſaedrons inſcribed circles Semidimetient.

he 25 Theoreme.

If two lines equall in power to the Spheres Diameter, retaine the proportion of a line (diuided by extreame and meane proportion) to his greater parte, the leſſer of thoſe lines is Icoſaedrons ſide: but if their proportion bee as the whole to the leſſer part, then is that leſſe part the ſide of Dodecaedron.

The 18 probleme

Arithmetically and Geometrically to ſearch out all the ſides, Diameters, perpendiculars, and lines Diagonall, with the baſes Semidiameters of all ſuch regular bodyes as ſhall circumſcribe or comprehend any ſphere whoſe Dimetient is knowne.

SEeing theſe bodyes as it is demonſtrated by *Euclide*, are of ſuch vniforme compoſition that they will bothe receiue an inward ſphere touching with his conuex ſuperfi ies euery of their baſes centers, and alſo an outwarde ſphere incloſing and with his concaue peripherie touching euerye of their Angles for reſolution of this Probleme it ſhall bee requiſite to ſhew how the Diameter of the containing ſphere may be founde by knowledge of the contayned ſpheres Dimetient: which doone, by the propoſition laſt paſt, the ſides, and ſemidiameters, may conſequently bee knowne. But becauſe theſe 5 bodyes being deſcribed without one ſphere, are not alſo inuironed of one, but haue three ſeuerall different comprehending ſpheres, the largeſt enuironing Tetraedron, the nexte encompaſſing the Cube and Octaedron, and the laſt incloſing Icoſaedron and Dodecaedron (for this as it is by demonſtration approoued of Euclide, ſo is it alſo apparant by theſe former problemes) it ſeemeth therefore requiſite to preſcribe three ſeuerall rules for the inquiring out of their Dimetients. And firſt for Tetraedrons Diameter, yee ſhall

onely increase the diameter giuen by 3, the producte is your desire. And for the spheres Dimetient that contayneth the cube and Octaedron, yee shall diuide the Diameter geuen by $\sqrt{\text{ʒ}}\ \frac{1}{3}$ the Quotient or resulting summe is likewise your demand. Or diuide the same Diameter by $\sqrt{\text{ʒ}}$ vniuers. $\frac{1}{3}$ — $\sqrt{\text{ʒ}}\ \frac{8}{90}$. So haue yee the Spheres dimetient that comprehendeth Dodecaedron and Icosaedron.

Example.

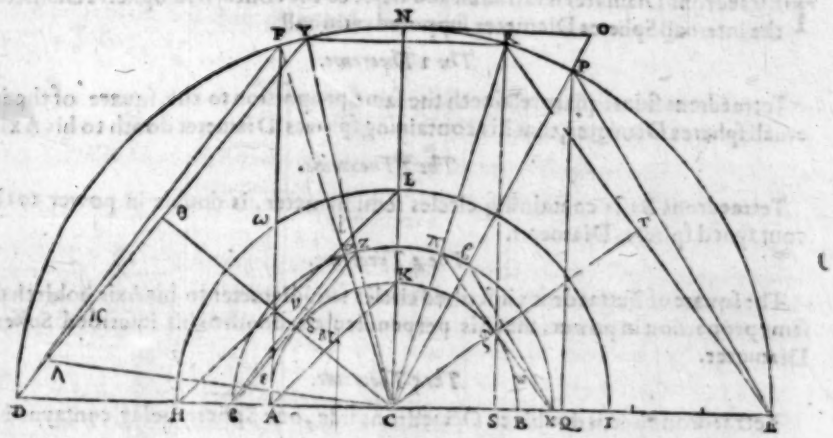
Admit the Diameter of the Sphere whereon these bodies shall be described 10, this augmented by 3, maketh 30, the Diameter of Tetraedron: Againe the square of 10 augmented by 3, bringeth 300, so is $\sqrt{\text{ʒ}}$ 300, the Diameter of Octaedron and the Cube: I diuide 100 by $\frac{2}{3}$ — $\sqrt{\text{ʒ}}\ \frac{4}{45}$ thereof ariseth 1500, — $\sqrt{\text{ʒ}}$ 1800000, I conclude therefore $\sqrt{\text{ʒ}}$ v. 1500 — $\sqrt{\text{ʒ}}$ 1800000, the Diameter of the sphere that shall include the Dodecaedron & Icosaedron, whose inscribed spheres Diameter is 10. and proceeding by the last probleme, yee shall finde the Dodecaedrons side $\sqrt{\text{ʒ}}$ v. 1150 — $\sqrt{\text{ʒ}}$ 1512500, Icosaedrons side $\sqrt{\text{ʒ}}$ vniuers. 1050 — $\sqrt{\text{ʒ}}$ 1012500, and so foorth of all the other solides sides, and semidiameters: for considering their operation is nothing differente from that was shewed in the 16 probleme, it were in vaine here againe to make therof a superfluous recitall.

Geometrically to perfourme the same.

Vpon A B the diameter giuen of the sphere (which all these regular bodies shall circumscribe) describe the semicircle, A K B, and vpon the same center C, describe the semicircle D N E, hauing his diameter D E triple to A B the diameter giuen, and from A, errrere the perpendicular A F, which diuided in two equall parts at ν, maketh A ν the lesse semidiameter of Tetraedrons base, and A F the greater: from F extende cordes to E D, E F is the tetraedrons side, and E A his perpendiculare. Nowe if ye fixe one foote of the compasse in C (and opening the other to ν) describe the semicircle H L I, it will touch the medietie of F E at G, whereby ye haue H I the diameter of Hexaedron and Octaedron, H L, the side of Octaedron, A B the side of the cube, A ν the semidimetient of Octaedrons basis conteyning circle, A K, the Semidiameter of the Cubes conteining circle, A C the Semidimetient of the Cubes inscribed Circle, and A M the medietie of A K is the octaedrons bases lesser Semidimetiente. Thus haue you founde the Diameters, sides, and circular Semidiameters, of these firste three bodyes: for the other two yee shall thus proceede, from N as yee were taughte in the firste booke, drawe N O paralele to C E equall to the medietie thereof, and (coupling C O together with a straighte line) from P where it cutteth the greatest Semicircles circumference let fall the Perpendicular P Q, and vpon S (leauing S Q equal to a 3 part of Q E) erect the perpendicular S R then from R and P extende straight lines to C & E, and to the middle of R E extend the right line C T, cutting the lest Semicircle in ε, from thence drawe a paralele to R E, cutting R C, C E ϛ, x, and (making C x a Semidiameter (describe the Semicircle β w x crossing C P in ι, from thence to x drawe the right line x ι, for that is the circumscribing Icosaedrons side, and ϛ x is the greater semidimetient of his base, w x the lesser, ϛ x the diameter of Icosaedrons comprehending sphere. This don from R drawe a paralele to the semicircles Diameter, crossing the greatest circumference in Y, fro thence to the end of the greatest, dimetient, drawe the line D Y, from whose medietie, drawe a straight line to the center C, then as was taught in the first booke, cut D ϛ in halfe at ϛ, and deuide D ϛ in extreame and meane proportion at λ by the first probleme, from these poynts λ F Y, to the center C extend right lines, cutting the circumference of the laste described Semicircle in the points α z, from them to β, draw lines again, & diuide the cord α in extreme & mene proportion at γ: likewise ϛ z is diuided in halfe at η, being both the contract of the lest circumference, & also the intersection made with θ C, but where λ C concureth with β z place this letter ι, so ι β γ the circumscribing dodecaedrons side β x his dimetient, ϛ η the semidiameter of his conteyning Circle, ϛ η the lesser semidiameter of the dodecaedrons pentagonall basis, and α his line Diagonall.

Thus

Thus haue ye in one figure all the sides and diameters both circular and spherall of all such regulare solides as comprehend or circumscribe the assigned sphere.



The diameter of the internall giuen Sphere 10

Tetraedrons.
- *Diameter D E*, 30
- *Syde E F*, $\sqrt{3}$ 600
- *Containing circles semidiameter, A F*, $\sqrt{3}$ 200
- *Inscribed circles semidimetient A*, $\sqrt{3}$ 50
- *Perpendiculare or Altitude, A E* 20

Hexaedrons
- *Diameter H I* $\sqrt{3}$ 300
- *Syde A B*, 10
- *Basis greater semidiameter, A K*, $\sqrt{3}$ 50
- *Basis lesser Semidimetient, A C* 5
- *Basis line Diagonall*, $\sqrt{3}$ 200
- *Altitude A B* 10

Octaedrons
- *Diameter H I* $\sqrt{3}$ 300
- *Syde H L*, $\sqrt{3}$ 150
- *Basis greater semid. A K* $\sqrt{3}$ 50
- *Basis lesse semidiameter, A M*, $\sqrt{3}$ $\frac{1}{2}$
- *Perpendicular A B*, 10

Icosaedrons
- *Diameter* x $\sqrt{3}$ v. 1500 — $\sqrt{3}$ 1800000
- *Syde* x. $\sqrt{3}$ vni 1050 — $\sqrt{3}$ 1012500
- *Basis greater semidiamet.* $\sqrt{3}$ v. 350 — $\sqrt{3}$ 112500
- *Basis lesse semidiameter* W, $\sqrt{3}$ v 87 $\frac{1}{2}$ — $\sqrt{3}$ 7031 $\frac{1}{4}$
- *Altitude* 10

Dodecaedrons.
- *Diameter* c x $\sqrt{3}$ v. 1500 — $\sqrt{3}$ 1800000
- *Syde* c $\sqrt{3}$ vni. 1250 — $\sqrt{3}$ 1512500
- *Basis greater semid* — $\sqrt{3}$ v. 350 — $\sqrt{3}$ 112500
- *Basis lesse semidiameter* $\sqrt{3}$ v. 37 $\frac{1}{2}$ — $\sqrt{3}$ 781 $\frac{1}{4}$
- *Altitude A B.* 10

Theoremes of the internal Spheres circumscribing Solides. 1.

TEtraedrons Diamater is rationall and triple to the contayned Spheres Diameter, the internall Spheres Diameter supposed rationall.

The 2 Theoreme.

Tetraedrons sides square retaineth the same proportion to the square of the internall spheres Diameter, that his containing spheres Diameter dooth to his Axis.

The 3 Theoreme.

Tetraedrons Basis containing circles semidiameter, is double in power to the contayned spheres Diameter.

The 4 Theoreme.

The square of Tetraedrons inscribed circles semidiameter to his Axis, holdeth the same proportion in power, that his perpendiculare dooth to his inscribed Spheres Diameter.

The 5 Theoreme.

Tetraedrons side, is double to Octaedrons side, one Sphere being contayned of them both.

The 6 Theoreme.

The square of Tetraedrons side, to the square of the Cubes side, retaineth the same proportion, that Tetraedrons comprehending spheres Diameter doothe to his Axis.

The 7 Theoreme.

The Diameter of Tetraedrons inscribed circle, is equall to the Diameter of Octaedrons containing circle.

The 8 Theoreme

Tetraedrons Diameter to the Diameter of the sphere that comprehendeth Octaedron and the Cube, hath the proportion of 3 to his Quadrate roote.

The 9 Theoreme.

Octaedron and the Cubes comprehending spheres diameter being rationall, the Diameter of Icosaedron and Dodecaedrons containing sphere is an irrationall Minor, bearing proportion to the forenamed Diameter, as $\sqrt{3}$ v,5 — $\sqrt{3}$ 20 vnto 1.

The 10 Theoreme.

Icosaedron and Dodecaedrons Axis being rationall, their comprehending spheres dimetient shall be an irrationall Minor, bearing proportion therevnto, as $\sqrt{3}$ vni. 60 — $\sqrt{3}$ 2880 vnto an vnitie.

The 11 Theoreme.

Icosaedrons Axis being rationall, his side is an Irrationall Apotome proportioned therevnto, as $\sqrt{3}$ v. 42 — $\sqrt{3}$ 1620 vnto an vnitie.

The 12 Theoreme.

Dodecaedrons Axis rationall, his side is a surde Minor proportioned to the axis, as $\sqrt{3}$ v,50 — $\sqrt{3}$ 2420 vnto an vnitie.

The 13 Theoreme.

Icosaedrons side being an Irrationall Apotome of the sixte order, Dodecaedrons side is an Irrationall Minor, retayning such Proportion thereto, as $\sqrt{3}$. v. 50 — $\sqrt{3}$ 2420 vnto $\sqrt{3}$ 27 — $\sqrt{3}$ 15

The 14 Theoreme.

Icosaedrons axis being rational, his basis containing circles semidiameter is an Apotome

potome of the first order bearing proportion to the axis, as 3—$\sqrt{3}$ 5. vnto an vnitie.

The 15 Theoreme.

Dodecaedrons axis being rational, his Pentagonall basis greater Semidiameter is an Apotome of the first order proportionall to his side, as $\sqrt{3}$ v. 7—$\sqrt{3}$ 45 to $\sqrt{3}$ v. 25—$\sqrt{3}$ 605,

The 18 probleme

The side of any Tetraedron giuen, to finde the sides, Diameters, and Axes, of all such Regulare bodies as may therein be described.

HAuing heretofore at large set foorth by Problemes sundrie wayes (the sides of these bodyes giuen) to finde the Semidiameters of their containing and contayned circles, the Diameters of their comprehending and comprehended spheres, with their contents superficiall and Solide: hauing also by Theoremes shewed manifolde diuersitie of proportions rationall and surde of these bodyes, their Superficies and lines compared with their comprehending and contained spheres, there remaineth onely now to conferre these bodyes mutually inscribed or circumscribed one with another, and to search out by the side of any one knowne, the sides and diameters both circulare and spherall, with the capacities superficiall and solide of all such bodyes as may within or without the same body bee described, I shall therefore first begin with Tetraedron, and so proceede with the rest. Tetraedron receyueth onelye Octaedron and Icosaedron, for the Cube and Dodecaedron cannot possiblye therein be so placed, that all their angles at one instant might exactly touche his superficies, the Tetraedrons side therefore giuen parted in two equall portions, either medietie is the inscribed Octaedrons side: Likewise the medietie of Tetraedrons sides square, is the square of Octaedrons Diameter, which diuided by 2 produceth a number, whose Quadrate roote is the Octaedrons Axis. For the inscribed Icosaedron yee shall diuide the medietie of Tetraedrons side by extreame and meane proportion, and double the square of the lesse portion the Quadrate roote of the product is the side. Or deduct one of these portions from the other, and adde the square of the remaines medietie to the square of Octaedrons sides medietie, the roote Quadrate of the product doubled, is the Icosaedrons Diameter: Or if from the last product, yee abate the thirde parte of Icosaedrons sides square, the roote Quadrate of the remaine is the Icosaedrons Axis.

Example,

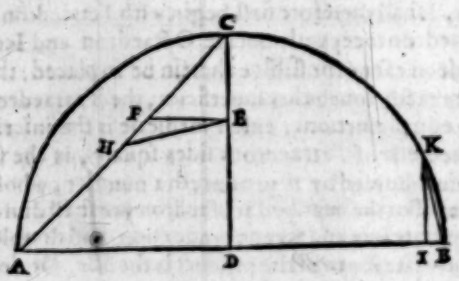
Tetraedrons side supposed, Octaedrons side is $\frac{1}{2}\sqrt{3}\ \frac{1}{2}$ *his Diameter, whose square diuided by 12 bringeth* $\frac{1}{24}$, *the roote being* $\sqrt{3}\ \frac{1}{24}$ *is the Octaedrons Axis. Likewise for Icosaedron the medietie of Tetraedrons side diuided in extreame and meane proportion by the first Probleme, maketh the lesser portion* $\frac{1}{4}$—$\sqrt{3}\ \frac{1}{16}$, *the square hereof doubled, hath for his roote* $\sqrt{3}$ vni. $\frac{1}{2}$—$\sqrt{3}\ \frac{1}{32}$, *so much is the inscribed Icosaedrons side. Again, the difference of tetraedrons sides medieties Portions diuided by extreame and meane proportion is* $\sqrt{3}\ \frac{1}{16}$—$\frac{1}{4}$, *the square of halfe this difference is* $\frac{1}{16}$—$\sqrt{3}\ \frac{1}{256}$, *which added to the square of Octaedrons sides medietie, produceth* $\frac{5}{16}$—$\sqrt{3}\ \frac{5}{16}$, *the roote thereof doubled, is* $\sqrt{3}$ v $\frac{2}{4}$—$\sqrt{3}$ 5 *the true quantitie of Icosaedrons dimetient. Now by subtracting* $\frac{2}{12}$—$\sqrt{3}\ \frac{48}{144}$ *the third part of Icosaedrons sides square, from* $\frac{5}{16}$—$\sqrt{3}\ \frac{1}{16}$ *the last product whose roote yee doubled to make the Diameter your remainder will be this number* $\frac{3}{16}$+$\sqrt{3}\ \frac{33}{144}$—+$\sqrt{3}\ \frac{9}{16}$ *whose roote vniuersall is the inscribed Icosaedrons Axis.*

The

The containynge Tetraedrons side,
- Tetraedrons
  - Diameter. $\sqrt{3}\frac{3}{2}$
  - Axis, $\sqrt{3}\frac{1}{24}$
- The inscribed Octaedrous
  - Side $\frac{1}{2}$
  - Diameter $\sqrt{3}\frac{2}{2}$
  - Axis, $\sqrt{3}\frac{1}{24}$
- The contayned Icosaedrons
  - Syde $\sqrt{3}$ vni. $\frac{2}{5}$ —— $\sqrt{3}\frac{1}{24}$
  - Diameter $\sqrt{3}$ vni. $\frac{1}{2}$ —— $\sqrt{3}$ 5
  - Axis, $\sqrt{3}\frac{1}{24}$

Or thus geometrically with out respect of number.

Admit A B the Tetraedrons side giuen, thereon I describe the semicircle A C B, erecting the perpendiculare D C, and drawing the line A C, that Corde A C, is the inscribed Octaedrons dimetiente, and A D his side. Now diuide D C by extreame and meane proportion as was taught in the firste Probleme at E, and from E erect the Perpendiculare E F, cutting A C in F, conclude F C the inscribed Icosaedrons side: againe parte A C in halfe at H, and from H to E extende a stra ght line, for that shall be the Icosaedrons semidiameter. For the Axis ye shall cut of from D B his twelfth parte, as ye were taught in the first booke: suppose it B I, vpon I erecte a Perpendiculare, cutting the circumference in K, so is the Corde K B the inscribed Icosaedron and Octaedrons Axis. As for their bases circulare semidiameters they are founde as was taught in the fifth and sixteenth Problemes, for asmuch as both their sides and dimetientes are knowen.



Theoremes of Tetraedrons inscribed solides. 1.

Tetraedrons side is double to the side of his contayned Octaedron.

The 2 *Theoreme.*

Octaedrons diameter is a meane proportionall betweene his side and the side of his contayning Tetraedron.

The 3 *Theoreme.*

If any equylater triangle be so described and situate within the Tetraedrons equiangle triangulare base, that the angles of the inscribed triangle diuide the sids of the base by extreame and meane proportion, the medietie of this inscribed Triangles side, is the side of this Icosaedron.

The 4 *Theoreme.*

Tetraedrons side deuided by extreame and meane Proportion, his Lesser parte is double in power to this Icosaedrons side.

The 5 *Theoreme.*

The dimetient of Octaedron diuided by extreame & meane Proportion maketh his lesser portion the Icosaedrons side.

The

The 6 Theoreme

Icoſ edrons Diameter is equall in power with Octaedrons ſide, and the ex eſſe or difference of his two partes, the one deducted fro the other.

The 7 Theoreme.

The ſide of Tetraedron being Rationall, his inſcribed Icoſaedrons ſide is an Apotome, bearing proportion to the Tetraedrons ſides, as $\sqrt{3}$ v. 1, $\frac{1}{4}$ — $\sqrt{3}$ $\frac{45}{64}$ vnto 1.

The 8 Theoreme.

The Axes of theſe inſcribed bodyes Octaedron and Icoſaedron are equall.

The 9 Theoreme.

The ſide of Tetraedron being rationall, the contained bodies axes are rationall, in power commenſurable, and beare proportion to the ſide as $\sqrt{3}$ 24 to 1.

The 10 Theoreme.

Octaedrons Diameter being Rationall, Icoſaedrons dimetient is an Irrationall *Minor*, proportioned to the Dimetient of Octaedron as $\sqrt{3}$ v. 5 — $\sqrt{3}$ 20 vnto an vnitie.

The 19 Probleme.

The ſide of a Cube giuen, to finde the ſides, Diameters and Axes of all ſuch regulare bodyes as may therein be deſcribed.

THe Cube is capable of three bodies, Tetraedron, Octaedron, and Icoſaedron, for the greateſt Dodecaedron that may within a Cube be imagined, will onely with 12 of his angles touch the Cubes ſuperficies, his other eight angles remaining within the body vnder euery of the Cubes angles, diuiding his ſemidimetients by extreame and meane proportion: this body therefore omitted bicauſe his inſcription is vnperfect, I ſhall giue rules for the other three, and firſt of Tetraedron, double the ſquare of the Cubes giuen ſide, the product roote Quadrate is the Tetraedrons ſide, the ſame giuen ſides ſquare tripled bringeth his comprehending ſpheres Diameters ſquare, which diuided by 36 yeeldeth in the Quotient a number, whoſe quadrate roote is the Tetraedrons axis. The Cubes ſides ſquares medieties roote Quadrate is the inſcribed Octaedrons ſide.

The Octaedrons Diameter is equall to the Cubes ſide.

The Cubes ſides ſquare diuided by 12, bringeth the contained Octaedrons Axis ſquare. For the contayned Icoſaedrons ſide, yee ſhall parte the contayning Cubes ſide giuen by extreame and meane proportion, as yee were taught in the firſte probleme, the greater part is your deſire. And if you adde the ſquare thereof to the ſquare of Hexaedrons ſide, the roote quadrate of the reſulting ſumme is the Icoſaedrons dimetient. And if ye ſquare the medietie of this dimetient, and from it deduct a third part of the ſquare of this inſcribed Icoſaedrons ſide, the roote quadrate of the remaine is the Icoſaedrons Axis.

Example.

Hexaedrons ſide giuen 1, his ſquare doubled is 2, $\sqrt{3}$ 2 is the contained Tetraedrons ſide, the giuen ſides ſquare tripled is 3, his quadrate roote is Tetraedrons diameter, that ſquare diuided 36 yeldeth $\frac{3}{12}$, $\sqrt{3}$ $\frac{3}{12}$ is the axis. Likewiſe $\sqrt{3}$ $\frac{1}{2}$ being the quadrate roote of halfe the cubes ſide is the inſcribed octaedrons ſide, the Octaedrons diameter being equal to the cubes ſide is 1 alſo, & the cubes ſide ſquare diuided by 12 yeldeth $\frac{1}{12}$, I conclud $\sqrt{3}$ $\frac{1}{12}$ the axis. Againſ fro Icoſaedron I diuid 1 by extreme & mene proportion, the greater part is $\sqrt{3}$ $\frac{1}{4}$ — $\frac{1}{2}$ ſo much is the

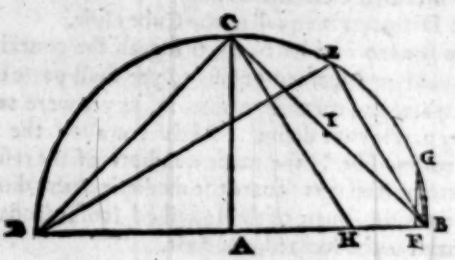
Ico-

Icosaedrons side, the square of this side added to the square of the Cubes side, maketh $2\frac{1}{2} - \sqrt{\frac{1}{2}}$ *whose roote vniuersall is the Diameter: from the square of this rootes medietie being* $\frac{1}{4} - \sqrt{\frac{1}{16}}$ *if ye deduct* $\frac{1}{3} - \sqrt{\frac{1}{72}}$ *the third parte of the sides square, there will remaine* $\frac{1}{12} + \sqrt{\frac{1}{144}} - \sqrt{\frac{1}{44}}$ *the roote vniuersall thereof is Icosaedrons axis.*

The Cubes side 1. His inscribed.

- Tetraedrons
  - Syde $\sqrt{2}$.
  - Diameter $\sqrt{3}$.
  - Axis $\sqrt{\frac{3}{8}}$
- Octaedrons
  - Side $\sqrt{\frac{1}{2}}$
  - Diameter. 1.
  - Axis $\sqrt{\frac{1}{12}}$
- Icosaedrons
  - Side $\sqrt{\frac{1}{4}} - \frac{1}{2}$
  - Diameter $\sqrt{\text{v.}2\frac{1}{2} - \sqrt{\frac{1}{4}}}$
  - Axis $\sqrt{\text{v.}\frac{1}{4} + \sqrt{\frac{1}{16}} - \sqrt{\frac{1}{12}}}$

Ye may also with the compasse find out all these sides, diameters, and axis, no regard had to any number so the containing Cubes side bee knowne. Admitte the Cubes side giuen A B, extende the same line out to D, and vppon A as a centre, making A B the Semediameter, describe the Semicircle B D C, and vppon A erecte the Perpendiculare A C, as yee were taught in the firste booke concurring with the circumference in C, againe as yee were taught in the firste Booke cutte off from A B $\frac{1}{24}$ *parte. Admitte it F B, vppon F erecte an other Perpendiculare crossinge the Circumference in G, diuide also C B in two equall partes in I, then open your compasse to the length of A B, and fixinge one foote in B, with the other crosse the circumference in E. Laste of all by the fyrste Probleme diuide A B by extreame and meane Proportion in H, so as A H may be his greater portion, and extende right lines from H to C, from B to C, from E to B D, and from G to B. Thus haue ye the Diameters sides and axes of those inscribed bodies, for C B is the inscribed Tetraedrons side, D E the Tetraedrons Dimetiente, G B the Tetraedrons axis, B I is the inscribed Octaedrons side, A C the Octaedrons Dimetiente, G B also Octaedrons axis, A H is the side of the inscribed Icosaedron, and C H his Diameter, as for the Axis yee maie Geometricallie finde it by the sixteenth Probleme, the comprehending Spheres Dimetient beeing knowen, whereof I minde not heere to make any newe recitall, considering it is sufficient plainely declared before.*



Theoremes of Hexaedrons inscribed regulare Solides.

The first Theoreme.

THe inscribed Tetraedrons side is double in power to the containinge Cubes side.

The

The 2 Theoreme.

The contayning Cube and his contained Tetraedron haue one comprehending ſphere, but their Axis are different, and their proportion in power triple.

The 3 Theoreme.

The containing cubes axis is equall to his inſcribed Octaedrons ſemidiameter, and their Axis in power retaine triple proportion.

The 4 Theoreme.

Hexaedrons ſide diuided by extreame and meane proportion, maketh his greater parte the Icoſaedrons ſide.

The 5 Theoreme.

The inſcribed Icoſaedrons dimetient is in power equall to his conteyning cubes ſide, and the greater portion thereof, it being diuided by extreame and meane proportion.

The 6 Theoreme.

The Cubes ſide being rationall, his inſcribed Icoſaedrons ſide is an *Apotome* bearing proportion to the cubes ſide, as $\sqrt{3}$ 1 $\frac{1}{4}$—$\frac{1}{2}$ vnto an vnitie.

The 7 Theoreme.

Hexaedrons ſide is meane proportionall betweene his inſcribed Tetraedron and Octaedrons ſide.

The 8 Theoreme

The Cubes ſide is equall in power to his Axis, and the conteyned Tetraedrons Semidiameter.

The 9 Theoreme.

Hexaedrons diameter to his inſcribed Tetraedrons perpendicular, retaineth the proportion of 3 to 2.

The 10 Theoreme.

The comprehending cubes ſide being rationall, his contayned Icoſaedrons ſide is an Irrationall Binomye, bearing proportion vnto the cubes ſide as $\sqrt{3}$ vniuerſalis $\frac{1}{2}+\sqrt{3}\ \frac{1}{172}$ to 1.

The 20 Probleme.

Octaedrons ſide giuen, to ſearche out all his contained bodyes, ſides, Diameters and Axes.

Odecaedron within this body may not aptly bee deſcribed, for as much as onely eight of his ſolide angles touche the ſuperficies of this body being ſituate in the centers of Octaedrons baſes, the other 12 falling quite within this body, not touching his ſuperficies on anye parte. But Tetraedron may be inſcribed and all his ſolide angles placed in the centers of Octaedrons triangles. Likewiſe the cubes ſolide angles take their place in the centers of his baſes, onely Icoſaedron hath his ſituation ſomwhat more ſtraunge and intricate, euery of his 12 angles reſting in a ſide of Octaedron, and diuide his 12 ſides by extreame and meane proportion. Now to attaine the diameters axes &c. peruſe theſe rules enſuing.

For Tetraedron.

Augment Octaedrons ſide by 2, and diuide by 3, the quotient is Tetraedrons ſide,

the square thereof augmented by 3, and parted by 2, yeeldeth his Diameter, which diuided by 6 declareth the axis.

The square of Octaedrons side multiplied by 2 and diuided by 3 produceth thys Cubes Diameters square, whych againe diuided by 3, sheweth his sides square, halfe the side is the Axis.

For Icosaedron.

Diuide the side giuen by extreame and meane proportion, the square of the lesser parte double, and from the producte extracte the roote quadrate, so haue ye the inscribed Icosaedrons side 3 deduct one of those former founde partes of the giuen side from the other, and square the difference, for that added to the square of Icosaedrons side bringeth the square of his dimetient. Nowe for the axis ye may deducte the third part of Icosaedrons sids square from the square of his Semidimetiente, the roote quadrate of the remainder is his Kathetus.

Examples of Tetraedron.

Octaedrons side giuen 1, increased by 2, and diuided by 3 maketh $\frac{3}{1}$, the Tetraedrons side, the square thereof being $\frac{9}{1}$ augmented by 3 and parted by 2, yeeldeth $\frac{3}{1}\sqrt{3}\,\frac{3}{1}$ is his diameter, whiche diuided by 6, bringeth $\sqrt{3}\,\frac{1}{14}$ the Axis.

Of the Cube.

The square of Octaedrons side 1, augmented by 2, and diuided by 3, produceth $\frac{3}{1}$ the diameters square, that againe diuided by 3, yeeldeth $\frac{3}{3}\sqrt{3}\,\frac{3}{1}$ is the cubes side, whose halfe being $\sqrt{3}\,\frac{1}{12}$ is the axis.

Of Icosaedron.

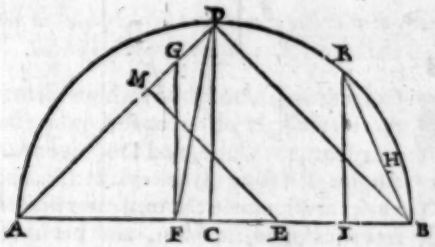
Octaedrons side 1 diuided by extreme and meane proportion, the greater part is $\sqrt{3}\,\frac{1}{4}-\frac{1}{2}$, the lesser $\frac{1}{2}-\sqrt{3}\,\frac{1}{4}$, this latter partes square doubled is $7,-\sqrt{45}$, the roote Quadrate vniuersall thereof is Icosaedrons side. Againe by subtraction of the partes yee shall finde the difference $\sqrt{3}\,5-2$, the square thereof added to the square of Octaedrons side giuen, maketh $10,-\sqrt{3}\,80$. I conclude $\sqrt{3}\,vni,10,-\sqrt{3}\,80$ the Icosaedrons diameter. Finallye for the axis I deducte $\frac{7}{4}-\sqrt{3}\,5$, the thirde parte of Icosaedrons sides square, from $\frac{1}{2}-\sqrt{3}\,5$, the semidimetient of Icosaedrons containing sphere, there remainet $b\,\frac{1}{4}\sqrt{3}\,\frac{1}{4}$, is the axis.

The comprehending Octaedrons side. 1

- *Tetraedrons*
  - *Syde* $\frac{3}{1}$
  - *Diameter* $\sqrt{3}\,\frac{3}{1}$
  - *Axis* $\sqrt{3}\,\frac{1}{14}$
- *Hexaedrons*
  - *Syde* $\sqrt{3}\,\frac{3}{1}$
  - *Diameter* $\sqrt{3}\,\frac{3}{1}$,
  - *Axis* $\sqrt{3}\,\frac{1}{12}$
- *Icosaedrons*
  - *Syde* $\sqrt{3}\,vni,7-\sqrt{3}\,45.$
  - *Diameter* $\sqrt{3}\,v10-\sqrt{3}\,80.$
  - *Axis* $\sqrt{3}\,\frac{1}{2}.$

Or thus without ayde of numbers, admit AB the Octaedrons side giuen, thereon (making C the mediety a center) I describe the semicircle ADB, and from C, I erecte the perpendicular CD,

C D, drawing lines from D to A B, the Semidiameter C B, I diuide in 3 equall partes at E and I, vppon I, I errere the perpendicular I K, concurring with the circumference in K. from E, I drawe E M paralele to D B, cutting A D in M, as was targht in the firste booke of this treatise: againe by the first probleme I diuide A B by extreame and meane proportion in F, and thereupon I raise the perpendicular F G, crossing A D in G, and from F to D, I extend a streight line, finallie I drawe the right line K B, cutting of a thirde parte at H, and thus is the figure fully finished, containing all these bodyes, sides, Diameters, and Axes: For A E is Tetraedrons side, K B his Semidiameter, H B his Axis, A M the cubes side, K B his Semidiameter, M D his axes, A G Icosaedrons side, F D his semidiameter, and K B his axes.



Theoremes of Octaedrons inscribed Regulare bodyes.

The first Theoreme.

THe conteining Octaedrons side beareth proportion to the side of his conteyned Tetraedron, as 3 to 2.

The 2 Theoreme.

Octaedrons side, to his inscribed cubes side, beareth the proportion of $\sqrt{3}$ '4 $\frac{1}{2}$ vnto 1.

The 3 Theoreme.

The side of Tetraedron, to the side of the cube, is as 2 vnto $\sqrt{3}$ '2.

The 4 Theoreme.

Octaedrons contayning dimetient, is triple in power to his inscribed Icosaedrons Axis.

The 5 Theoreme.

The containing Octaedrons diameter is triple to the conteyned cubes side.

The 6 Theoreme

The Axis of these three inscribed bodyes are proportional, and the cubes Axis is the meane proportion betweene the other two.

The 7 Theoreme.

The side of Octaedron diuided by extreame and meane proportion, that quantitie which is double in power to the lesser parte, is equall to the inscribed Icosaedrons side.

The 8 Theoreme.

Icosaedrons Diameter is equall in power to the side of the conteyning Octaedron, and the difference or excesse of Octaedrons sides partes being diuided in extreame and meane proportion.

The 9 Theoreme.

Octaedrons dimetient diuided by extreame and meane proportion maketh his lesser segment the inscribed Icosaedrons side.

The 10 Theoreme.

Octaedrons side being Rationall, Icosaedrons side is an Apotome proportioned to the Octaedrons side, as $\sqrt{3}$ vniuers. 7 — $\sqrt{3}$ 45 vnto an vnitie.

The 21 probleme

Icosaedrons side geuen, in line or number to set foorth all the sides, Diameters and Axes of his contained regulare bodyes.

Icosaedron is a bodye of such vniforme capacitie, that hee receiueth all the other foure regulare bodyes, whereof Tetraedron, the Cube, and Dodecaedron, haue their solide Angles all resting in the centers of his bases, & Octaedrons angles are situate in the medieties or middle sections of Icosaedrons opposite sides, and his three diameters crosse themselues at right angles vpon the center of his spheres. Now for the inuention of their sides, Diameters, and Axes, note the rules ensuing.

Of Tetraedron.

Diuide the giuen side by extreame and meane proportion, adioyning therevnto his greater part the product square, and from it subtract the thirde parte of the square of the geuen side, the roote Quadrate of the remainder is the Tetraedrons containing spheres Diameter, and the third part of his square doubled is the square of the Tetraedrons side, whose third parte deducted fro the fourth parte of the Diameters square, leaueth the square of the Axis, or diuide the Dimetient by sixe, so haue yee the Axis also.

Of Octaedron.

The side of Icosaedron giuen, being parted by extreame and meane proportion, hauing his greater parte to him adioyned, yeeldeth the Octaedrons dimetient, whose squares medieties roote is the side, and the third parte of his sides square abated from the square of the Semidimetient, leaueth the square of his Axis.

Of Hexaedron.

The Cubes Diameter is equall to the Diameter of Tetraedron, which found as before, yee shall diuide the square thereof by 3, the Quotients roote Quadrate is the Cubes side, whose medietie is his Axis.

Of Dodecaedron

His Diameter is equall to the Diameter of the inscribed cube, and his side is the third part of Octaedrons Dimetient, which yee were tofore taught to find. Now for the Axis it behoueth you first to search out the Semidiameter of Dodecaedrons containing circle, which yee are taught to doe in the fifth and tenth Problemes, the square of this Semidiameter deducted fro the square of Dodecaedrons Semidimetient, leaueth the square of his Axis,

Example,

Example,

Suppose the comprehending Icosaedrons side 12, this diuided by extreame and meane proportion, maketh his greater parte $\sqrt{\text{ʒ}}$ 180 — 6, *which added vnto* 12, *bringeth* $\sqrt{\text{ʒ}}$ 180 + 6, *the square thereof is* 216 + $\sqrt{\text{ʒ}}$ 25920, *from this deduct the third part of Icosaedrons sides square there remaineth* 168 + $\sqrt{\text{ʒ}}$ 25920 *the roote Quadrate vniuersall thereof is Tetraedrons Diameter, the third part of that square is* 56 + $\sqrt{\text{ʒ}}$ 2880, *which doubled, maketh* 112 + $\sqrt{\text{ʒ}}$ 11520, *so is* $\sqrt{\text{ʒ}}$ v. 112 + $\sqrt{\text{ʒ}}$ 11520 *the Tetraedrons side. Now for the Axis yee must subtract* 37 $\frac{1}{3}$ + $\sqrt{\text{ʒ}}$ 1280 *being the third parte of Tetraedrons sides square from* 42 + $\sqrt{}$ 1620, *for so much is the fourth parte of Tetraedrons dimetients square, there remaineth* 4 $\frac{2}{3}$ + $\sqrt{\text{ʒ}}$ 1620 — $\sqrt{\text{ʒ}}$ 1280, *the roote vniuersall thereof is the Axis. Or diuide* $\sqrt{\text{ʒ}}$ v. 168 + $\sqrt{\text{ʒ}}$ 25920 *by* 6, *thereof commeth* $\sqrt{\text{ʒ}}$ v. 4 $\frac{2}{3}$ + $\sqrt{}$ 20, *the Axis also, agreeing with the former operation, as by reduction of the former Trimonye to a Binomye yee shall perceiue. This example might suffice considering the other rules are as euident and easie to bee practised as this, for more plainnesse beholde the table folowing, where yee shall finde the number of all the sides, Diameters and Axes of these inscribed bodyes ready supputate, which may serue your turne as well for better vnderstanding of the rules, as also for examination and triall of your operations.*

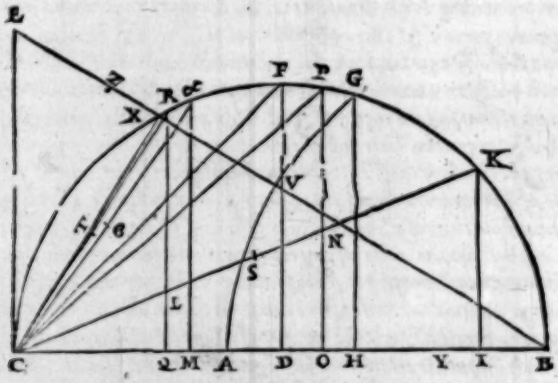
Icosaedrons inscribed Regular solides sides.

Icosaedrons dia. $\sqrt{\text{ʒ}}$ v. 310 + $\sqrt{\text{ʒ}}$ 25920	*Tetraedrons*	*Syde* $\sqrt{\text{ʒ}}$ v. 112 + $\sqrt{\text{ʒ}}$ 11520 *Diameter* $\sqrt{\text{ʒ}}$ v. 198 + $\sqrt{\text{ʒ}}$ 25920 *Axis* $\sqrt{\text{ʒ}}$ v. 4 $\frac{2}{3}$ + $\sqrt{\text{ʒ}}$ 20
The comprehending Icosaedrons side 12.	*Octaedrons*	*Side* $\sqrt{\text{ʒ}}$ v. 108 + $\sqrt{\text{ʒ}}$ 6480 *Diameter* $\sqrt{\text{ʒ}}$ v. 216 + $\sqrt{\text{ʒ}}$ 25920 *Axis,* $\sqrt{\text{ʒ}}$ v. 18 + $\sqrt{\text{ʒ}}$ 188
	Hexaedrons	*Syde* $\sqrt{\text{ʒ}}$ v. 56 + $\sqrt{\text{ʒ}}$ 2880 *Diameter* $\sqrt{\text{ʒ}}$ v. 168 + $\sqrt{\text{ʒ}}$ 25920 *Axis* $\sqrt{\text{ʒ}}$ v. 14 + $\sqrt{\text{ʒ}}$ 180
Icosaedrons Axis $\sqrt{\text{ʒ}}$ v. 42 + $\sqrt{\text{ʒ}}$ 1620	*dodecaedrons*	*Syde,* $\sqrt{\text{ʒ}}$ v. 24 + $\sqrt{\text{ʒ}}$ 320 *Diameter* $\sqrt{\text{ʒ}}$ v. 168 + $\sqrt{\text{ʒ}}$ 25920 *Axis,* $\sqrt{\text{ʒ}}$ v. 18 + $\sqrt{\text{ʒ}}$ 64 $\frac{4}{5}$ + $\sqrt{\text{ʒ}}$ v. d. 8 $\sqrt{\text{ʒ}}$ $\frac{1184}{405}$

Geometrically without aide of Arithmeticall Calculation, yee shall thus searche the quantitie of euery side, Diameter, or Axis. Admit AB the containing Icosaedrons side, this line I diuide by extreame and meane proportion in γ, *and extending forthe AB, I make AC equall to A*γ, *and vpon CB as a Diameter I describe the semicircle GFB, whose center is D, then opening my Compasse to AB, and fixing one foote in B, with the other I drawe the Arke AG, crossing the Semicircle in G, from thence letting fall the Perpendicular GH, from HB I cut of the third part IB, and IC I diuide in three equall partes with the pointes at QO. Now vpon QDOI, I ereare Perpendiculars, crossing the circumference of the semicircle in RFPK, and from CI extend cordes to RFKP, noting the intersections of CK, and QR, P, O, with LN: I parte also LN in halfe at S, and CR at T, this done I raise a Perpendicular againe vpon C, making CE equal to AB the side giuen, and drawing a right line from E to B, I marke his intersection made by FD with the letter V, finally from C let fall a perpendicular right line to EB, crossing it in* x, *thus cutting of from EX & CB their third parts at M & Z, erecting vpon M the perpendicular M* α, *& diuiding the Corde C* α *in halfe at* β, *yee haue in this figure the true lengthes*

and

and quantities of euery side, Diameter, or Axis of all such Regular Solides as are within that Icosaedron, whose side is giuen, comprehended.



The inscribed regulare Solides lines.

- A B *The comprehending Icosaedrons side geuen* 1
- E B *The comprehending Icos. Diameter* $\sqrt{3}\,v.\frac{1}{2}+\sqrt{3}\,\frac{1}{2}$
- S K *The comprehending Icosaedrons Axis,* $\sqrt{3}\,v.\frac{7}{24}+\sqrt{3}\,\frac{1}{24}$
- C P *Tetraedrons side,* $\sqrt{3}\,v.\frac{2}{3}+\sqrt{3}\,\frac{1}{3}$
- C K *Tetraedrons Diameter,* $\sqrt{3}\,v.1\,\frac{1}{2}+\sqrt{3}\,\frac{1}{4}$
- S N *Tetraedrons Axis,* $\sqrt{3}\,v.\frac{7}{11\,2}+\sqrt{3}\,\frac{1}{11\,4}$
- C R *Hexaedrons side,* $\sqrt{3}\,v.\frac{2}{12}+\sqrt{3}\,\frac{1}{12}$
- C K *Hexaedrons Dimetient,* $\sqrt{3}\,v.1\,\frac{1}{2}+\sqrt{3}\,1\,\frac{1}{4}$
- C T *Hexaedrons Axis* $\sqrt{3}\,v.\frac{7}{12}+\sqrt{3}\,\frac{1}{12}$
- C F *Octaedrons side,* $\sqrt{3}\,v.\frac{1}{4}+\sqrt{3}\,\frac{1}{12}$
- C B *Octaedrons Dimetient,* $\sqrt{3}\,\frac{1}{4}+\frac{1}{2}$
- C T *Octaedrons Axis* $\sqrt{3}\,v.\frac{1}{8}-\sqrt{3}\,\frac{3\,1}{2\,3\,1\,2}$
- C M *Dodecaedrons side* $\sqrt{3}\,\frac{1}{12}+\frac{1}{2}$
- C K *Dodecaedrons Diameter* $\sqrt{3}\,v.\frac{1}{2}+\sqrt{3}\,\frac{1}{4}$
- V Z *Dodecaedrons Axis,* $\sqrt{3}\,v.\frac{1}{2}+\sqrt{3}\,\frac{1}{12\,2}+\sqrt{3}\,v.\,D.\frac{1}{12}\sqrt{3}\,\frac{1}{12\,2}$

Theoremes of Icosaedrons inscribed bodyes.

The first Theoreme.

THe containing Icosaedrons Dimetient is in power equall with his side and the Diameter of his inscribed Octaedron.

The second Theoreme.

The inscribed Octaedrons dimetient, is in power equall to the Dimetient of the contained Cube, and the Diameter of Icosaedrons contained circle.

The third Theoreme.

Octaedrons Dimetient, (diuided by extreame and meane proportion) his greater segment is the comprehending Icosaedrons side.

The fourth Theoreme.

Icosaedrons Dimetient in power is equall with the diameter of his containing circle, and the dimetient of his contained cube.

The

The 5 Theoreme.

Tetraedrons side, is double in power to the side of the contained cube.

The 6 Theoreme.

Tetraedrons Diameter, is triple in power to the inscribed Cubes side.

The 7 Theoreme.

Dodecaedrons inscribed side is the third parte of such a line as diuided by extreame and meane proportion maketh his greater parte the contayning Icosaedrons side.

The 8 Theoreme.

Octaedrons Diameter retaineth the same proportion to the side of Dodecaedron that the Diameter of Tetraedron doth in power to the side of the Cube.

The 9 Theoreme.

Icosaedrons Diameter retaineth the same proportion to the Dimetient of Octaedron, that the side of Icosaedron beareth to the Semidiameter of that circle, whereon Icosaedron is framed.

The tenth Theoreme.

Icosaedrons Axis is triple in power to the Axis of the contayned Cube.

The 22 Probleme.

The side of any Dodecaedron giuen, both Arithmetically and Geometrically to searche out the sides, Diameters and axis of all the regulare bodies therein described.

SVch is the resemblance and mutuall conformitie betweene Icosaedron and this bodye, that the 12 solide Angles of Icosaedron will precisely rest in the centers of Dodecaedrons Pentagonall bases, the angles of the inscribed Octaedron haue their place in the meditie of the sixe opposite side of Dodecaedron, which coupled togither with right lines make 12 sides containing his 8 triangles, and 3 Diameters, crossing themselues at right angles on the center of his spheres. But the solide Angles of the internall Cube and Tetraedron are situate in the angles of the contayning Dodecaedron, and one sphere comprehendeth them all three. As for the longitude of their Axes, sides, and Diameters, peruse these preceptes following.

Of Tetraedron

DIuide the side giuen by the first Probleme into extreame and meane proportion, and to the whole side adioyne his greater part, the square of the resulting quantitie tripled maketh the Tetraedrons Dimetients square, which augmented by 2 and diuided by 3 bringeth the square of Tetraedrons side, which diuided by 24 yeeldeth in the Quotient the square of Tetraedrons Axis: extract the rootes Quadrate of these squares, so haue yee the desired lines.

Of

Of Hexaedron.

DIuide Tetraedrons sides square by 2, so haue yee the square of the Cubes side, which augmented by 3 sheweth the Diameters square, and that diuided by 12 produceth the axis square, extracte the Zenzike rootes of these numbers, so haue ye the lines.

Of Octaedron.

DOdecaedrons side giuen added to the side of Hexaedron late founde, maketh the Diameter of Octaedron, the roote the quadrate of his squares medietie is the side, and the roote quadrate of the Dimetients twelfth part is the axis.

Of Icosaedron.

THe side of Dodecaedron beeing giuen, firste searche the Diameter of his containing Cyrcle by the fyfth and tenthe Problemes, and deducte the square thereof from the Square of the Cubes Dimetiente founde as is before declared, the roote quadrate of the remaine is the Icosaedrons Dimetiente: Nowe for his side yee shall adde the Squares of the Cubes side and Dodecaedrons sides togeather, the roote vniuersall of the producte yee shall reserue for a Diuisor, then multiplye Icosaedrons Dimetiente by Dodecadrons side, and the producte diuide by your reserued Diuisor, the quotiente is the Icosaedrons side, the Square of this side Deducte from triple the Square of Icosaedrons Semidimetiente, and fro the thyrde parte of the remaynder extracte the Zenzike roote, for that is the Icosaedrons Axis.

Because these rules of them selues are apparaunt inough, I shall onely adioyne an example for the last, with a table contayninge the numbers ready calculate of all the rest, which shall supplie the place of examples for the other.

An example of Icosaedron.

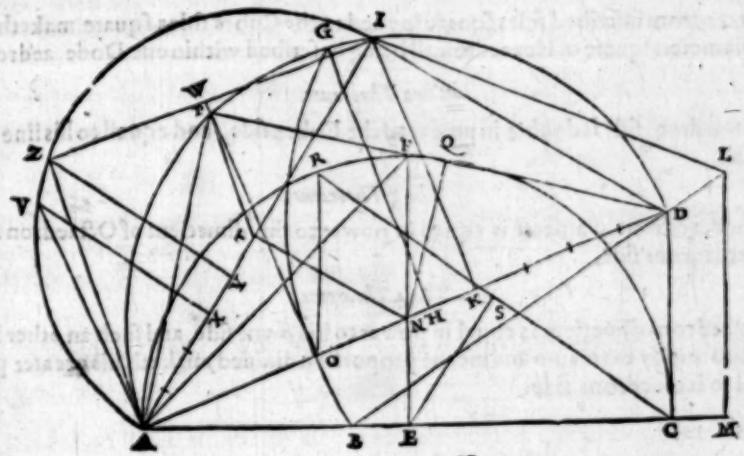
The Dodecaedrons side giuen 20 diuided by extreame and meane proportion maketh hys greater parte $\sqrt{\text{ʒ}}\,500-10$, *which added vnto 20, is* $\sqrt{\text{ʒ}}\,500+10$, *the square thereof tripled is* $1800+\sqrt{\text{ʒ}}\,1800000$, *so is the square of the Cubes dimetient, and from it deductinge* $800+\sqrt{\text{ʒ}}\,128000$ *Dodecaedrons basis containing circles diameters square, there will remaine* $1000+\sqrt{\text{ʒ}}\,968000$, *the roote quadrate vniuersall thereof is the diameter of the contained Icosaedron, which multiplied by the dodecaedrons side createth* $\sqrt{\text{ʒ}}\,v.400000+\sqrt{\text{ʒ}}\,1548800000$ *and that diuided by* $\sqrt{\text{ʒ}}\,v.1000+\sqrt{\text{ʒ}}\,200000$ *(for so much ariseth for a diuisor by addition of the Cubes sides square to the square of Dodecaedrons side) there will resulte of that diuision* $\sqrt{\text{ʒ}}\,v.280+\sqrt{\text{ʒ}}\,72000$, *so much is the contained Icosaedoons side, whose square deducted fro triple the square of Icosaedrons semidiameter* $\frac{37600}{80}+\sqrt{\text{ʒ}}\,\frac{70560000}{320}$, *whose thyrd partes roote is* $\sqrt{\text{ʒ}}\,v.\frac{1880}{13}+\sqrt{\text{ʒ}}\,\frac{392000}{16}$ *so much is the contained Icosaedrons axis, and that doubled sheweth the quantitie of his contained spheres dimetient.*

Dodecaedrons	*Diameter* $\sqrt{\text{ʒ}}\,v.1800+\sqrt{\text{ʒ}}\,1800000$
	Axis $\sqrt{\text{ʒ}}\,v.250+\sqrt{\text{ʒ}}\,60500.$
The contayned Tetraedrons	*Syde* $\sqrt{\text{ʒ}}\,v.1200+\sqrt{\text{ʒ}}\,800000$
	Diameter $\sqrt{\text{ʒ}}\,v.1800+\sqrt{\text{ʒ}}\,1800000$
	Axis $\sqrt{\text{ʒ}}\,v.50+\sqrt{\text{ʒ}}\,1388\frac{8}{9}$
The inscribed Hexaedrons	*Syde* $\sqrt{\text{ʒ}}\,v,600+\sqrt{\text{ʒ}}\,200000$
	Diameter $\sqrt{\text{ʒ}}\,v.1800+\sqrt{\text{ʒ}}\,1800000$
	Axis $\sqrt{\text{ʒ}}\,v.50+\sqrt{\text{ʒ}}\,12500$

The

The containing Dodecaedrons side 20.	The included Octaedrons	Syde $\sqrt{3}$ v. 200 + $\sqrt{3}$ 450000
		Diameter $\sqrt{3}$ v. 1400 + $\sqrt{3}$ 1800000
		Axis $\sqrt{3}$ v. 116⅔ + $\sqrt{3}$ 12500
	The internall Icosaedrons	Syde $\sqrt{3}$ v. 280 + $\sqrt{3}$ 72000
		Diameter $\sqrt{3}$ v. 1000 + $\sqrt{3}$ 568000
		Axis $\sqrt{3}$ v. 156⅔ + $\sqrt{3}$ 24500

For the Geometricall ſearching out of all theſe lines, beholde the figure following, where A B is the Dodecaedrons ſide giuen, B C ſuch a line as being diuided into extreame and meane proportion maketh his greater parte equall to B A, and therefore the inſcribed Cubes ſide A C is the inſcribed Octaedrons diameter, and A F C a Semicircle deſcribed thereon, the centre beeing E, D C is equal to A B, and perpendiculare to A C. A D is the diameter of Dodecaedron, the contayned Cube, and Tetraedron, A G D a Semicircle deſcribed vpon his medietie the centre N, B G is an arke deſcribed vpon the centre D, G H is a perpendiculare let fall on A D, H K is the fifth part of H D, K I an other perpendiculare ereared on that point cutting the Semicircle in I. A I is the Diameter of the inſcribed Icoſaedron, and A V I a Semicircle thereon deſcribed, B L equall to A I, L M a parallele to D C falling perpendicularly vpon A C extended to M, A V is equall to L M, it is the contained Icoſaedrons ſide, V X a perpendiculare from V to A I, X Y is a thirde parte of A X, Y Z a perpendiculare to A I, cutting the Semicircle at Z, Z I is the contained Icoſaedrons inſcribed ſpheres Diameter, W I his Medietie is the Icoſaedrons axis, A O is a thirde parte of A D, O P a perpendiculare vpon A D, P D the inſcribed Tetraedrons ſide, P A the Cubes ſide, O R a parallele to D C, R C the Diameter of the Inſcribed Octaedrons contayning circle, S his middle point, E S Octaedrons Axis, E F a perpendiculare ereared on the diameter, A C cutting his Semicircles circumference in F, A F Octaedrons ſide, T W is the ſemidiameter of Icoſaedrons contayning circle, T N the Semidiameter of Dodecaedrons contayning circle, O P the ſemidimetient of Tetraedrons baſe, and D Q the circles ſemidiameter that contayneth the Cubes ſquare. Thus haue ye in one figure all the ſides, diameters, and axes, both of the containing Dodecaedron, and his inſcribed Tetraedron, Octaedron, Hexaedron, and Icoſaedron, with the ſemidiameters alſo of the circles inuironing their baſes, but leaſt the intricate croſſing and concurring of lines and arkes might breede confuſion to ſuch as haue not beene trained in geometricall demonſtration, I ſhall adioyne a table containing the ſides, axes, and diameters both circulare and ſpherall of euery body particularlie by it ſelfe, expreſſed alſo exactly with numbers and Algebraicall characters the containing Dodecaderons ſide admitted an vnitie wherein the skilful Arithmetrician ſhall finde matter abundance to exerciſe or delight himſelf.



V

The

The comprehending Dodecaedrons
- Syde AB, 1
- Basis line Diagonall, $\sqrt{3}.1\frac{1}{3}+\sqrt{3}.\frac{1}{3}$ BC.
- Containing spheres Diameter, $\sqrt{3}.v.4\frac{1}{3}+\sqrt{3}.11\frac{1}{3}$ AD.
- Basis containing circles Semid. $\sqrt{3}.v.\frac{1}{3}+\sqrt{3}.\frac{1}{15}$ TN.
- Axis, $\sqrt{3}.vni.\frac{1}{3}+\sqrt{3}.\frac{1}{135}$, AT.

Dodecaedrons inscribed regulare

Tetraedrons
- Syde $\sqrt{3}.vni.3+\sqrt{3}.5$ DP.
- Diameter, $\sqrt{3}.vni.4\frac{1}{2}+\sqrt{3}.11\frac{1}{2}$. AD.
- Basis greater semid. $\sqrt{3}.v.1.+\sqrt{3}.\frac{1}{3}$ OP.
- Axis, $\sqrt{3}.vni.\frac{1}{4}+\sqrt{3}.\frac{1}{176}$, ON.
- Perpendiculare $\sqrt{3}.vni.2+\sqrt{3}.2\frac{2}{3}$ OD.

Octaedrons
- Syde, $\sqrt{3}.v.1\frac{1}{4}+\sqrt{3}.\frac{41}{16}$. AF.
- Diameter $\sqrt{3}.vni.\frac{7}{2}+\sqrt{3}.\frac{41}{2}$, AC.
- Basis greater semid. $\sqrt{3}.v.\frac{7}{12}+\sqrt{3}.\frac{41}{144}$, RS.
- Axis, $\sqrt{3}.v.\frac{7}{24}+\sqrt{3}.\frac{7}{24}$ ES.

Hexaedrons
- Syde $\sqrt{3}.vni.\frac{1}{2}+\sqrt{3}.\frac{1}{4}$ AP.
- Diameter, $\sqrt{3}.vni.4\frac{2}{3}+\sqrt{3}.11\frac{2}{3}$ AD.
- Basis Diagonall, $\sqrt{3}.vni.3+\sqrt{3}.5$ PD.
- Containing circles Semid. $\sqrt{3}.v.\frac{1}{4}+\sqrt{3}.\frac{1}{16}$ DQ.
- Axis, $\sqrt{3}.vni.\frac{1}{8}+\sqrt{3}.\frac{1}{64}$ NQ.

Icosaedrons
- Syde $\sqrt{3}.vni.\frac{7}{10}+\sqrt{3}.\frac{9}{50}$ LM.
- Dimetient, $\sqrt{3}.v.2\frac{1}{3}+\sqrt{3}.\frac{131}{30}$, AI.
- Basis cont. circles semid. $\sqrt{3}.v.\frac{7}{10}+\sqrt{3}.\frac{9}{250}$, TW.
- Axis $\sqrt{3}.v.\frac{47}{110}+\sqrt{3}.\frac{49}{110}$ VI.
- The Circles semidiameter whereon Icos. is framed, $\sqrt{3}.v.\frac{1}{3}+\sqrt{3}.\frac{131}{300}$ VX.

Theoremes of Dodecaedrons

inscribed regulare Solides.

The first Theoreme.

TEtraedrons inscribed sides square ioyned to the Cubes sides square maketh the Diameters square of Hexaedron, all being described within one Dodecaedron.

The 2 Theoreme.

Tetraedrons side is double in power to the Cubes side, and equall to his line Diagonall.

The 3 Theoreme.

Dodecaedrons diameter is equall in power to the dimetient of Octaedron and Dodecaedrons side.

The 4 Theoreme

Icosaedrons dimetient is equal in power to his owne side, and such an other line also (as being by extreame and meane proportion diuided) maketh his greater part equall to Icosaedrons side.

The

The 5 Theoreme.

A line equall in power to Dodecaedron and his inſcribed Cubes ſide, hath ſuch proportion to Icoſaedrons Diameter, as Dodecaedrons ſide, hath to the ſide of his contayned Icoſaedron.

The 6 Theoreme.

Octaedrons diameter diuided by extreame and meane proportion maketh his greater parte the Cubes ſide, and his leſſer the ſide of Dodecaedron.

The 7 Theoreme.

Hexaedrons ſides ſquare is equall to the ſquare of Dodecaedrons ſide, and a right angled Parallelogramme contained of the Cubes ſide, and Dodecaedrons ſide.

The 8 Theoreme.

Octaedrons Diameters ſquare is equall to the ſquare of Tetraedrons ſide, and a right angled Parallelogramme contayned of Dodecaedrons ſide and his inſcribed Cubes ſide.

The 9 Theoreme.

Octaedrons Diameter being by extreame and meane proportion diuided the exceſſe or difference of theſe partes ſquares deducted fro the ſquare of the Diameter, leaueth the ſquare of the inſcribed Tetraedrons ſide.

The 10 Theoreme.

Hexaedrons Diameter is equall in power to Dodecaedrons Baſis contayning circles Diameter of the inſcribed Icoſaedron.

The 11 Theoreme.

The exceſſe of Dodecaedrons baſis contayning circles Diameters ſquare aboue the ſquare of his line Diagonall, added vnto his contained Spheres Diameter, createth the ſquare of his inſcribed Tetraedrons ſide.

The 12 Theoreme.

Dodecaedrons dimetient is equall in power with theſe three lines, the greater Diameter of his owne baſis, the Diameter of Icoſaedrons contained ſphere, and the greater diameter of Icoſaedrons baſis.

The 13 Theoreme.

Octaedrons dimetient retaineth the ſame proportion to the Diameter of his baſis, that Dodecaedrons diameter doth to the ſide of his inſcribed Tetraedron.

The 14 Theoreme.

Octaedrons containing circles Diameter retayneth the ſame proportion to his contayned ſpheres Dimetient, that Tetraedrons ſide dooth to the ſide of the Cube.

The 15 Theoreme.

Dodecaedrons ſide being rationall, the ſide of his contained Icoſaedron is an irrationall Binomie of the 3 order, bearing proportion to the contayning dodecaedrons ſide, as $\sqrt{\sqrt{\frac{9}{20}}+\sqrt{\frac{1}{20}}}$ to an vnitie.

The 16 Theoreme.

Octaedrons diameter, to the ſide of Icoſaedron, is proportioned in power, as 5 to 1.

The 17 Theoreme.

Octaedrons Diameter beareth ſuch proportion to the dimetient of Icoſaedron, as a Corde Pentagonall dooth to a Corde Hexagonall of the ſame circle.

The 20 Theoreme.

A line in power equall to the sides of Dodecaedron and the Cube to the Diameter of Icosaedron, retayneth such proportion as the lesse segment of Dodecaedrons basis perpendiculare (diuided in extreame and meane proportion) doth to the lesse Semidiameter of the same basis.

The 21 Theoreme.

Icosaedrons side holdeth the same proportion to Octaedrons dimetient, that the Semidiameter of Dodecaedrons contayned circle dooth to the Perpendiculare of his base.

The 22 Theoreme.

If any line be diuided in extreame and meane proportion, and from the whole yee abate the medietie of the greater parte, the remaine being by extreame and meane proportion againe diuided, the lesser segment of this last diuided line, to the medietie first abated, retaineth such proportion, as Dodecaedrons side, to the side of his inscribed Icosaedron.

The 23 Theoreme.

If Icosaedrons side doubled be diuided in extreame and meane Proportion, and to the greater parte the Icosaedrons side adioyned, the whole line thereof resulting, is the Diameter of the inscribed Octaedron.

The 24 Theoreme.

The contayned Icosaedrons semidiameter to the lesse Semidimetient of the comprehending Dodecaedrons base, retaineth such proportion as Dodecaedrons basis line Diagonall to the side.

The 25 Theoreme.

Icosaedrons inscribed diameter to the dimetient of Dodecaedrons contayning circle, beareth such proportion, as the medietie of Dodecaedrons basis Diagonall, to the greater portion of his side by extreame and meane Proportion diuided.

The 23 probleme

The side, Diameter or Axis of any regulare bodye knowne, to searche out all those forenamed lines in any regulare bodye that shall include or circumscribe that proponed Solide.

ALthough this question of circumscribing bodyes might bee diuided into 5 Chapters, and in euery one as many different rules, precepts, and Theoremes taught, as was in the former of bodyes inscribed, yet for breuitie sake, I thinke best to remit the more ample handling hereof to the ingenious student, who comparing the rules, and well waying the Theoremes alreadie giuen, shall easily applye them to this purpose, and inuent many moe perhaps of greater facilitie and no lesse certaintie: and in this Chapiter I will onely open one way, leauing a large field for others to inuent and exercise themselues in at pleasure. It shall therefore be requisite when the side, Diameter or Axis of any regulare body is proponed, to consider by the fiue Problemes last past, what bodies may conteyne or circumscribe the same, and resorting

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ting to their peculiar Chapters, searche out the numbers appropriate to the conteyning and conteyned Solides sides, Axes and Diameters, which found by three quantities knowne, vsing the rule of proportion, yee may readily finde the fourth; as by the Example shall more plainely appeare.

Example.

Suppose the side of a Tetraedron giuen 10, *for the sides, Diameters and Axis. I repaire first to the* 19 *Probleme, where I finde the conteyning cubes side, being* 1, *the conteined Tetraedrons side* $\sqrt{3}\cdot 2$. *saying therefore by the rule of proportion* $\sqrt{3}\cdot 2$, *the conteined Tetraedrons side found heretofore in the* 19 *Probleme, giueth* 1 *for his conteining cubes side, what yeeldeth* 10 *the side giuen your fourth proportionall number will be* $\sqrt{3}\cdot 50$, *the containing Cubes side. Likewise in the* 22 *Probleme, I finde the conteining Dodecaedrons side being* 1, *the contained Tetraedrons side* $\sqrt{3}\cdot v.3+\sqrt{3}\cdot 5$, *augmenting therefore* 10, *the side giuen by Tetraedrons side in that Probleme founde, and diuiding by* $\sqrt{3}\cdot$ vni. $3+\sqrt{5}$, *your Quotient will be* $\sqrt{3}\cdot 62\frac{1}{2}-\sqrt{3}\cdot 12\frac{1}{2}$, *so much conclude the side of a Dodecaedron that shall conteyne or comprehend this Tetraedron, whose side is* 10. *In like manner may yee searche out the other sides, Diameters, and Axes, of all the comprehending bodyes, whereof I leaue to giue any farther examples, these two being sufficient to the ingenious to proceede with like order in the rest.*

But for such as not contented with one kinde of working will delight themselues in the diuersitie of rules and kindes of calculation, I haue thought good to adioyne these Theoremes ensuing, which well wayed and compared with such as are already past, shall yeeld matter abundantly for the inuention of many more conclusions and strange operations, than hitherto hath beene vsed or published by any.

Theoremes of these bodies mutually circumscribed and conferred with their inscribed regular bodyes.

The first Theoreme.

TEtraedron may be conteyned or circumscribed of all the other foure regulare bodies, and his side being rationall, his containing Octaedrons side is also rationall. proportioned therevnto, as 3 to 2.

The 2 *Theoreme.*

Tetraedrons comprehending cubes side is equall to the dimetient of his inscribed Octaedron.

The 3 *Theoreme.*

Tetraedrons conteyning Octaedrons side to the side of his inscribed Octaedron is triple.

The 4 *Theoreme.*

Tetraedrons side being rationall, his encompassing Icosaedrons side is an Apotome, and triple to Tetraedrons inscribed Icosaedrons side.

The 5 *Theoreme.*

Tetraedrons side being rationall, his circumscribing Dodecaedrons side is an Apotome of the 6 order, proportioned to the side as $\sqrt{3}\cdot$ vni. $\frac{1}{2}-\sqrt{3}\cdot\frac{1}{12}$ vnto an vnity.

The

The 6 Theoreme.

Hexaedron hath onely 3 circumscribing regular bodies, for no Tetraedron may be so placed about a cube but that his superficies shall either cut or not touche some of the Hexaedrons angles.

The 7 Theoreme.

Hexaedrons conteyning Octaedrons side is triple to his contayned Octaedrons side, and to the Hexaedrons side it beareth such proportion, as Tetraedrons diameter to his containing circles Semidimetient.

The 8 Theoreme.

Hexaedrons side being rationall, his comprehending Icosaedrons side, is an irrationall Minor, proportioned to the cubes side, as $\sqrt{3}$ v. $31\frac{1}{2}$ $\sqrt{3}$ $911\frac{1}{4}$ vnto 1.

The 9 Theoreme.

Hexaedrons side being rationall, his comprehending Dodecaedrons side is an Apotome of the sixt order proportioned to the side as $\sqrt{3}$ v. $1\frac{1}{2}$ — $\sqrt{3}$ $\frac{1}{4}$ to an vnity.

The 10 Theoreme.

Hexaedrons Dimetient is meane proportionall betweene his side and the Dimetient of his contayning Octaedron.

The 11 Theoreme.

Octaedron may be comprehended of all the other regular bodyes, and his side is the medietie of his encompassing Tetraedrons side.

The 12 Theoreme.

Octaedrons externall cubes side is equall to his diameter, and double in power to his side.

The 13 Theoreme.

Octaedrons side being rationall, his including Icosaedrons side is an *Apotome* of the 6 order, hauing proportion to Octaedrons side, as $\sqrt{3}$ vniuers. 3 — $\sqrt{5}$ vnto 1.

The 14 Theoreme.

Octaedrons side being rationall, his enuironing Dodecaedrons side is an *Apotome*, proportioned to the Octaedrons side, as $\sqrt{3}$ vni. 7 — $\sqrt{3}$ 45 vnto 1.

The 15 Theoreme.

Octaedrons comprehending Dodecaedrons side is equall to his contayned Icosaedrons side.

The 16 Theoreme.

Icosaedron may be comprehended perfectly of all the other regulare solides, and his containing Tetraedrons side is triple to the side of his contained Tetraedron.

The 17 Theoreme.

Icosaedrons side being rational, his comprehending cubes side is a Binomye, and the Icosaedrons diameter in power equal to them bothe,

The 18 Theoreme.

Icosaedrons side rationall his encompassing Octaedrons side is a Binomye, and equall to the medietie of his externall Tetraedrons side.

The 19 Theoreme.

Icosaedrons side rationall, his comprehending dodecaedrons side is an irrational Apotome, bearing proportion to the side as $\sqrt{3}$ v. $17\frac{1}{2}$ — $\sqrt{3}$ $281\frac{1}{4}$ vnto 1.

The

The 20 Theoreme.

Icoſaedrons comprehending cubes ſide is double in power to his comprehended Octaedrons ſide.

The 21 Theoreme.

Dodecaedron can perfectly be comprehended of no regulare bodie ſaue onely of Icoſaedron, whoſe ſide reteyneth ſuch proportion to the ſide of Dodecaedron, as $\sqrt{3}$ vni. 13 $\frac{1}{2}$ — $\sqrt{}$ 101 $\frac{1}{4}$ vnto an vntitie.

The 22 Theoreme.

Dodecaedron and Icoſaedron hauing equall ſides, the cubes ſide that containeth Icoſaedron is equall to the cubes ſide conteyned of Dodecaedron.

The 23 Theoreme.

Dodecaedron and Octaedron hauing equall and rational ſides, the ſide of Octaedrons comprehending Icoſaedron is an Apotome of the ſame order, that Dodecaedrons inſcribed Tetraedrons ſide is a Binomye, and their names or componing quantities equall.

The 24 Theoreme.

Dodecaedron and the cube hauing equall ſides, whether they be rational or ſurd, the Dodecaedrons conteyning Icoſaedrons ſide is triple to the cubes comprehending Dodecaedrons ſide.

The 25 Theoreme.

Dodecaedrons and Tetraedron hauing equall ſides, whether they be rationall or irrationall, the Dodecaedrons conteyning Icoſaedrons ſide to the Tetraedrons comprehending Dodecaedrons ſide, retayneth ſuch proportion, as Tetraedons diameter dooth to the leſſe Semidiameter of his baſis.

The 24 Probleme.

The ſide, Diameter, Axis, or altitude, of any regular body, or any ſemidiameter, perpendicular or line Diagonall of their baſe giuen, to ſearche out the content Superficiall and Solide, not only of that body, but alſo of any other regular ſolide that ſhall inſcribe or containe that body or any of his ſpheres.

FOr reſolution of this Probleme I might preſcribe rules innumerable of moſt ſtraunge and intricate operation, ſuch is the varietie of proportions betweene theſe bodies ſides, dimetients, Axes, and other their ſuperficiall and ſolide lines, that if I ſhould ſet foorth for euerye peculiar queſtion that might heerein be propoſed, but the tenth parte of ſuch preepts as offer themſelues vpon conſideration of theſe ſolides formes, proportions and nature, this onely Chapter ſhould growe to an huge treatiſe. But to conclude it in ſhorteſt forme and feweſt wordes, the whole varietie may be reduced to theſe principall points, what maner line is propoſed or giuen, and what content or capacitie is demaunded. If the line giuen be none of the bodies ſides, but ſome Diameter Axis, perpendiculare or other line before named, yee haue in the 6 Probleme and thoſe other foure that immediatlie enſue, the proportion of all ſuch lines to the ſolides ſides in Rationall and Irrationall

numbers

numbers expressed, so that by conuersion of the proportions there found, vsing the rule of proportion, as hath beene partlye before declared, and shall in this Chapiter by example be somewhat more plainely shewed, yee may finde out the correspondent side of that solide, whose Diameter, Axis or other line is giuen.

Then if it be the content superficiall or solide of the same body that is required, ye shall resorte to the 10 probleme, and the other foure following, where ye shal finde in the correspondente Chapiter by the side tofore knowne, the meanes how to searche his forenamed contentes.

But if those contentes be not demaunded of the same body, they are eyther of bodies inscribed or circumscribed, and that either of the proponed regulare solide, or some of his spheres, for bodies described within his spheres, it behoueth ye first by the side known to learne the Diameter of that Sphere, and then repaire vnto the sixtenth Probleme, where ye shall be taught to finde all the inscribed Solides sides, but if it bee an inscribed body of your proponed Solide, then haue yee preceptes for the inuention of hys syde in the eyghteenthe Probleme, or some other of the foure immediatelye insuinge. Likewise if it bee the contente of a circumscribinge bodye that is required, that body doth either circumscribe some Sphere, and then haue yee Preceptes in the seuenteenth Probleme for the inuention of his side, or else it containeth the Solide, whose side is knowen, and then may ye by the rules giuen in the last Probleme search out his side, and the side of euery Solide so founde (resorting to the sixthe, seuenthe, eight, ninth and tenth Problemes) ye shall there in his answerable Chapiter, receiue rules for the inuention of his capacitie superficiall and Solide, or else worke for euery seuerall body according to the rules ensuing.

Rules for the content Superficiall and Solide of the fiue Regular bodies.

For Tetraedron.

AVgment the square of the side founde by $\sqrt{\jmath}$ 3, and the Cube of the side by $\sqrt{\jmath}$ $\frac{1}{72}$, the first producte is the Superficies, the seconde is the Solide concent or Tetraedron.

For Octaedron

MVltiplie the sides square by $\sqrt{\jmath}$ 12, and the sides cube by $\sqrt{\jmath}$ $\frac{2}{27}$, so haue yee two productes equall to the two contentes Solide and Superficiall of Octaedron.

For Hexaedron.

THe square of the side augmented by 6, giueth his Superficies, which augmented by $\frac{1}{7}$ of the side, produceth the crassitude.

For Icosaedron.

INcrease the sides squared square by 75, the roote Zenzike of the producte is the Superficies, annd the Cube of the side augmented by $\sqrt{\jmath}$ v.2 $\frac{11}{72}$ + $\sqrt{\jmath}$ 5 $\frac{141}{172}$ yeedethe the crassitude.

For Dodecaedron

MVltiplie the sids square by $\sqrt{\jmath}$ v.225 + v.40500, so haue ye the superficies, or augment the sides Cube by $\sqrt{\jmath}$ v.89$\frac{1}{4}$ + $\sqrt{\jmath}$ 86$\frac{21}{24}$, the producte is the crassitude.

For more plainnesse I shall adioyne v. questions and examples of these principall varietyes, whereunto all other peculiar questions may be referred.

The

The fyrst question.

I haue a Dodecaedron, whose comprehending spheres diameter I know to be fiue, I demaund his capacitie superficiall and solide.

COnsidering the Dimetient of this solide is knowne vnto mee, I repaire vnto the tenth probleme, where I finde the Dodecaedrons side being 10 the comprehending Spheres Diameter √ʒ v. 450 + √ʒ 112500, conuerting therefore the limits or bounds of this proportion, I say thus √ʒ v. 450 + √ʒ 112500 geueth 10, what yeeldeth 5, working by the rule of proportion, yee shall finde the fourth proportionall number √ʒ 10 $\frac{5}{12}$ — √ʒ 2 $\frac{1}{12}$, so much is the side of the proponed Dodecaedron, which knowne I resort to the fifteenth Probleme, there am I willed to searche out the Semidiameter and line Diagonall of his basis, and the Axis or Semidimetient of his inscribed sphere his Axis is √ʒ v. 2 $\frac{1}{12}$ + √ʒ 3 $\frac{17}{36}$ his containing circles Semidimetient √ʒ v. 4 $\frac{1}{6}$ — √ʒ 3 $\frac{17}{36}$, his basis line Diagonall is √ʒ 8 $\frac{1}{3}$, so that by multiplication and diuision of these numbers according to the Precepts there giuen, I finde the Dodecaedrons superficies √ʒ v. 7812 $\frac{1}{2}$ — √ 1220703 $\frac{1}{4}$ and his crassitude √ʒ vn. 1808 $\frac{67}{216}$ + √ʒ 654097 $\frac{25403}{46956}$ — √ʒ 523278 $\frac{1033}{11664}$. In like manner if ye augment 12 $\frac{1}{2}$ — √ʒ 86 $\frac{39}{36}$ the square of the Dodecaedrons founde side, in 225 + √ʒ 40500. Or √ʒ v. 3038 $\frac{7}{36}$ + √ 4709502 $\frac{1033}{1296}$ √ʒ 8477105 $\frac{5}{144}$ — √ʒ 5128125 $\frac{3125}{11664}$ being the Cube of the same side, in √ʒ v. 29 $\frac{1}{2}$ + √ʒ v. 861 $\frac{1}{64}$, for so are ye taught to doe by the rule of this Chapiter, the first Product will bee the Dodecaedrons superficies, and the last his Crassitude, exactly agreing with your former operations.

The 2 Question,

A Cube is proponed, whose Diameter is the Zenzike roote of 108. I would knowe the Superficiall and solide contents of such a Tetraedron as this Cubes contained sphere should circumscribe.

BIcause the Diameter of this Cube is giuen, I resort to the sixteenth Probleme, by the rules there prescribed, I finde the Axis 3, which doubled maketh 6, the Diameter of this Cubes contained sphere, and considering the Tetraedron, whose capacitie is required, must be circumscribed of this sphere, I search out againe by the same Chapiter this spheres inscribed Tetraedrons side, finding it the Zenzike roote of 24 yee may also thereby learne the Axis and containing Circles Semidiameter, and so consequently the capacities of this Tetraedron, whereof yee haue examples in the eleuenth Probleme: Or by the first rule of this Probleme if ye augment 24 the square of Tetraedrons side by √ʒ 3, there ariseth √ʒ 1728 for his Superficies, and by multiplication of √ʒ 13824 the Cube of this inscribed Tetraedrons side by √ʒ $\frac{12}{72}$, there amounteth √ʒ 192, so muche conclude the Tetraedrons Crassitude.

The third queſtion

I demaund the Superficiall and Solide capacitie of a Dodecaedron, circumſcribing ſuch an Octaedrons containing ſphere, as hath for his ſide this Irrationall Maior $\sqrt{3}$ *v.* $20+\sqrt{3}\ 387\frac{1}{7}$.

BY the eigth Probleme I finde this Octaedrons comprehending ſpheres Dimetient $\sqrt{3}$ v. $40+\sqrt{3}\ \frac{2744}{7}$ the medietie hereof being $\sqrt{3}$ vniu. $10+\sqrt{3}\ \frac{686}{7}$ is the Axis of the Dodecaedron, whoſe contents are required, which knowne by the rules of the ſeuenteenth Probleme, I finde the ſide $\sqrt{3}$ v. $500-\sqrt{3}\ 234256$, and by extraction of this irrationall numbers roote, it appeareth 4, whoſe ſquare augmented by the roote Zenzike vniuerſall of $225+\sqrt{3}\ 40500$ produceth $\sqrt{3}$ v. $57600+\sqrt{3}\ 2654208000$, ſo much is the Dodecaedrons ſuperficies. Likewiſe by multiplication of $\sqrt{3}$ vniu. $\frac{225}{8}+\sqrt{3}\ \frac{50625}{64}$ in the Cube of 4, there ariſeth $\sqrt{3}$ v. $\frac{8503056}{8}+\sqrt{3}\ \frac{2299446451000}{64}$ for the craſſitude of this Dodecaedron.

The fourth queſtion.

There is a Dodecaedron, whoſe ſide is this Irrationall Apotome $\sqrt{3}$ *v.* $1120-\sqrt{3}\ 1152000$, *my deſire is to knowe his inſcribed Icoſaedrons ſuperficies and Craſſitude.*

FOraſmuch as this Icoſaedron whoſe contents are required, is deſcribed within the Dodecaedron, whoſe ſide is knowne, I ſearch out by the rules giuen in the xxii. Chapiter, the ſide of his contained Icoſaedron. Or with more ſpeede yee may thus worke, ſaying a containing Dodecaedrons ſide being an vnitie, geueth for the ſide of his contained Icoſaedron $\sqrt{3}$ v. $\frac{7}{10}+\sqrt{3}\ \frac{9}{10}$, what ſhall $\sqrt{3}$ *vniuerſalis* 11200 $-\sqrt{3}$ 1152000, by multiplication of theſe latter numbers yee ſhall produce $\sqrt{3}$ v. $784-\sqrt{3}\ 518400$, which yee ſhall finde (by extraction of the vniuerſall Zenzike roote) 8, the ſide thus founde, yee may ſearche out his contents by the fifteenth Probleme, or by the fourth rule tofore preſcribed thus, I augment 64 by $\sqrt{3}$ 75 there amounteth $\sqrt{3}$ 307200 the inſcribed Icoſaedrons ſuperficies. Or increaſe 512 the ſide cubically multiplyed by $\sqrt{3}$ vniu. $2\frac{11}{17}+\sqrt{3}\ 5\frac{241}{1156}$ there amounteth $\sqrt{3}$ vniu. $637155\frac{1}{3}+\sqrt{3}\ 37282702222\frac{1}{3}$ the deſired Craſſitude.

The fyfth Queſtion,

An Icoſaedron is offered, whoſe baſis containing Circles Semidiameter is this ſurde Binomye $\sqrt{3}$ *vni.* $14\frac{14}{17}+\sqrt{3}\ 204\frac{8}{9}$ *the Superficies and craſſitude of his containing Dodecaedron is required.*

COnſidering the Semidiameter of this Icoſaedrons baſis is giuen, I repare to the 9 Probleme, wherein I receaue rules for the inuention of the comprehending Dodecaedrons ſide, or more particularly to ſet foorth the whole operation,

I am

operation, I am referred to the 9 Probleme, where I perceaue by the 9 Theoreme, that the square of this knowne Semidiameter tripled, maketh the square of the Icosaedrons side: I augment therefore the giuen Binomyes square by 3, there ariseth $44\frac{4}{9}+\sqrt{3}\cdot\frac{16864}{10}$, the roote Zenzike vniuersall thereof is the Icosaedrons side. And for his containing Dodecaedrons side, I worke by the precepts of the last Chapter, saying: $\sqrt{3}$ v. 280 $+\sqrt{3}$ 72000 giueth 20, what yeeldeth $\sqrt{3}$ vni. $44\frac{4}{9}+\sqrt{3}\cdot\frac{16864}{10}$ your fourth Proportionall number will be 8, Or thus by the fifteenth Theoreme of the 22 Probleme: Diuide $\sqrt{3}$ vni. $44\frac{4}{9}+\sqrt{3}\cdot\frac{16864}{10}$ by $\sqrt{3}\cdot\frac{9}{20}+\sqrt{3}\cdot\frac{5}{20}$, so will the Quotient be 8 also, and that is the comprehending Dodecaedrons side. Now for his contents Solide and Superficiall, I repare to the fifteenth Probleme, where I am referred to the 10 Probleme, and working by the precepts there giuen, I finde the Dodecaedrons basis line Diagonal $\sqrt{3}$ v 96 $+\sqrt{3}$ 5120, his basis greater Semidiameter $\sqrt{3}$ vni. 32 $+\sqrt{3}\cdot\frac{4096}{20}$, his Axis $\sqrt{3}$ vni. 40 $+\sqrt{3}\cdot\frac{409616}{320}$, triple the Semidiameteris $\sqrt{3}$ vni. 238 $+\sqrt{3}\cdot\frac{3317776}{20}$, the line Diagonall increased by 5. maketh $\sqrt{3}$ vni. 2400 $+\sqrt{3}$ 320000, these multiplyed togither, create this surde rote $\sqrt{3}$ v 691200 $+\sqrt{3}$ 530841600000 $+\sqrt{3}$ 265420800000 $+\sqrt{3}$ 95551488000, resting betweene the Zenzike rootes of 1745904, and 1745905, being very nighe 1321$\frac{761}{2642}$, so much is the desired contente Superficiall. Againe by multiplication of $\sqrt{3}$ vniuers. 96 $+\sqrt{3}$ 5120 the Line Diagonall in $\sqrt{3}$ vniuers. 32 $+\sqrt{3}\cdot\frac{409616}{20}$ there amounteth $\sqrt{3}$ vniuers. 4096 $+\sqrt{3}$ 5242880 $+\sqrt{3}$ 1887436$\frac{4}{5}$, which increased by $\sqrt{3}$ vni. 40 $+\sqrt{3}\cdot\frac{409616}{320}$, the Axis produceth $\sqrt{3}$ vni 16384 $+\sqrt{3}\cdot\frac{1020819240576}{320}$ $+\sqrt{3}\cdot\frac{6651204144820448}{3200}$, and this multiplyed by 5, createth $\sqrt{3}$ vni. 7700480 $+\sqrt{3}$ 5919001804800 the containe Dodecaedrons content Solide. The like is brought to passe by the last rules, for 64 the Dodecaedrons sides square augmented by $\sqrt{3}$, vni. 225 $+\sqrt{3}$ 40500 maketh $\sqrt{3}$ v. 921600 $+\sqrt{3}$ 679477248000 the Superficies and 512 the Cube of 8 multiplyed in $\sqrt{3}$ vni. 29$\frac{1}{8}+\sqrt{3}\cdot\frac{151225}{64}$ createth $\sqrt{3}$ vniuer. 7700480 $+\sqrt{3}$ 5919001804800 the Solide capacitie of a Dodecaedron described within the Icosaedron, whose side was giuen exactly agreeing with the former calculations.

Theoremes of the Superficiall and Solide quantities of these regulare bodyes conferred by mutuall inscription and circumscription, and first:

Of Tetraedron

The first Theoreme.

TEtraedrons Superficies is double to the Superficies of his inscribed Octaedron.

The 2 Theoreme.

Tetraedrons Superficies to the Superficies of his containing Cube beareth proportion, as 1 to $\sqrt{3}$ 3.

The 3 Theoreme.

Tetraedrons circumscribing Cubes content superficiall, to the content superficiall of his containe Octaedron, retayneth such proportion as the same Octaedrons superficies dooth to the square of his side.

The 4 Theoreme.

Tetraedrons incompassing Octaedrons superficies contayneth the Superficies of his inscribed Octaedrons 9 times.

The 5 Theoreme.

One base of Tetraedrons containing Icosaedron, is equall vnto 9 of his inscribed Icosaedrons Triangulare bases.

The 6 Theoreme.

Tetraedron to his contayned Octaedron retaineth such proportion as their sides.

The 7 Theoreme.

Tetraedrons contayning Octaedron to the Cube described without the same Tetraedron, retayneth such proportion as the perpendiculare of the contayning Octaedrons basis to the lesse semidiameter of the Tetraedrons base.

The 8 Theoreme.

Tetraedrons circumscribing Octaedron containeth his inscribed Octaedron 27 times.

The 9 Theoreme.

Tetraedrons contayning Cube holdeth the same proportion to Tetraedrons inscribed Octaedron, that the Cubes Diameter dooth to the Octaedrons Axis.

The 10 Theoreme.

Tetraedrons comprehending Icosaedron, contayneth his inscribed Icosaedron 27 times.

Of Hexaedron

The 11 Theoreme.

HExaedrons superficies to the superficies of his inscribed Tetraedron, holdeth such proportion as Tetraedrons dimetient doth to the Cubes side.

The 12 Theoreme.

Hexaedrons Superficiall content to the content Superficiall of his inscribed Octaedron, retayneth the proportion of the Cubes Diameter to Octaedrons Semidimetient.

The 13 Theoreme.

Hexaedrons contayning Octaedrons contente superficiall hath suche Proportion to the Superficiall quantitie of his inscribed Tetraedron, as the Octaedrons contayned Spheres Dimetientes square to the greater Semidiameters square of Tetraedrons base.

The 14 Theoreme.

Hexaedrons comprehending Octaedrons superficies contayneth his inscribed Octaedrons Superficies 9 times.

The 15 Theoreme.

The proportion of Hexaedrons comprehending Dodecaedrons superficies, to the Superficies of his inscribed Icosaedron, is compounded of double, the proportion betweene a Corde Trigonall and a Corde Pentagonall of one Circle, adioyned to the proportion of the Pentagonall, Diagonall to the same Corde Trigonall.

The

The 16 Theoreme.

Hexaedrons ſolide capacitie to his inſcribed Tetraedron, beareth ſuch proportion as the Hexaedrons comprehending ſpheres Semidimetient dooth vnto the Tetraedrons Axis.

The 17 Theoreme.

Hexaedron to his contayned Octaedron, retaineth the Proportion of his Diameter to the Octaedrons Axis.

The 18 Theoreme.

Hexaedrons comprehending Octaedron to his contayned Tetraedron, beareth the proportion of 27 vnto 2.

The 19 Theoreme.

Hexaedrons externall Octaedron contayneth his internall Octaedron 27 times.

The 20 Theoreme.

Hexaedrons comprehending Dodecaedrons ſolide capacitie to the contente ſolide of his inſcribed Icoſaedron, retayneth the ſame Proportion that the line Diagonall of the Dodecaedrons Baſis doth vnto that parte of his ſide wherevnto the whole ſide is proportioned as the ſquare of a Corde Trigonall to the ſquare of a Corde Pentagonall.

Of Octaedron

The 21 Theoreme.

Octaedrons baſe doubled, is equall vnto 9 of his contayned Tetraedrons triangular baſes.

The 22 Theoreme.

Octaedrons ſuperficies to the content ſuperficiall of his inſcribed Cube, beareth ſuch proportion as the Octaedrons axis to the Cubes ſides third part.

The 23 Theoreme.

Octaedrons circumſcribed Tetraedrons ſuperficies contayneth the Superficies of his inſcribed Tetraedron 9 times.

The 24 Theoreme.

Octaedrons comprehending Cubes Diagonall lines ſquare, is triple to the ſuperficies of his comprehended Cube.

The 25 Theoreme.

Octaedrons externall Tetraedrons ſuperficies to the ſuperficiall quantitie of his internall Cube, is proportioned, as $\sqrt{3}$ 27 vnto an vnitie.

The 26 Theoreme.

Octaedron is proportioned to his inſcribed Tetraedron, as $1\ 3\frac{1}{3}$ vnto 1.

The 27 Theoreme.

Octaedron to his inſcribed Cube retayneth double the Proportion of theyr ſides.

The 28 Theoreme.

Octaedrons external Tetraedron to his internall Tetraedron, retaineth tripled the proportion of their ſides that is 27 vnto 1.

The

The 29 Theoreme.

Octaedrons circumscribing Cube to his inscribed Cube, retayneth double the proportion of the comprehending Cubes Diameter to the internall Cubes side.

The 30 Theoreme.

Octaedrons comprehending Dodecaedron to his included Icosaedron, beareth such Proportion as the Square of a Corde Trigonall dooth to the greater parte of a Pentagonall Cordes square (of one Circle) diuided by extreame and meane Proportion.

Of Icosaedron

The 31 Theoreme.

Icosaedrons comprehending Tetraedrons Superficies contayneth the Superficies of his internall Tetraedron 9 times.

The 32 Theoreme.

Icosaedrons contayning Hexaedrons Superficies to the superficies of his contaitayned Octaedron is proportioned as the Cubes Diameter to his Axis.

The 33 Theoreme.

Icosaedrons circumscribing Octaedrons superficies to the superficies of his internall Cube, beareth such proportion as 9 vnto $\sqrt{3}$ 12.

The 34 Theoreme.

Icosaedrons comprehending Tetraedrons superficies is double to the superficies of his comprehending Octaedron.

The 35 Theoreme.

Icosaedrons comprehending Octaedrons superficies hath the same proportion to his inscribed Tetraedrons superficies that Octaedrons lesser Spheres Dimetientes Square hath to the Square of the greater Semidiameter of Tetraedrons base.

The 36 Theoreme.

Icosaedrons comprehending Tetraedron contayneth his incloſed Tetraedron 27 times.

The 37 Theoreme.

Icosaedrons contayned Tetraedron is a third parte of his contayned Cube.

The 38 Theoreme.

Icosaedrons incompassing Tetraedron to his incloſed Cube, holdeth the proportion of their Diameters squares.

The 39 Theoreme.

Icosaedrons externall Hexaedron contayneth his internall Octaedrons 6 times.

The 40 Theoreme.

The proportion of Icosaedrons solide capacity to his cõprehended Dodecaedron, is equall to the proportion of the Icosaedrons Semidiameter to the Dodecaedrons Axis

Axis, and the difference of a Pentagonall Diagonals proportion to his Corde Trigonall of one selfesame circle deducted from the proportion of these Solides Diameters ioyned together.

Of Dodecaedron

The 41 Theoreme.

THe proportion of Dodecaedrons superficies to his inscribed Icosaedrons Superficies, is compounded of the proportion of Dodecaedrons basis Pentagonall Diagonall, to the Corde Trigonall of his contayning Circle, and the proportion of Dodecaedrons greater Spheres Semidiameter to his inscribed Icosaedrons Axis.

The 42 Theoreme.

If from the proportion of Dodecaedrons externall Icosaedrons greater Spheres Semidiameter to Dodecaedrons Axis, yee deduct the proportion betweene the Dodecaedrons Pentagonall basis Diagonall and his contayning circles Corde Trigonall, there remaineth the proportion of their contents superficiall.

The 43 Theoreme.

The Superficies of Dodecaedrons contayning Icosaedron to the Superficies of his contayned Icosaedron, retayneth quadruple the proportion of Dodecaedrons comprehending spheres Semidiameter to his Axis.

The 44 Theoreme.

Dodecaedrons contayned Cubes Superficies and side, and his contayned Tetraedrons Superficies and Diameter, are reciprocally proportionall, that is to saye, the Cubes Superficies retayneth the same proportion to the Tetraedrons Superficies that the Tetraedrons Diameter doth to the Cubes side.

The 45 Theoreme.

Dodecaedrons contayned Icosaedron and Octaedron haue equall Superficiall quantities.

The 46 Theoreme.

The proportion of Dodecaedron to his inscribed Icosaedron is equall to the proportion of Dodecaedrons greater Spheres Semidiameter to Icosaedrons Axis, and the proportion of Dodecaedrons basis Diagonall line to the inscribed Icosaedrons side.

The 47 Theoreme.

The proportion betweene Dodecaedron and his comprehending Icosaedron, is componed of the proportion betweene Icosaedrons containing spheres Dimetient and Dodecaedrons internall Spheres Diameter, added to the proportion betweene Icosaedrons side, and Dodecaedrons basis line Diagonall.

The 48 Theoreme.

Dodecaedrons inscribed Icosaedron to his inscribed Octaedron, retayneth such proportion, as Icosaedrons Axis doubled to the Diameter of Octaedrons contayned Sphere.

The

The 49 Theoreme.

Dodecaedrons comprehended Tetraedron to his internall Hexaedron, beareth the proportion of Tetraedrons contayned spheres dimetient to the Cubes comprehending spheres Diameter.

The 50 Theoreme.

Dodecaedrons comprehending Icosaedron to his contayned Icosaedron holdeth sixefolde the proportion of his contayning and contained spheres Diameters.

The 25 probleme

A Metamorphosis or transformation of the fiue regulare bodyes.

HItherto haue I onely intreated of the fiue regulare bodyes, Theorically and practically opening sundrie means to search out the proportion and quantities of their sides, Diameters, Axes, Perpendiculares, altitudes, and contents both superficiall and Solide, and that not onely in these solides considered by themselues, but also conferred with other, aswell by inscription as circumscription, both of themselues and their spheres: so as I suppose hardely any question may be proponed concerning these bodyes, which by the former Problemes or Theoremes may not bee resolued. Yet before I finish this Treatise I thought good to adioyne one Chapter of the transformation of these Solides into such bodyes (as though they may not be termed regulare, for that Euclide hath demonstrate onely fiue, and no moe possiblye to be found or imagined) yet haue they such vniforme composition and conuenience with these solides, that they are not onely enuironed with equilater and equiangle Superficies as they be, but also haue all their sides equall, and one comprehending sphere exactly and at once touching all their solide angles, they are also capable of the regulare bodyes, and onely herein different, that whereas the regulare solides be inuironed with one kind of plaines and receiue one internall sphere, and so consequentlye one Axis. These transformed bodyes are incompassed with seuerall kinde of plaines, and haue seuerall Axes and contained spheres, of which their diuersitie ariseth manifold most straung, rare, and different kinds of proportions, than may in a very large volume (I will not saye be demonstrate, but onely by Theoremes) be declared. Transformed regulare solydes I call them, both to auoide the forging of new names, which I should be inforced to vse for distinction sake, and also bicause they seeme to be created by the vniforme section of the regulare bodies, and may by addition of certaine proportionall equall Pyramides, be reduced to the regulare solides, and are in sundrye proportions and proprieties so agreeable and resemblante to those regulare Solides, whose names they beare, that they seeme onely to lose the ferme, and yet still to retaine the nature of them. I meane not heere in so ample manner as the noueltie of the matter requireth to intreate of them, but onely by Diffinitions and Theoremes open so much as may be sufficient to explane the composition, forme, nature, and proportion of these, and also giue light to the ingenious infinitely to proceed for inuention of the like, whose vse and appliance may be manifolde to conclusions no lesse straunge than necessarie: but thereof in due place for the nature of these

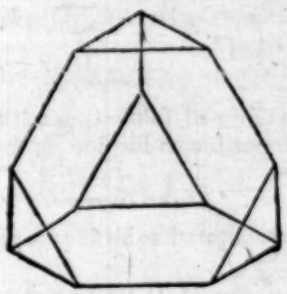
transf-

transformed Solides peruse their seuerall Diffinitions and Theoremes immediatly ensuing.

Of Tetraedron transformed.

The first Diffinition.

TEtraedron transformed, is a Solide incompassed with foure equall Equilater Triangles, and foure equall Equiangle Playnes Hexagonall, hauing equall sides with the Triangles.



The first Theoreme.

THis Solide hath 18 equall sides, 36 plaine Angles, and 12 solide Angles, and may be circumscribed of a sphere as exactly as any of the forenamed regulare bodies: the right line drawne from this Spheres center to one of the solide Angles I call the transformed Tetraedrons Semidiameter.

The 2 Theoreme.

This Solide also receiueth two intrinsical Spheres, the lesser touching all the centers of his Hexagonall playnes, the greater touching the centers of the Trigonal Playnes, and making circulare sections concentricall with the Hexagonall bases, the Semidiameters of these Spheres I call the Solides Axes.

The 3 Theoreme.

Within this Solide may Octaedron be described, and his sixe Angles shall rest in the medieties of those his sides that are peculiare to the Hexagonall plaines and not common both to the Hexagonall and Trigonall bases.

The 4 Theoreme.

Within this Solide may two Tetraedrons of different quantitie be described, the greater hauing his Solide angles resting in the centers of the Trigonal playnes, and the lesser hath his angles situate in the Hexagonall bases centers.

The 5 Theoreme.

Icosaedron may within this transfigured Tetraedron bee described, hauing his

ſolide Angles placed in the Hexagonall playnes, and foure of his Trigonal baſes concentrical, with thoſe Hexagonal baſes wherein they are ſituate.

The 6 Theoreme.

This Solides ſemidiameter is equall in power to the Semidiameter of his Triangulare plaines containing circle, and his greater Axis.

The 7 Theoreme.

The Square of this Solides ſides deducted from the ſquare of his Semidiameter, leaueth the ſquare of his leſſe axis.

The 8 Theoreme.

The ſquares of the Axes deducted one from an other, the Zenzike roote of the remayne is the Semidiameter of the Hexagonall baſis concentricall circle to fore mentioned in the ſecond Theoreme.

The 9 Theoreme.

The altitude of this tranſformed ſolide is equall to his Axes bothe ioyned together, and beareth proportion to his ſide, as foure vnto the roote Zenzike of ſixe.

The 10 Theoreme.

This Solide Semidiameter beareth to his ſide the proportion of $\sqrt{3}$'11 to $\sqrt{3}$'8.

The 11 Theoreme.

The greater Axis to the leſſer is proportioned as 5 vnto 3.

The 12 Theoreme.

The medietie of this Solides altitude is equall to the Semidiameter of the concentricall circle made in the Hexagonal baſes by the peripherie of the Solides greater internall ſphere.

The 13 Theoreme.

The leſſe ſemidiameters of this Solides Hexagonal and Trigonal baſes are proportioned one to an other, as the ſquares of their greater Semidiameters.

The 14 Theoreme.

The greater Diameter of this Solides Hexagonall baſis, to the greater Dimetient of his Trigonall baſe, is in power triple.

The 15 Theoreme.

The ſide of this Solide is equall to the ſide of his leſſer inſcribed Tetraedron, and to his greater internall Tetraedrons ſide, it retayneth ſuche proportion as 3 to 5.

The 16 Theoreme.

This Solides inſcribed Octaedrons ſide is triple to the medietie of his containing tranſſigured Tetraedrons ſide.

The 17 Theoreme.

The ſide of this Solides inſcribed Octaedron (being by extreame and meane proportion diuided) the leſſer ſegments ſquare is halfe the ſquare of the inſcribed Icoſaedrons ſide.

The

The 18 Theoreme.

The ſuperficies of this tranſformed Tetraedron is equall to bothe the Superficies of an Octaedron and Icoſaedron, hauing equall ſides with this Solide ioyned together.

The 19 Theoreme.

This Solides Craſſitude is equall vnto two Pyramides, whereof the one hath for his baſe a Triangle whoſe ſide is double to the ſide of this Solide, and his Axis equal to the greater Axis of this body, the other is an Hexagonal Pyramis: whoſe baſe is an equiangle Hexagonum, hauing his ſide double to the ſide of this ſolide, and his altitude equall to this tranſformed Tetraedrons leſſer Axis.

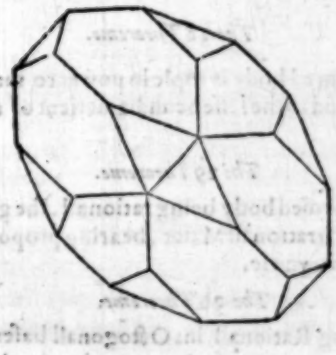
The 20 Theoreme.

This tranſſigured ſolide may be reſolued into 23 equall Tetraedrons, euerye of them of equall ſides vnto this Figure, and all ioyned together make his exacte Craſſitude, which is proportioned to the Cube of his ſide, as 23 to the Quadrate roote of 72.

Of the transfygured Cube.

The ſecond Diffinition.

A Tranſformed Cube is a Figure Geometricall enuironed with 6 Equiangle Octogonall and 8 Equilater Triangular plaines or baſes, whoſe ſides are all equall.



The 21 Theoreme.

This figure hath 36 equall ſides, 72 plaine Angles, and 24 Solide Angles, it may be incompaſſed of a ſphere, exactly touching with his concaue peripherie euery of their ſolide angles, and the ſemidimetient of that comprehending Sphere is called alſo this Solides Semidimetient.

The 22 Theoreme.

This tranſſigured bodye is alſo capable of two internall ſpheres, the leſſe touching onely the centers of the Octogonall playnes, the greater bothe touching exactly

act'y with his conuex circumference the Triangulare bases centers, and also cutting the playnes Octagonall describing therein concentricall circulare sections, the Semidiametients of these spheres are the Axis of the Solide.

The 23 Theoreme

Within this Solide may a Tetraedron be described, whose Angles shall rest in the centers of the Trigonall bases.

The 24 Theoreme.

Octaedron may also herein bee placed, his angles resting in the centers of the Octogonall playnes.

The 25 Theoreme.

Hexaedron may likewise be inscribed by drawing straight lines conioyning all the Trigonall bases centers.

The 26 Theoreme.

This transformed Figure receyueth also Icosaedron, whose 12 Solide Angles are placed in his 6 Octagonall Bases, euerye of them receyuing two of his Angles.

The 27 Theoreme.

Dodecaedron may not exactly be within this figure described, for 12 of his Angles being situate in the 6 Octagonall bases, diuiding their lesse Semidiameters by extreame and meane proportion, the other 8 Angles will not touche his superficies, but rest precisely in the greater Axes of this Solide, directlye vnder the centers of the 8 Trigonal playnes, vniformly and proportionally cutting euery of these forenamed greater Axes.

The 28 Theoreme.

The side of this transfigured body is triple in power to the greater Semidiameter of his Triangulare base, and to the lesse Semidiametient of that base it is proportioned as 1 vnto $\sqrt{3}\,\frac{1}{12}$.

The 29 Theoreme.

The side of this transformed body being rationall, the greater Semidiameter of his Octogonall bases is an Irrationall Maior, bearing proportion to the Solies side, as $\sqrt{3}$ vni. $1+\sqrt{3}\,\frac{1}{2}$ vnto an vnitie.

The 30 Theoreme.

This Solides side beeing Rationall, his Octogonall bases lesse Semidimetiente is an Irrationall Binomye, retayning such proportion to the side, as $\sqrt{3}$ vniuer. $\frac{1}{4}+\sqrt{3}\,\frac{1}{2}$ to 1.

The 31 Theoreme.

This solide side being Rationall, his greater Axis is an irrationall Binomye proportioned to the side, as $\sqrt{3}\,\frac{1}{2}+\sqrt{3}\,\frac{1}{4}$ vnto an vnitie.

The 32 Theoreme.

The lesse Axis of this Figure is equall to the lesse Semidimetient of his Octogonall

genall Basis, and retayneth vnto his side the proportion of $\sqrt{3}\cdot\frac{1}{2}+\frac{1}{2}$ to an vnitie.

The 33 Theoreme.

This transformed Figures side being Rationall, his comprehending spheres Dimetient is an Irrationall Maior, proportioned to the side, as $\sqrt{3}$ vniuer 7. $+\sqrt{3}$ 32 to 1.

The 34 Theoreme.

This Solides contayned Tetraedrons side is proportioned to the greater Axis, as $\sqrt{3}\cdot\frac{2}{3}$ to an vnitie.

The 35 Theoreme.

The side of the inscribed Tetraedron is double in power to the side of the inscribed Cube.

The 36 Theoreme.

This transfigured Cubes lesse altitude is double in power to the side of his inscribed Octaedron.

The 37 Theoreme.

The Octogonall bases lesse Diameter diuided by extreame and meane proportion, maketh his greater segment, the inscribed Icosaedrons side.

The 38 Theoreme.

The side of this transformed Solide, beeing by extreame and meane Proportion diuided, if to the lesse portion yee adioyne a line in power double, that whole line thereof resulting, is the side of the tofore mentioned inscribed Dodecaedron.

The 39 Theoreme.

The Cube of the transfigured Solides Octogonall bases lesser Diameter, hath a superficies exceeding the superficial content of all this Solides Octogonal bases, by a superficies proportionate to the superficial quantitie of his Trigonall bases, as the Cubes Dimetient to his side.

The 40 Theoreme.

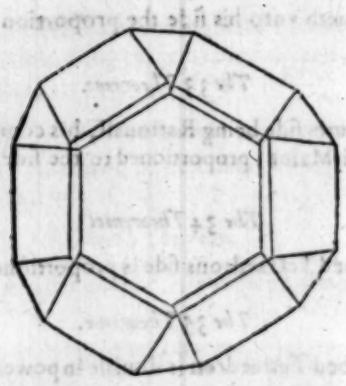
The Cube whose side is equall to the lesse altitude of the transfigured Hexaedron, exceedeth in Solide capacitie this transformed Figure, by the Solide content of an Octaedron described vpon the same side with this transfigured Hexaedron.

Of Octaedron transformed.

The 3 Diffinition.

A Transfigured Octaedron is a Geometricall Figure incompassed with 14 bases, whereof 8 are equall Equiangle Hexagonall plaines, and the other 6 are equall squares.

The



The 41 Theoreme.

A transformed Octaedron hath 36 equall sides, 72 playne Angles, and 24 Solide Angles, and maye bee inclosed with a contayning Sphere exactly touching all his Angles.

The 42 Theoreme.

This Solide also receyueth two Internall Spheres, the one onely touching all the Hexagonal Bases centers, the other both touching all the centers of the Squares and also with his peripherie cutting the Hexagonall playnes, delineating in them concentricall circles, the semidiameters of these spheres are called the Axes.

The 43 Theoreme.

Within this Solide may Tetraedron bee described, and his Angles situate in the centers of the Hexagonall basis.

The 44 Theoreme.

This Figure also receyueth the Cube, with his 8 solide Angles resident in the 8 centers of his Hexagonall playnes,

The 45 Theoreme.

The inscribed Octaedrons 6 solide Angles rest in the 6 centers of the Quadrate plaines,

The 46 Theoreme.

This solides inscribed Icosaedron hath all his Angles situate in those his 12 sides which are peculiare to the Hexagonall bases, not participating with the Quadrate plaines.

The 47 Theoreme.

This solides side is equall to the greater semidiameter of his Hexagonall basis, and double in power to the greater semidimetients of his Quadrate playnes.

The 48 Theoreme.

The greater dimetient of the Hexagonal Basis, is equal in power to his lesse Diameter and sides

The 49 Theoreme.

The lesse Diameter of the Hexagonal Basis, is triple in power to his lesse Diameter of the Quadrate basis.

The 50 Theoreme.

The greater Axis of this Solide, is double in power to his side.

The 51 Theoreme.

This Solides Semidimetiente, is equall in power to his lesse Axis, and side:

The 52 Theoreme.

The Semidimetient of the concentricall circle, is equall to the greater Semidimeter of the quadrate plaines, and the square thereof added to the square of the lesse Axis produceth the square of the greater.

The 53 Theoreme.

The inscribed Tetraedrons side is double to the side of this transfigured Solide.

The 54 Theoreme.

The side of the inscribed Cube, is meane proportionall betweene the side of the inscribed Tetraedron, and the side of this Solide.

The 55 Theoreme.

The Diameter of this body is fiuefolde in power greater than the side of his inscribed Cube.

The 56 Theoreme.

This Solides inscribed Octaedrons side, is equall to the side of his inscribed Tetraedron, and double in power to the inscribed Cubes side, or the greater Axis, his equall.

The 57 Theoreme.

If the greater Diameter of this transformed Figures quadrate bases bee diuided by extreame and meane proportion, triple the lesse parte is the inscribed Icosadrons side.

The 58 Theoreme.

The side of an Octaedron transformed beeing rationall, the righte line that matcheth in power his Superficies, is an Irrationall called of Euclide, *Potens rationale & mediale*, proportioned to the square of the side, as $\sqrt{3}$ v. $\sqrt{3}$ 432 + 8 vnto an vnitie.

The 59 Theoreme.

This transfigured Octaedron maye intellectually bee diuided into 48 Pyramides rising fro the Superficies, and concurring with their toppes or vertices in the center of this Solide, whereof 8 are Hexagonall, and 6 Tetragonall, according to the Bases from whence they ascende, and the Hexagonall Pyramides ioyned together, are Triple to the Solide capacitie of the Tetragonall Pyramides.

The 60 Theoreme.

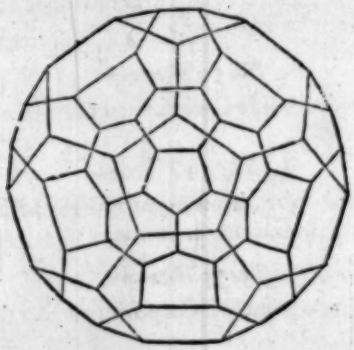
Octaedron transformed to the Cube of his side retayneth the Proportion, of $\sqrt{3}$ 28 vnto an vnitie, and may be resolued into 24 equall Octaedrons, hauing euerye of their sides equall to this transformed Figure.

Of

Of the transfygured Icoſaedron.

The fourth Diffinition.

Icoſaedron tranſſigured, is a Solide bodye incompaſſed with 32 Equiangle and Equilater baſes, whereof 20 are Hexagonall, and the other 12 Pentagonall plainet.



The 61 Theoreme.

This tranſformed figure hath 90 equall ſides, 180 plaine Angles, and 60 Solides, one comprehending ſphere inuironeth all his Angles, and the Semidiameter of that ſphere, is the Semidimetient of this ſolide.

The 62 Theoreme.

This Solide alſo conceiueth two internall ſpheres, the one onely touching all the Hexagonall baſes centers, the other both touching all the centers of the Pentagonall baſes, and alſo cutting the plaines Hexagonall, deliniating in them concentrical circles, the ſpheres ſemidimetients are this Solides Axes.

The 63 Theoreme.

Within this body may Tetraedron bee deſcribed, and his Angles ſituate in 4 of the Hexagonal baſes centers.

The 64 Theoreme.

This ſolides inſcribed Cube hath his ſolide Angles alſo placed in the centers of his Hexagonal plaines.

The 65 Theoreme.

Icoſaedron tranſſigured, receiueth an internal Octaedron, whoſe ſolide angles reſt in the medieties of 6 ſuch ſides of his Hexagonall baſes, as communicate not with his Pentagonal plaines, but are ſituate Paralel, Perpendiculare, and oppoſite one to to an other.

The 66 Theoreme.

Icoſaedron may alſo herein be inſcribed, his 12 ſolide Angles reſident in the Centers of his 12 Pentagonall baſes.

The

The 67 Theoreme.

Dodecaedron may also perfectly be described within this Soldie, all his Solide Angles being placed in the centers of all the Hexagonal bases.

The 68 Theoreme.

The side of this tranformed figure being Rationall, his Semidiameter is an Irrational Maior, proportionate to his side, as $\sqrt{\mathfrak{z}}$ v. $3\frac{5}{8}+\sqrt{\mathfrak{z}}\,6\frac{21}{64}$ to an vnitie.

The 69 Theoreme.

The greater semidimetient of the Hexagonal bases, is equal to the side of this transfigured body: and to the greater Semidiameter of the Pentagonal plaines, it beareth proportion as 1 vnto $\sqrt{\mathfrak{z}}$ v. $\frac{1}{2}+\sqrt{\mathfrak{z}}\,\frac{1}{25}$.

The 70 Theoreme.

The lesse Semidimetients of these Pentagonal and Hexagonal bases, are proportionate, as $\sqrt{\mathfrak{z}}$ v. $\frac{1}{8}+\sqrt{\mathfrak{z}}\,\frac{4}{45}$ vnto an vnitie.

The 71 Theoreme.

This transfigured solides side being Rational, his greater Axis is a Binomye proportioned to the side, as $\sqrt{\mathfrak{z}}$ v. $3\frac{1}{8}+\sqrt{\mathfrak{z}}\,\frac{1681}{320}$ vnto an vnitie.

The 72 Theoreme.

The side of this Solide being Rational, his lesser Axis is also a Binomye, and to the side proportionate as $\sqrt{\mathfrak{z}}$ v. $\frac{21}{8}+\sqrt{\mathfrak{z}}\,\frac{405}{64}$ vnto an vnitie.

The 73 Theoreme.

The greater Axis in power exceedeth the lesser, by the Semidimetient of the concentrical circle, which retaineth the same proportion to the side of this transformed Solide, that $\sqrt{\mathfrak{z}}$ v $\frac{1}{2}-\sqrt{\mathfrak{z}}\,\frac{1}{20}$ dooth vnto an vnitie, and it is the greater part of this Solides Pentagonall bases Semidiameter, diuided by extreame and meane proportion.

The 74 Theoreme.

This transformed figures inscribed Tetraedrons side, is double in power to the side of the inscribed Cube, and to the side of this Solide, it beareth the proportion of $\sqrt{\mathfrak{z}}$ v. $7+\sqrt{\mathfrak{z}}\,45$. vnto 1.

The 75 Theoreme.

The side of Octaedron described within a transformed Icosaedron, whose side is Rational, shall be a Binomye proportional to the Rational side, as $\sqrt{\mathfrak{z}}$ vniuers. $6\frac{3}{4}+\sqrt{\mathfrak{z}}\,\frac{405}{16}$ to 1.

The 76 Theoreme.

This transformed figures side being Rationall, his inscribed Isaedrons side is an Irrationall Binomye, retaining such proportion to this transformed Solides side, as triple his greater altitude to the Diameter of his comprehending Icosaedron, that is as $\sqrt{\mathfrak{z}}$ v. $4\frac{1}{5}+\sqrt{\mathfrak{z}}\,3\frac{1}{5}$ to an vnitie.

The 77 Theoreme.

The side of a Dodecaedron described within this transfigured Icosaedron, being diuided by extreame and meane proportion, maketh his greater part this transformed solides side, and is proportioned therevnto, as $\sqrt{\mathfrak{z}}$ v. $\frac{3}{2}+\sqrt{\mathfrak{z}}\,\frac{5}{4}$ vnto an vnitie.

The 78 Theoreme,

The superficies of this transformed Icosaedron, is equal to the Superficial capacities of one Dodecaedron, and 6 Icosaedrons, hauing their sides all equall to the side of this transformed Figure.

The 79 Theoreme.

The superficies of a transfigured Icosaedron, is equall to the Superficial capacitie of a Dodecaedron and an Octaedron of equal sides to this Solide, and the superficies of such a transformed Tetraedron, as hath a side equall to bothe the sides of those two bodyes ioyned together.

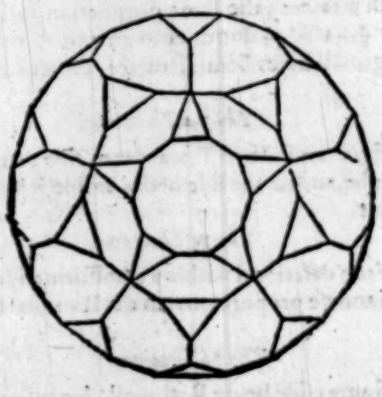
The 80 Theoreme.

A transfigured Icosaedron may be resolued into 12 Pentagonal and 20 Hexagonal Pyramides, concurring with their toppes or vertices all in the centers of this transformed body, and their bases the Pentagonal and Hexagonal Equiangle superficies tofore mentioned in the diffinition of this Solide. The 12 Pentagonal Pyramides Crassitude, to a Dodecaedron created vpon this Transfigured Solides side, is proportioned, as his greater Axis, to that line, whose greater Segment is this Pentagonal basis lesse Semidimetient. And the Crassitude of the other 20 Hexagonal Pyramides, is $\frac{3}{7}$ of this transfigured Solides containing Icosaedron.

Of Dodecaedron transformed.

The fift Diffinition.

A Transformed Dodecaedron is a masse or Solide Figure, comprehended of 12 Equiangle Decagonal, and 20 Equilater Triangular bases.



The 81 Theoreme.

A transfigured Dodecaedron hath 90 equal sides, 180 plaine Angles, and 60 Solide Angles, comprehended of one sphere, with his concaue Peripherie touching all those Angles, whose Semidimetient is the Semidiameter of this transformed Solide.

The 82 Theoreme.

This transformed figure hath two internal Spheres, the greater with his connex superficies touching all the Trigonal bases centers, and cutting the bases or plaines Deca-

Decagonal, deſcribing in them concentrical circles, the leſſe ſphære onely toucheth all the Decagonal baſes centers, reſting wholy within this Solide, and the Semidimetients of theſe Sphæres, are called the Axes of this tranſformed Figure.

The 83 Theoreme.

This tranſſigurate body, receiueth an internall Tetraedron, whoſe Solide angles reſt in the Centers of his Trigonal Baſes.

The 84 Theoreme.

Hexaedron may alſo within this body be deſcribed, his Angles likewiſe remaining in the Trigonal baſes Centers.

The 85 Theoreme.

Within a tranſſigured Dodecaedron may an Octaedron be framed, and his Angles will be reſident in the medieties of thoſe his ſides that be peculiar to the Decagonal, not communicating with the Trigonal baſes, but oppoſite, Parallel, and Orthogonal, one to an other.

The 86 Theoreme.

Icoſaedron within this tranſſigurate body may be deſcribed, and all his 12 angles in the Centers of his 12 Decagonal baſes ſituate.

The 87 Theoreme.

Dodecaedron may alſo within this tranſſigurate bodye bee deſcribed, all his 20 Solide Angles placed in the 20 Trigonal baſes Centers,

The 88 Theoreme.

This Solides Semidiameter is æqual in power to his leſſe Axis, and the greater Semidiameter of his Decagonal baſes,

The 89 Theoreme.

The ſide of this Solide being Rational, his Decagonal baſes leſſe Semidiameter is an Irrational Maior, proportioned to the greater Semidimetient of the ſame baſes, as $\sqrt{3}$ v. $\frac{1}{4}+\sqrt{3}\frac{1}{4}$ vnto $\sqrt{3}$ v. $1\frac{1}{4}+\sqrt{3}$ $1\frac{1}{4}$.

The 90 Theoreme.

The ſide of this tranſſigurate body, is triple in power to the greater Semidiameter of his Trigonall baſes, and to the leſſe Semidimetient of the ſame baſes, it is proportioned, as the Diameter of a Cube to his Axis.

The 91 Theoreme.

The Dimetient of this Solide, is equall in power to theſe three, his Side, his leſſe Altitude, and the leſſer Diameter of his Decagonal baſes.

The 92 Theoreme.

The ſquare of this Solides greater Axis, added to the thirde parte of his ſides Square, maketh the Square of his Semidimetient.

The 93 Theoreme.

The ſquare of the leſſe Altitude of this Solide, deducted from the ſquare of the greater, leaueth the ſquare of the Decagonal baſis concentricall circles Diameter.

The 94 Theoreme.

Twentie direct or vpright Equilater triangular *Pyramides* may vpon theſe ſolides 20 Triangular baſes of ſuch altitude be erected, and to this Solide in ſuch ſorte adioyned

ioyned, that the whole body thereof resulting, shall be a Dodecaedron: whose side conteyneth in power fiuefold the side of this transformed Figure.

The 95 Theoreme.

Euery of these adioyned *Pyramides* sides, are equall to the greater portion of this transfigured Dodecaedrons side, diuided by extreame and meane proportion.

The 96 Theoreme.

The square of this transformed figures Trigonal bases greater Semidimetient, deducted from the square of his sides greater portion diuided by extreame and meane proportion, leaueth the square of his adioyned *Pyramides* altitude.

The 97 Theoreme.

The greater Altitude of this Solide, is equall in power to the sides of his inscribed Cube, and Tetraedron, and if the power thereof be diuided in two lines retayning the proportion of a line diuided by extreame and meane proportion to his lesse parte, the lesser of these lines is this transfigured Solides inscribed Dodecaedrons side.

The 98 Theoreme.

This transformed Dodecaedrons inscribed Octaedrons Dimetient, is equall in power to the inscribed Icosaedrons Diameter, and the lesse Dimetient of his Decagonall bases, and the power of the lesse Altitude of this Solide, (beeing diuided in two right lines retayning extreame and meane proportion) the lesser of those lynes is the inscribed Icosaedrons side.

The 99 Theoreme.

The Superficies of a transfigured Dodecaedron doth exceed the Superficies of an Icosaedron made vpon this transformed Solides side, by the Superficies of an Equiangle Decagonum, whose side retaineth the same proportion to the side of this Solide, that the side dooth to his Triangular bases lesser Semidiameter, and the Superficies of all this Solides Decagonal bases, to the superficies of all his Trigonall bases, retaineth the proportion of his Decagonal basis lesser Diameter, to the semidimetient of his Trigonal bases contained circle.

The 100 Theoreme.

A transformed Dodecaedron may be resolued into 32 Pyramides, rising from his Equilater Bases, and concurring with their toppes or Vertices at his Center, whereof 12 are Decagonal, and 20 Trigonall, according to the bases from whence they rise, and the proportion of all the Decagonal Pyramides Crassitude, to the Solide content of all the Trigonal, is compounded of the proportion betweene the Decagonal Basis lesse Diameter, and the Trigonal basis Semidiameter, added to the proportion of the lesse Altitude to the greater.

Theoremes of these transformed Bodyes conferred both with their circumscribing regular Bodyes, and also betweene themselues.

The first Theoreme.

THe square of a transformed Tetraedrons Diameter, added to 8 times the square of his side, produceth the square of his comprehending Tetraedrons Diameter.

The

The 2 Theoreme.

The Superficies of a transfigured Tetraedron, is proportioned to the Superficies of his comprehending Tetraedron, as 7 to 9.

The 3 Theoreme.

If the side of a transformed Octaedron, to the side of a transformed Tetraedron, retaine the proportion of $\sqrt{3}\,7$ vnto $\sqrt{3}\,12$, then is the Superficiall capacitie of the transformed Octaedrons Hexagonal plaines, equall to the whole superfices of Tetraedron transformate.

The 4 Theoreme.

The Solide capacitie of a transformed Tetraedron, to his circumscribing Tetraedron, holdeth the proportion of 23 vnto 27.

The 5 Theoreme.

The Solide quantitie of a transfigured Tetraedron, to the content solide of a transfigurate Octaedron framed on the same side, is proportioned as 23 to 96.

The 6 Theoreme.

Hexaedron transformed, may of a Cube be circumscribed, and his side deducted from the Cubes side, leaueth a line double in power to the transformed Hexaedrons side.

The 7 Theoreme.

If ye adioyne $\sqrt{3}\,\frac{1}{2}$ of a transfigured Hexaedrons side vnto his greater Axis, the product is the Semidimetient of his containing Cube.

The 8 Theoreme.

The containing Cubes superficies, exceedeth the content superficial of his inscribed transformed Hexaedrons Octogonall bases, by the square of a line retaining such proportion to the transfigured Solides side, as a Cubes Diameter, to the greater semidimetient of his Quadrate basis.

The 9 Theoreme.

A transfigured Cube, with foure Tetraedrons made on his side, are equall to the Crassitude of his comprehending Hexaedron.

The 10 Theoreme.

A transformed Tetraedron, and transfigured Hexaedron of equall sides, haue the differences betweene their Crassitudes and the solide capacities of their comprehending regular bodyes equall.

The 11 Theoreme.

A transfigured Octaedron, may within an Octaedron be described, and the Dimetient of that containing Octaedron to the side of the transformed bodye, retaineth the same proportion, that the side of a transfigured Cube dooth to the default or difference betweene his greater Axis, and the Semidimetient of his comprehending Hexaedron.

The 12 Theoreme.

A transformed Octaedron, and a transformed Tetraedron hauing equall sides, their comprehending regular Solides sides are equal also, and their Superficial quantities in double proportion.

The

The 13 Theoreme.

Octaedrons ſuperficies, dooth ſurmount the content ſuperficial of his inſcribed Octaedron tranſſigured by the difference of an Equiangle Hexagonum, (whoſe ſide is double to the tranſformed Solides ſide (from the ſquare of the ſame tranſfigured ſolides leſſe Altitude.

The 14 Theoreme.

If two meane proportional lines be found between the ſides of Octaedron tranſformed, and his circumſcribed regular body, the Octaedron hauing to ſide the leſſe of thoſe meane proportionall lines, deducted from the Craſſitude of the containing Octaedron, leaueth the Solide quantitie of the tranſſigured body.

The 15 Theoreme.

If two meane proportional lines be found betweene the ſide of a comprehending Octaedron, and the greater Axis of his inſcribed tranſſigured Octaedron, and likewiſe two other meane proportionals betweene the inſcribed tranſformed Solides ſide and his Quadrate baſes greater Diameter: A Cube hauing for his ſide the greater of the former two meane proportionall lines, is equal to the circumſcribing Octaedron. And if from that Cube ye deduct a Cube, hauing his ſide the leſſer of the latter meane proportionals, the reſidue or remaining quantitie, is the Craſſitude of the inſcribed tranſſigured Octaedron.

The 16 Theoreme.

Icoſaedron tranſformed may be incloſed with an Icoſaedron, & all the 180 plaine angles of the tranſſigured body, concurring and reſting in the 30 ſides of the containing Icoſaedron, whoſe ſide is triple to the ſide of the contained tranſformed body.

The 17 Theoreme.

The Semidimetient of the comprehending Icoſaedron is equall in power to the leſſer Axis, and a meane proportionall betweene the tranſſigured body, and his containing Icoſaedrons ſides.

The 18 Theoreme.

The greater Diameter of a tranſſigured Icoſaedrons Pentagonal Baſis, diuided by extreame and meane proportion, and the greater parte adioyned to his greater altitude, produceth his comprehending Icoſaedrons Dimetient.

The 19 Theoreme.

The Superficies of an Icoſaedron ſurmounteth the Superficial quantitie of all his inſcribed tranſſigured Icoſaedrons Hexagonall plaines, by a Superficies retaining ſuch proportion to the ſuperficiall capacitie of all his Pentagonal plaines, as the leſſe Dimetient of the comprehending Icoſaedrons baſis dooth to the leſſe Diameter of the tranſformed Solides Pentagonall baſis.

The 20 Theoreme,

Two ſuch lines being found, that the firſt bee to a tranſſigured Icoſaedrons ſide double, and the ſecond line equall to the greater ſegment of this tranſformed Icoſaedrons Pentagonall baſis greater Semidiameter parted by extreame and meane proportion, a Pentilater *Priſma*, hauing for his altitude this ſecond line, and for the ſide of his Pentagonal Equiangle baſis the firſt line, ſhall be equal to the exceſſe or difference of the containing Icoſaedron aboue the craſſitude of his inſcribed tranſſigured Icoſaedron.

The

The 21 Theoreme.

Dodecaedron transfigured, may of a Regular Dodecaedron be circumscribed, whose side being diuided in two lines reteyning double extreame and meane proportion, if from the greater ye deduct the lesser, the remaine is the side of the inscribed transfigured Dodecaedron.

The 22 Theoreme.

The side of Dodecaedron transformed, being subtracted from the side of his comprehending Regular Dodecaedron, and from the square of that remaines medietie againe deducting one third part of the square of the transfigured solides side, there remaineth a Superficies, whose Quadrate roote or line matching it in power, added to the greater Axis of the transfigured body, maketh the Semidimetient of his comprehending regular Dodecaedron.

The 23 Theoreme.

The superficiall content of all transformed Dodecaedrons Decagonal Bases, to the superficiall capacitie of all his Trigonal plaines, retaineth such proportion as the lesse Diameter of the Decagonal bases, to the lesse Semidimetient of his Triangular bases, and if vnto the Perpendicular of the Triangular bases ye adioyne triple the Decagonall bases lesser Dimetient. The square whose side is meane proportionall betweene that resulting line, and the circuit or Perimetrie of one Decagonal base, is equall to the vniuersall superficies of the transfigured Dodecaedron.

The 24 Theoreme.

The Superficies of a comprehending regular Dodecaedron, surmounteth the superficial quantitie of all his contained transfigurate Dodecaedrons Decagonall Bases, by a superficies, retaining the same proportion to the Superficiall content of all his Trigonal plaines, that the lesser of two such right lines retaining double extreame and meane proportion (as ioyned together make the lesser part of the comprehending Dodecaedrons Pentagonall Perpendiculare diuided by extreame and meane proportion) doth vnto the lesse Semidiameter of the transformate Dodecaedrons Trigonall bases.

The 25 Theoreme.

A transformed Dodecaedrons comprehending Dodecaedrons Diameter, being into two right lines retaining quadruple extreame and meane proportion diuided, and the lesser of them againe into two lines, retaining double extreame and meane proportion parted: A Trigonal Pyramis, hauing for his Equilater Triangular bases side, a line double to the transfigured Dodecaedrons side, and his Altitude fiuefold the lesse line produced by the latter section of the Dodecaedrons Diameters lesser part, shall be equall to the excesse or difference, whereby the containing regular Dodecaedron, surmounteth the solide capacitie of his inscribed transfigurate Dodecaedron.

Heere ensue the Additions, Diffinitions, Theoremes, and Problemes, for the better vnderstanding of such Mathematicall *Martiall Discourses as the Author heereafter meanes to Publish.*

To the fauourable Studious Ingenious Reader

touching these Diffinitions *and* Theoremes *concerning the newe Science of great* Artillerie.

THe Auncient *Philosophers* and *Mathematicians* that were either the first Founders or amplifiers of any Science with newe and rare Inuentions, were euer enforced at the first to vse manye newe and vnaccustomed names to distinguish and breefely to expresse such seuerall subiects as the Methodes of their discourses did occasion them to handell. Let no man therefore marueil, if my selfe being now to Intreate of so newe and rare a Science as this of great *Artillerie* bee also enforced to vse sundrie strange tearmes, not vnderstande perhaps of the verye Artificers themselues that most Manedge that kinde of Engine. For albeit in all such vsual actions as Cannoniers commonly practize, they haue wordes good inough to expresse their owne meanings, (And therein I vse also the same, except I finde them verye improper or misaplied) yet in the remote and hidden mysteries of Randonnes and of the seuerall Proportions of the mettaline Bodies and Soules or Cylinders of all seueral Peeces and of the strange varietie of the Circuites of all Bullets in the Ayre, by reason of the repugnance or variance of the violent and naturall motions: In these Mysteries I say scarsly thought on, and farre beyond the compasse of ordinary Cannoniers without exquisite knowledge in the *Mathematicall Sciences* to intermeddle withall. I finde not anye wordes in vse that can serue my purpose, and haue therefore chosen such as in my iudgement are most proper and effectual, and to take awaye all obscuritye, haue set downe euery of their *Diffinitions* that are not in my *Pantometria*, alreadye sufficiently expl ined: which Booke together with my *Stratiotices*, amplyfied with new Additions, I haue new published as Pathes to leade my ingenious Countrimen to the vnderstanding also of this newe Science, when I shall publish the same in her best perfection. And in the meane time as many yeares sithence I propounded publikely sundry Questions concerning this facultie, to stirre vp bothe Theorical and Practical spirites of all sortes more profoundly to searche the hidden secrets of this Arte. So haue I nowe thought good to Imprint also these halfe hundered *Theoremes*, resoluing the most part of all those my former Propositions, wherein I must neuertheless admonish the reader that I haue enterlaced a verie fewe that are not in the seuere examination of Geometrical Demonstration perfectly exact, albeit they approche so nighe as ordinarie experiments will hardly discouer the difference, which I haue of purpose doone, the rather to animate Mathematical Practisioners by triall to finde them out, and thereby perhaps also to discouer other more rare Secrets then I haue hitherto thought vpon. For by experience my selfe haue founde manye rare Mysteries in that Science, which I should neuer once haue amused on, if suche Scales and Theorikes (as my Father by his long painefull chargeable experiences in great Ordinance ioyned with his Mathematicall Science first inuented) had beene reduced to their accomplished perfection. And yet without those first principles left mee I must truely confesse I might haue spent the greatest part of my life in meere obscuritie, albeit I had by Office or otherwise enioyed the Manedging

of all sorts of great Artillerie at my owne pleasure charge free. The burden whereof no doubte hath beene the cheefest cause that so many rare Mathematicians as *Tartalea* the *Italian* and diuers others of these late ages in *Europe* haue mistaken so much, and performed so little toward the accomplishment of that Science. And although by publishing the same my Treatize of Martiall Pyrotechnie and great Artillery in the Latin toong, I should I knowe greatlye amplifie myne owne Fame, and the admiration of such rare Mathematicians as at this daye liue in seuerall Nations of Christendome, from whome I haue for farre inferior Inuentions Imprinted in my Treatize, Intituled *Ala seu scala Mathematica*, already receiued no small applause. Yet if I publish the same at all, I doe constantly resolue to doe it onely in my Natiue Language: Aswell to make the benefite thereof the more priuate to my Countreymen, as also to make thereby other Nations to affect as much our Language as my selfe haue desired to learne the Highe Dutche: for the workes onely of two Famous men that wrote their Bookes in that toong, which cannot possiblye by anye other that haue not the same Spirites and perfection that was in the verie Authors be Translated: as appeareth euidently by diuerse Treatizes vtterlye peruerted by the mistakings and misconceauings of the Translators.

These *Praeludia* therefore of a matter more serious and Important, good Reader accept in good parte, so shall I bee animated the more speedelye to imparte the rest, which some contrarye effect hath hitherto detayned from thee.

Certaine Diffinitions, taken out of my thirde *Booke of* Pyrotechnie *Militarie, and great* Artillerie, *for* explanation of these *Theoremes* ensuing.

The first Diffinition.

Forasmuch as by the direction of the hollowe Cylinder or Truncke of the Peece, the violence of all shot of great Artillerie is not onely directed but also increased I call that hollowe Cylinder of the Peece her Soule.

The 2 Diffinition.

The Mettalline substance of the Peece of what kinde, shape or proportion soeuer I call the Bodye *of the Peece.*

The 3 Diffinition.

This Soule *in all principall Peeces of Batterye, is euer a perfect uniforme Cylinder, comprehended with a Circuler columne, and two equall Circles, whereof the one I call the Head, the other the Base.*

The

The fourth Diffinition.

That direct line which by Mathematicall *Imagination doth conioyne the Centers of the two circles and is the perfect and true direction of all Shot made out of great Ordinance, I terme the Axis of that peeces Soule.*

The fift Diffinition.

Of this soule or hollowe Cylinder one parte is proportionally euer to be limitted for the Poulder and Bullet and that parte I call for distinction sake the charged Cylinder.

The 6 Diffinition.

The other emptie parte which is more properly called the Soule, because it bothe directeth and according to the difference of her proportion encreaseth or diminisheth the violence and force of the shotte I call also the vacant Cylinder.

The 7 Diffinition.

But in all such peeces as haue the Chamber for their Poulder either a lesse Cylinder or in forme Conicall or Campanall I terme the vacant Cylinder, onely the Soule of the peece, and the other her chamber be it Conicall or Campanall or Cylinder.

The 8 Diffinition.

Of the peeces body there is one parte of Counterpeyze called of Mathematitians Centrum Grauitatis *whereabout there are to all peeces bodyes in their foundings adioyned two stayes which I call the Peeces Eares.*

The 9 Diffinition.

That parte of the Peeces body where the touchehole is or ought to be made to geue fire in the discharging of all Ordenance, I call the Coyle of the peece.

The 10 Diffinition,

That parte where the body of the Peece is leaſt neere the Head or Mouth. I call the Necke of the peece, and at these three points the Coyle the Eares and Necke, the Bodyes mettalline of all peeces are to be measured to knowe whether they holde their proportion of perfection for that kinde and purpose they are destinate.

The 11 Diffinition.

Anye peece is saide to lye Pointe blancke with any marke, when the Axis of her Soule directeth perfectly to the very middell or Center of that marke.

The 12 Diffinition.

And a marke is sayde to lye within Pointe Blancke when the Peece beeing directed with her conuenient Bullet and charge is able to reache and strike that marke.

The 13 Diffinition.

A marke is said to lye within the mettall of the Peece, when the Peece beeing directed not by the Axis of the Soule, but by the Cornice *or vpmost ring of her head and Coyle is able to reache the marke.*

The 14 Diffinition.

The difference of these two ranges, I call the difference of the leuell ranges of the Soule and Body of any peece.

The 15 Diffinition.

The difference of theſe two meaſured in the body of the Peece I meane the exceſſe wherby the Semidiameter of the Ringe or Cornice of the Head dooth exceed the Cornice of the Coyle I call the Anomalye or Difference of the Soule *and* Bodye *of all Peeces truly founded, and of common Cannoniers is called the Peeces* Diſparte.

The 16 Diffinition.

The Axis of the Bodye *of any peece, I terme that ſtraight line which paſſeth betweene the Centers of the two vtmoſt Circular Cornices at the coyle and Head of the Peece, the which in all Peeces truely founded is alſo the verie ſame Axis of the* Soule.

The 17 Diffinition.

If the two Axes differ the Peece is falſe founded, and then they are either Parallele or make an Angle, if they be Parallele their difference I terme the Diſtance of the Axes *of that Peeces* Bodye *and* Soule.

The 18 Diffinition.

If they bee not Parallele, their Angles of variation are conſidered two wayes, that is to ſay in Altitude and Latitude, and thoſe Angles accordinglye named the Anomalie *Angles of Altitude or Latitude of thoſe Peeces.*

The 19 Diffinition.

There is in the Axis of euery Peece one onely pointe, wherevpon if the Bodye *of the Peece ſhould be ſuſpended, ſhe ſhall hang in a perfect Counterpoyſe, and that pointe I call the* Center *of her Gravity or pointe of Counterpoyſe.*

The 20 Diffinition.

If the Peece ſhould be deuided into two partes by a plaine Perpendiculare, cutting the Axis in that pointe of Counterpeyze, *the ſection will be circular, and for Diſtinction ſhalbe called the* Counterpeyze Section *of that Peece.*

Diffinitions taken out of my fyft Booke *of Martiall* Pyrotechnye *and great Artillerie.*

Firſte.

FOraſmuch as euery Bullet violently throwne out of any peece of Ordinance at any Angle of Randon paſſeth a good diſtance directlye without anye great variation from the right line pointed out by the Axis, and then falleth into a *Curue* Arke, and laſt of all finiſheth either in a right line or in a *Curue*, aproching nighe a right againe,

For Diſtinction ſake.

The

The first Diffinition.

THe first parte of the violent course of Gunners, commonly termed the peeces poinct blanke reache, I call the Direct Line of the Bullets circuite.

The 2 Diffinition.

The second parte beeing a Curue Circuite, beginning at the foresaide declination from the Axis, ascending to the highest altitude aboue the Horizon, And ending at a like Altitude to his beginning I terme for Distinction sake his middell Helicall or Conicall Arke.

The 3 Diffinition.

The rest euen to the Horizontall plaine againe I call his Declining Line.

The 4 Diffinition.

The Altitude of any Bullets circuite I call that line Perpendicular, which by Immaginiation Mathematicall, falleth from the Bullet at his verie highest of his Mount Perpendicularly downe to the plaine Horizontall: which line of Altitude coupled together, with the right linet from the top and foote, concurring at the center of the peeces circular Base, doth make a right Angled Triangle.

The 5 Diffinition.

The Horizontall line of that Triangle I call the Base.

The 6 Diffinition.

The other sloope line is the line Hypothenusall.

The 7 Diffinition.

The peeces direct line of that circuite which is alwayes aboue the Hypothenusall for distinction sake, I call the line Diagonall, for that there are seuerall of these Diagonall lines to all Angles of Randon, and together with the line Horizontall, doe comprehende the Angle of Mounte.

The 8 Diffinition.

The peeces leuell Horizontall range, I terme the distance betweene the Peece and the first graze or bounde of the Bullet, when the peece at her discharge lyeth leuell vpon her cariadge not mounted vpon anye loftie Platforme, but such as lyeth euen with the true Horizontall plaine whereon the Bullet must playe.

The 9 Diffinition.

All other Ranges made by the Bullet on any Horizontall plaine, when the Peece is mounted at any seuerall Randones, I terme the Ranges Horizontall.

The

The 10 Diffinition.

And because euery Peece hath some Grade certaine of the Quadrant whereunto mounted she maketh her vtmost Horizontall Range in such sorte as if yee mounte the Peece higher the Bullet shall flye a shorter Distance, and the Horizontall Ranges returne lesse and lesse againe. That pointe of vtmost randone Horizontall, I terme the Tropike *point or grade,*

The 11 Diffinition.

And because in Mounting the Peece Higher and Higher abone the Trepike grade, the Ranges growe still shorter and shorter, so that at length yee shall come to that grade which shall cause the Bullet to range exactly his first leuell Horizontall Range, that Grade or pointe for Distinction sake I call the Grade Æquorizontall.

The 12 Diffinition.

A declining plaine I terme any plaine grounde that lyeth not Leuell with the Horizontal plaine but descendeth downeward, making an Acute Angle with the plaine Horizontall and that Angle I call the Angle of Declination.

The 13 Diffinition.

An Enelining plaine I call that which from the Peece ascendeth vp, making likewise an Acute Angle with the plaine Horizontall, which Angle I call for Distinction sake the Angle of Inclination,

The 14 Diffinition.

And because the Bullet violently threwne out of the Peece by the furie of the Poulder hath two motions, the one violent, which endeuoreth to carry the Bullet right out in his Line Diagonall, according to the direction of the Peeces Axis from whence the violent motion proceedeth, the other Naturall in the Bullet it selfe, which endauoureth still to carrye the same directly downeward by a right line Perpendiculare to the Horizon. For Distinction sake I call the first his Diagonall motion Violent, *and the second his* Perpendicular motion Naturall, *and the compounded motion of these two his* Mixte helicall Circuite.

The 15 Diffinition.

And as for the discouerye of the strange varietie of the Planets courses in the Heauens, Astronomers are enforced to vse sundrie kindes of supposed Circles Excentricall and Concentricall, with their Epicicles mooning regularly sometimes on their owne Centers, and sometimes on Centers of Æquation, *which they terme the Planets Theorike. So likewise in the discouering of the Reciprocall most strange varietie of these Bullets Helicall Circuites in the Ayre, making continuall alteration according to the Quantitie and Qualitie of the Angles of Randon or Angles of Inclination or Declination of the Plaines whereon the Bullets playe, or Angles comprehended betweene the Diagonall violent and Perpendicular naturall motions: Being I say likewise enforced to vse portions of Circles, some Concentricall, and some Excentrical, some vniformelie deuided on their owne Centers, and some from Centers Equant in more strange manner to discouer and finde out these Helicall motions, I doe likewise terme the same* Theorikes *or* Scales.

The 16 Diffinition.

And that which serues to discouer the different violence of all Peeces at Pointe blanke, howsoeuer

how soeuer Mounted (by me termed the lines direct of the Bullets Circuites, and lines Diagonall) I call for Distinction sake the Theorike *of lines Diagonall.*

The 17 Diffinition.

The other that discouereth how high Bullets at all Randons can Mounte possibly aboue the Horizon, I terme the Scale *or* Theorike *of Altitudes.*

The 18 Diffinition.

The thirde which discouereth the varietie of Ranges, of all Peeces at all degrees of Randome, I call the Theorike *or* Scale *of* Randons.

The 19 Diffinition.

The fourth and fift which discouereth the varietie and alteration of all ranges, by reason of the Inclination or Declination of any markes from the plaine Horizontall of that Platforme where the Peeces playe, I call the Theorike *of* Inclining Plains.

The 20 Diffinition.

The sixth composed of all these, and by conference of all their partes together framing a Theorike of perfection, discouering in all plaines Horizontall or varrying for all kinde of Peeces and Bullets, what soeuer their Ranges at all Randones, the Altitude of their Circuites, together with their lines Diagonall and Hypothenusall shall hee named the Theorike *of* Artillery generall.

Theoremes concerning the newe Science

of great Ordinance, *resoluing the most part of my Artillery Questions in* Stratioticos *proponed.*

The first Theoreme.

THere are three cheefe most materiall and efficient causes of the greater violence of any shotte made out of great Ordinance, *viz.* the *Poulder*, the *Peece*, and the waight of the *Bullet*.

The 2 Theoreme.

Poulder is compounded of three Principles or Elements, *Salpetre*, *Sulphur*, and *Cole*, whereof *Salpetre* is it that giues the cheefest violence.

The 3 Theoreme.

Albeit *Salpetre* bee indeede the onelye or most materiall cause of the violence. And that Poulder commonly founde moste forcible, that is ritchest of *Petre*, yet is there a certaine proportion of Perfection of these three Componentes. And that in such sorte as if you adde more or lesse *Petre* the violence shall abate.

The 4 Theoreme.

Although Poulder be alſo the moſt efficient cauſe of the Force & violence of any ſhotte, yet is there ſuch a proportionall charge of Poulder to be found for euery ſeueral Peece, in regarde of the proportion of the charged and vacant Cylinders, as giuing more or leſſe, then the ſame proportionall charges it ſhall diminiſh and not increaſe the violence of the ſhotte.

The 5 Theoreme.

If any two Bullets of equall quantitie, but vnequall waight be let fall from anye loftie place to the Horizon, the more waightie ſhall euer fall the more ſwiftly: albeit not proportionally to their waighte, which Axiome is indeede erronious, albeit a great Philoſopher haue auerred the ſame.

The 6 Theoreme.

If two equall Bullets of different waight, be ſhotte out of one and the ſame Peece directly to the Zenith, bothe Bullets being of maſſie mettall and charged with one quantitie and kinde of Poulder, the lighter ſhall alwaies out flie the heuier. But ſuch kinde of Bullets they may bee charged withall, as the Heuier ſhall out flie the Lighter, although they bee bothe diſcharged with the ſame Peece, and quantitie of the ſame Poulder.

The 7 Theoreme.

There is ſuch a conuenient waight to be found of the *Bullet*, in reſpect of the *Poulder* and *Peece*, as the Bullets mettall being either heuier or lighter then that waight, ſhall rather hinder then further the violence or farre randge of the ſhot.

The 8 Theoreme.

There is ſuch a conuient Proportion to be founde of the length of euery Peece to his *Bore* or Bullets Diameter in reſpect of the Poulder, and waight of the Baull, as either encreaſing or diminiſhing that proportion it ſhall abate alſo and hinder the violence of the ſhotte,

The 9 Theoreme.

This Proportion exactly founde in any one Peece, dooth not holde in all other, and yet the difference and alteration ſuch, as may be reduced to Rules certaine,

The 10 Theoreme.

Beſides theſe 3 moſt materiall cauſes of violence, the Randons alſo and different Mountes of Peeces, cauſe a great alteration not onelye of the farre ſhooting of all Peeces, but alſo of their violent Batterie. And albeit the different alterations are verye intricate and ſtrange, yet haue they a *Theorike* certaine.

The 11 Theoreme.

There are alſo many other Accidentall alterations happening by reaſon of the winde, the thickneſſe or thinneſſe of the Ayre, the heating or cooling of the Peece, the different manner of charging by Raminge faſt or loaſe of the Poulder, by cloſe or looſe rouling or lying of the Bullet, by the vnequal Recule of the Peece, either by reaſon of the vnequality of the Platforme or Wheeles, or by the vneuen lying of the Peece in his Carriage, or diformitie of the Axtree, with diuers other ſuch like, whereof no rules certaine can be preſcribed, to reduce theſe vncertaine differences to any certaine proportions: but all theſe are by Practize, Diſcretion, and Iudgement to be conſidered and vniformely guided and performed in their beſt perfection.

The

The 12 Theoreme.

Any Peece mounted 90 grades aboue the Horizon, throweth his Bullet moſt violentlie immediatly after the diſcharge, and then the motion groweth ſlower till the Bullet be come to his vtmoſt Altitude, and then by Perpendiculare falling encreaſeth by little and little, his ſwiftneſſe againe euen till it come to the Horizon. But at all other Randons it falleth not ſo out.

The 13 Theoreme,

Albeit in the ſubtiltie of *Geometricall* Demonſtration no parte of the Bullets violent motion can be truely auerred a right or direct line, ſaue onely the Perpendiculare: yet in theſe experiments *Mechanicall*. That firſt part of the violent motion I meane ſo farre as the Peece is ſaide to carry *Poynte Blancke*, beeing ſo neere the direct, is and may well be teemed the direct line. As all water leuels are accounted in all Mechanicall opperations, the Perfecteſt leuels and directeſt lines. Albeit the ſubtiltie of Geometricall Demonſtration, doth finde them not right or direct, but Curue or Circuler.

The 14 Theoreme.

When any Peece is mounted directly to the Zenith. Then doth his Motion violent (being in that ſcituation directly oppoſite to the naturall) cary the *Bullet* in a perfect right line, directly vpward, till the force of the violence be ſpente, and the Naturall motion haue gotten the victorie. And then dooth the Naturall returne the *Bullet* downward againe by the very ſame Perpendicular line. And ſo is the whole motion of the *Bullet* in this caſe a verie direct Perpendicular to the Horizon.

The 15 Theoreme.

But if any Peece be diſcharged vpon any Angle of Randon, albeit the violent motion contend to carry the *Bullet* directly by the line Diagonall. Yet the Perpendicular motion being not directly oppoſite, dooth though vnſenſiblye euen from the beginning by little and little drawe it from that direct and Diagonall courſe. And as the violent dooth decaye, ſo dooth the naturall encreaſe: and of theſe two right lined motions is made that mixt Curue Helical Circuite of the *Bullet*.

The 16 Theoreme.

Any Peece therefore diſcharged at any Mounte or Randon, firſt throweth foorth her *Bullet* directly a certaine diſtance, called of Gunners their *Pointe Blancke Range*, and then it maketh a Curue declining Arke, and after finiſheth either in a direct line, or nighe enclining towards it.

The 17 Theoreme.

The further that any Peece ſhooteth in her direct line, commonlye called *Poynte Blancke*, the deeper alſo ſhe pearceth in her Battery, if the Bullet be not of ſubſtance brickle or frangeable.

The 18 *Theoreme.*

The more Ponderous a *Bullet* is the more it ſhaketh in Battery, albeit it pearce not alwayes ſo deepe as the lighter or leſſer ſhotte conueniently charged.

The 19 *Theoreme.*

Any two Peeces of Battery Ordinance, charged with one kinde of Bullet and ſhotte into one Rampire of maſſie vniforme kinde of Subſtance, ſhall euer make their Profundities of pearcing Proportionall to their leuell Randges Horizontall, if they be diſcharged either leuell ot at one grade of Randon, and at like diſtance.

The 20 *Theoreme.*

Any two Peeces of Batterie diſcharged into any Rampire of vniforme maſſie ſubſtanee, ſhall euer make their Pearcing depthes proportionall to their lines Diagonall, albeit theſe Peeces be diſcharged from different Randons, ſo as they batter at like diſtance.

The 21 *Theoreme.*

As *Archimedes* line Helicall or Spirall, is made by the direct motion of a pointe carried in a right line, while that right line is Circularly turned as Semidiameter vpon his Circles Center. So is this Artillery Helicall line of the Bullets Circuite created onely by two right lined motions becomming more or leſſe *Curue* according to the difference of their Angles occaſioned by the ſeuerall Angles of Randon. Whereupon by demonſtration Geometricall a *Theorike* may bee framed that ſhall deliuer a true and perfect deſcription of thoſe *Helicall* lines at all Angles made beeweene the Horizon and the Peeces lines Diagonall.

The 22 *Theoreme.*

Theſe direct or Diagonall lines, are alwayes longeſt when the Peeces Axis is directed to the Zenith. And alwayes as the Peeces Axis declineth more and more to the Horizon. So doe the Diagonall lines growe ſhorter, and at the leuell Horizontall ſhorteſt of all.

The 23 *Theoreme.*

Theſe direct lines Diagonall, albeit they encreaſe in length at euery grade of Randon from the Horizon to the Zenith, yet is not their encreaſe vniforme or proportionall either to their degrees of Randon or Horizontall Ranges, nor yet to their Circuites or Altitudes, and yet ſuch as may be reduced to a *Theorike* certaine.

The 24 *Theoreme.*

Theſe middle *Curue* Arkes of the Bullets Circuites, compound of the violent and naturall motions of the Bullet, albeit they be indeed meere Helicall, yet haue they a very great reſemblance of the Arkes Conicall. And in Randons aboue 45. they doe much reſemble the Hyperbole, and in all vnder the *Ellepſis*: But exactlye they neuer accorde, being indeed *Spirall* mixte and *Helicall*.

The 25 *Theoreme.*

Any Peece diſcharged at any one Randon with like Bullets and ſeuerall charges of Poulder ſhall make bothe their lines Diagonall and Curue Circuites of diffe-

rent

rent longitude, but the Curue Arkes shall alwayes be as Paralleles, and their Longitudes Proportionall to their liues Diagonall.

The 26 Theoreme.

The last declining line of the *Bullets* Circuite, albeit it seeme to aproache somewhat to the nature of a direct line againe, yet is it indeed still *Helicall* and mixte, so longe as there remaineth any part of the motion violent. But after that is cleane spent, the rest of his coruse to the Horizon is direct and Perpendicular, and a perfect right line indeed, which is best discerned in those Grades of Randon, which are betweene the Zenithe and the Mount or Randon Equorizontall.

The 27 Theoreme.

This declining line dooth alwayes make a greater and greater Angle with the Horizon, as you raise the Peece to a greater Mounte, till you come to the Mounte Equorizantall, about which Pointe the same declining line becommeth Perpendicular before the *Bullet* fall to the Horizon.

The 28 Theoreme.

The Horizontall Ranges in all Peeces mounted from the Horizon towarde the Zenith, dooth not still encrease but at euery grade of Randon are longer, till you come to the Pointe or mounte *Tropicall*, commonlye called the vtmost Randon, which hath beene generally thought to be the grade 45. but is not so. And from that *Tropicall* grade vpwarde the Ranges decrease againe till you come to the grade Aequorizontall, so called because the *Bullet* then falleth a like distance to the leuell Ranges.

The 29 Theoreme.

This Aequorizontal Grade is as farre distant from the Zenithe, as that Grade is from the Horizon, which shall cause the Peece to shoote in the Horizontall plaine a distance equal to his highest Altitude or longest line Diagonal.

The 30 Theoreme.

The Mounting of any Peece aboue his Aequorizontal grade doth stil decrease her Horizontall Ranges euen till it come to the Zenithe. But in a Proportion different from any of the former, her *Bullet* ending euerye of those Circuites in a direct line Perpendicular.

The 31 Theoreme.

The Gradual encrease and decrease of these Ranges Horizontal, albeit they are equal in the Quadrant, yet are they neither equal nor proportional in the Horizon, neither the Ranges nor their Interualls. Neither compared between themselues nor yet conferred with the Chordes or Sinus of their Arkes. And yet is there such a kind of Proportional encrease and decrease of the Proportion of their Interualles, as may be reduced to a *Theorike* certaine.

The 32 Theoreme.

The *Tropicall* grade commonly called the vtmost Randon, is not as hath beene generally supposed the Medium or Middle between the Horizon & the Zenith, *viz.* 45, but rather betweene the Horizon and the grade Aequorizontal, which wil fall out much nigher 50 from the Zenithe, and 40 from the Horizon.

The 33 Theoreme.

The higheſt *Altitude* of anye *Bullets* Circuite is fardeſt diſtante from the Peece, when ſhee is diſcharged at her vtmoſt Randon or point *Tropicall*, and at all other Randons either aboue or beneath that *Tropicall Pointe*. That higheſt *Altitude* is euer leaſt diſtant, and the baſes of theſe Triangles do euer encreaſe to the Randon *Tropicall*, and decreaſe after euen as the Horizontall ranges, but in Proportion more different euery one from other.

The 34 Theoreme.

The *Altitudes* of the Circuites of Randons do not encreaſe and decreaſe as their Ranges *Reciprocally*, but from the Horizon in euery grade to the Zenith, doe ſtill encreaſe, but yet neither equallye nor Proportionallye, neither conſetred betweene themſelues, neither yet with the lines or Cordes of their Arkes of Randon. And yet the encreaſe and decreaſe of their Interualles Proportions ſuch, as may be reduced to a *Theorike* certaine.

The 35 Theoreme.

The *Hypothenuſall* lines of all theſe different Circuites carrie a mixte proportion of the compoſition of the Proportions of theſe *Altitudes* and baſes by addition of their Squares. But are not proportionall to the lines Diagonall of their correſponding Angles of Randon.

The 36 Theoreme.

Any two Peeces of Ordinance being mounted to anye one Grade of Randon ſhall make their Horizontall Randges of their Bullets Proportional to the *Altitudes* of their Circuites.

The 37 Theoreme.

The Ranges Horizontall of any two Peeces diſcharged at one Randon, wil bee alwayes proportionall to their lines Diagonall of the ſame Peeces Circuites.

The 38 Theoreme.

The Horizontall leuell Randges of any two Peeces of Artillery are euer proportionall to the vtmoſt Randges Horizontall of the ſame Peeces.

The 39 Theoreme.

Anye two Peeces whatſoeuer diſcharged at one Randon doe euer make their lines Diagonall, and lines of Altitude Proportionall howſoeuer the Proportions of their charges vary.

The 40 Theoreme.

Any two Peeces whatſoeuer diſcharged at one grade of Randon vpon any enclining or declining Plaine ſhall neuertheleſſe make their Ranges proportionall to their

their lines Diagonall and Altitudes of those their different Ranges. Albeit the Peeces be charged with a different kinde of Proportion of Poulder and Bullet, so as the shot be made in a faire Calme daye, as is in these cases alwaies presupposed because for such vncertaine Accidents there cannot certaine Rules Artificial be prescribed.

The 41 Theoreme.

One Peece discharged at seuerall Randons vnder the vtmost Randon being a like charged and discharged, and the Peece also of one temper at bothe times, shall euer make seuerall Randges. But if she be discharged at seuerall Randons, the one aboue the *Tropike* point, the other vnder. Then may their Ranges be equall notwithstanding their Randons, *Lines Diagonall*, *Altitudes*, *Bases*, and *Lines Hypothenusall*, be all different.

The 42 Theoreme.

When anye Peece (being twise discharged at seuerall Randons, the one aboue the other, beneath the *Tropike* pointe) shall make the same or equall Randges in an *Horizontall* plaine. The middle grade betweene those seueral Mountes is very nigh the grade of vtmost Randon: and the Peece Mounted to that middle grade, shall then make verye nighe his vtmost *Horizontall* Randge.

The 43 Theoreme.

The grade of vtmost Randon or pointe *Tropicall* of any Peece in a Playne *Horizontall* shall not be the *Tropicall* grade of that Peece, in a Plaine declining or enclining, but an other Peculiar to that Angle of Inclination or Declination.

The 44 Theoreme.

Any Peece discharged at his grade of vtmost aduantage *Horizontal* vpon a Plaine enclining, shall not make so great a Randge as on his plaine *Horizontall*: But contrariwise on a Plaine discending shall make a farther Range.

The 45 Theoreme.

A Peece discharged first at his dew leuel, and againe at his Æquorizontal grade, albeit in the plaine *Horizontall* they make equal Randges, yet in Plaines declining shal they not so do, but alwayes the Leuel Randges shall euer out shoote in all declining Plaines the Randge of that grade Æquorizontall.

The 46 Theoreme.

A Peece discharged at any grade from the Zenithe to the grade Æquorizontal, shal alwayes make a greater Range in any Plaine enclining or declining, then on the Plaine *Horizontall*.

The

The 47 Theoreme,

In all Plaines enclining at all Randons betweene the *Horizontall* Leuel and pointe *Tropicall*, all Peeces ſhoote farther in their Plaines *Horizontall*, then on any Playnes enclining, and contrarye wiſe in Playnes Declining, But aboue the *Tropike* grade not alwayes ſo, but ſometimes, and not alwayes contrary.

The 48 Theoreme.

In any Plaine whether it be enclining or declining, if anye Peece of Ordinance be diſcharged being *Parallele* or Equediſtant to that plaine, and the firſte graze or bounde noted. If the ſame Peece be with like charge vniformely charged and diſcharged at ſuch an highe grade of Randon as may cauſe the *Bullet* Range the former Diſtance: That middle grade of the Quadrant, which lyeth betweene theſe two Mountes, ſhalbe very nighe the grade of vtmoſt aduantage, for that enclining or declining plaine. The which in all plaines enclining, wil bee aboue the vtmoſt Range *Horizontall*, and in all declining vnder.

The 49 Theoreme.

In all enclining or declining playnes, as the grade *Tropike* of greateſt aduantage dooth varie. So dooth alſo the Proportions of their Randges, at euerye grade of Randon differ, whether they be accounted from the Zenithe, Horizon or Playnes, enclining or declining. But yet in ſuch an aſſured and certaine manner as may be reduced to a *Theorike* perfite.

The 50 Theoreme,

In all Grades of Randons, and in all manner of Peeces, whether the playnes be *Horizontall* or varyé by Inclination or Declination, the *Diagonall* Lines are ſtil proportionall to thoſe of the plaines *Horizontall* reſpectiuelye taken by Graduation from the Zenithe, in all Peeces whatſoeuer. But the Lines of *Altitudes*, their *Baſes* and Lines *Hypothenuſall* are euer different in euery ſeuerall Angle bothe of Inclination and declination, and varye by ſuch a different Proportion from the *Horizontall*, as they are to be diſcouered by a ſeueral Methode of Calculation.

The 51 Theoreme.

Such *Theorikes*, *Scales*, and *Inſtruments*, may bee framed for the Inuention of theſe ſtrange Proportions of *Altitudes*, *Lines Diagonall*, and *Ranges Horizontall*, as thereby with the ayde of *Calculations Arithmeticall*, and ſome Rules *Geometricall*, a man may exactly and readely diſcouer the true Circuites and Ranges of the *Bullets* of al Peeces of Ordinance whatſoeuer, mounted howſoeuer, and vpon all grounds or plaines enclining or declining, that can be Imagined, as ſhortly to the world by Gods grace ſhalbe made apparant.

Certaine

Certaine Geometricall Problemes of great vse

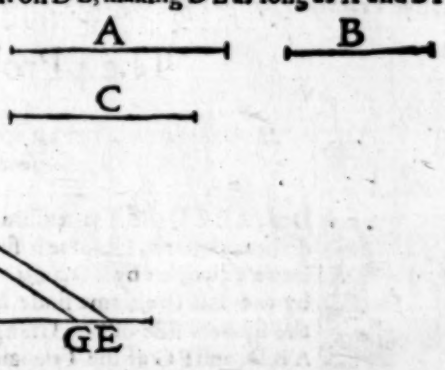
and Importance for the vnderstanding of my other Treatize of Architecture Militarie and Nauticall and great Artillery.

The fyrst Probleme.

To any two right lines geuen to make two other proportionall, wher of any one shall be equall to any other right line assigned.

Dmit the two right lines first giuen A and B, C a third also assigned to whome or to his equall I desire an other retayning the same proportion of A to B.

To performe this Probleme by the first Chapter of my first Booke I erect D F Perpendicular on D E, making D E as long as A and B F as B, and then if it be desired that C shall be the lesse of the two proportional sought, cut of his equall from D F, if greater from D E, as nowe I make D G equall to C, last of all by the 3 Chapter of the same my firste Booke. I drawe a parallele to E F from G, cutting D F in H. So shall D G to H D retayne the very same proportion that A doth to B and D G the greater equall to C.

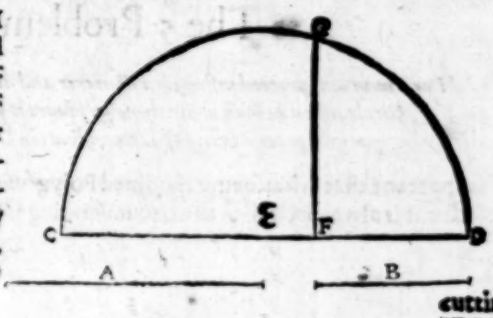


This Arte serueth generallye to make 2, 3, 4, or as manye moe lines as you list proportional to any other assigned, as the ingenious ful easily may conceiue.

The 2 Probleme.

How to finde out a meane Proportionall betweene any two right lines giuen.

Dmitte A B, the two right lines giuen to finde a meane or middle proportionall betweene them. I make C D equall to A B, that is C F equall to A and F D to B then by the 4. Chapter deuide C D in halfe at E, and vpon E as a Center describe the Semicircle C G D, then vpon F by the first Chapter of Longimetra, erect a Perpendiculare on C D



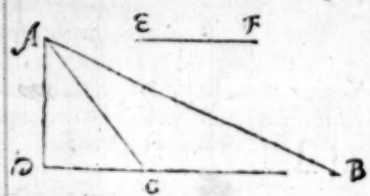
cutting

cutting the Circumference at G, ſo ſhall F G be the meane proportionall betweene A and B, which by this Probleame was required.

The third Probleme.

How to ſquare anye Triangle that ſhall bee proponed.

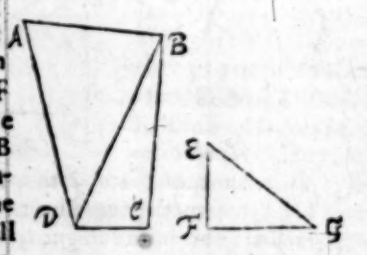
Dmitte the Triangle A B C whereto I deſire to deliuer an equall ſquare. Firſt therefore I produce the ſide B C, and then from A, I let fall the Perpendicular A D, this doone by the laſt Probleme, I ſearche out the middle Proportional betweene A D and the Moitie of C B, admit that E F, I conclude therefore E F the deſired ſquares ſide, which ſhall be exactly equall to the Proponed Triangle.



The 4 Probleme.

How to ſquare all Trapezia or right lined Polygona *whatſoeuer.*

Dmit A B C D the Trapezium, I deſire to ſquare, I reſolue it firſt into 2 Triangles by B D, and then by the laſt Probleme finde E F the ſquares ſide of the Triangle A B D, and F G of the Triangle B D C. Theſe two ſides I place perpendicularly enuironing the right Angle E F G, drawing the lyne Hypothenuſall E G, whoſe ſquare ſhall be equall to that Trapezium. In this ſort if there were an other Triangle adioyned to this Trapezium, to make it Pentagonall, it were but on E G to erect a Perpendicular at E, equal to the ſide of that Triangles ſquare, and ſo ſhould the Hypothenuſall of that newe Rectangle Triangle bee the ſide of a ſquare equall to the Pentagonum, and in this ſorte we may proceed to ſquare Hexagonal, Heptagonal, Octogonal, and conſequently by the ſame rule and reaſon, all right lined Figures, whether they be Æquiangle or otherwiſe howſoeuer.

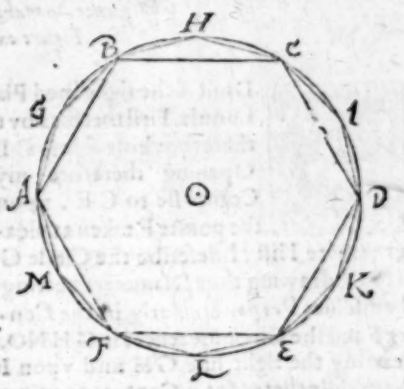


The 5 Probleme,

How a man may proceed infinitely ſtill neerer and neerer to the ſquaring of the Circle, albeit he ſhall neuer thereby ſquare it perfectly, according to the preciſe verity of Demonſtration Geometricall.

IT is apparant, that whatſoeuer right lined Polygonium yee ſhall within anye Circle deſcribe, it is alway leſſe then the circumſcribing Circle, As in the Circle ⊙ ye may behould

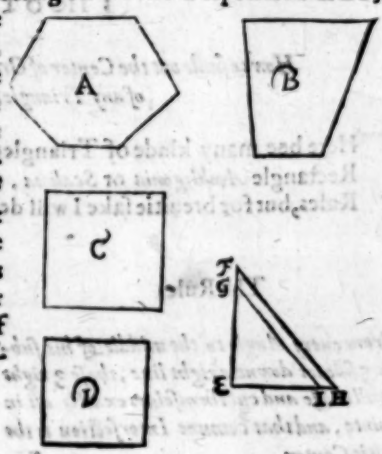
behold the *Hexagonum* A B C D E F greater then the Triangle A C E and yet leſſe then the Circle, but if theſe Circumferences bee againe diuided into halfe, in the pointes G H I K L M, yee frame a new *Polygonum* of 12 Sides leſſe then the Circle, but yet more nigh in quantitie then the former, And by like ſubdiuiſion yee may proceede on to 24, and ſo forwarde infinitely framing *Polygona* ſtill neerer and neerer in content to the Circle, but neuer exactly equall, and ſeeing by the former *Probleme* ye are taught how to ſquare all ſortes of *Polygona*, ye may by this meanes finde ſquares ſtill infinitely nigher in quantitie to the Circle, but neuer exactly æquall. By which Arte *Archimedes Siracuſanus* firſt found out the Proportion of the *Peripherie* to the *Diameter*, to bee leſſe then $3\frac{1}{7}$ and more then $3\frac{10}{71}$, and therefore not as $\sqrt{9.10}$ to an vnitie, which ſome ſuppoſed to be the vndoubted Proportion.



The 6 Probleme,

Howe to finde out the Proportion of any two right lined plaine Superficies whatſoeuer, be they Triangles, Trapezia, or other Polygona, and to giue their true Proportion in right lines.

ADmit A and B the right lined plaines, whoſe Proportion I ſhould expreſſe in right lines. Firſt therefore by the 4. Probleame I ſquare theſe Plaines, making the ſquare of C æquall to the Pentagonall ſuperficies A and D to B, then ſecting on E L the Perpendicular E F, I make E F equall to the ſide of the ſquare C, and E G, E H, eyther of them equall to the Side of the Square D, then drawing a ſtraighte Line from F to H by the thirde Chapter of the firſt Booke from G, I drawe a Parallele thereto, cutting E H in I, I concludde therefore E F ſhall hould the ſame Proportion to E I that the Pentagonum A doth to the Quadrangle B, and this rule is generall for all Playnes whatſoeuer, for in Squares as in all other like Figures rightlined, the Proportion of their Playnes is alway double to the correſpondent Sides.

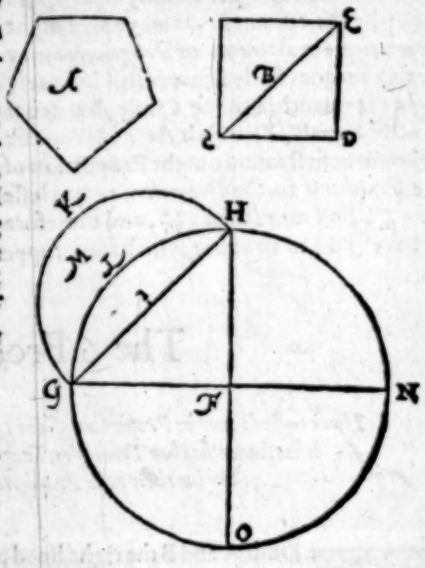


C c The

The 7 Probleme.

To any Triangle, Quadrangle, or other rightlined Plaine whatsoeuer, to make a Lunula or Lunular Figure exactly æquall.

Admit A the rightlined Plaine whereunto I would make an æquall Lunula. First therfore by the 4 Probleme, I make the Square B æquall thereto, whose side is C D and his Diameter or Diagonall line C E, Opening therefore my Compasse to C E, vpon the pointe F, taken at pleasure where I list, I describe the Circle G H N O, drawing the 2 Diameters, cutting themselues Perpendicularly in the Center F and the Circumferences in G H N O, drawing the right line G H and vpon I the middle thereof as a Center, opening my Compasse to G or H, I describe the Semicircumference G K H, so haue I made the Lunula M comprehended of the 2 Arkes G K H, and G L H exactlye æquall to A the Rightlined Playne proponed.



A Note.

Heereby the Possibilitie of squaring the Circle is apparant seeing to any right lined Plaine, a Curue of circular Arkes compoſed, may bee made æquall, which made a great Philosopher say, that Conclusion was possible, though not yet discouered.

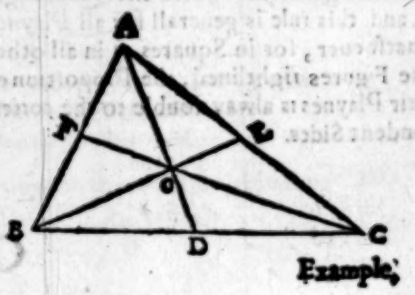
The 8 Probleme,

How to finde out the Center of Granitie, or Counterpeyze Center of any Triangle whatsoeuer.

THere bee many kinde of Triangles *æquilater Isosceles Scalenum*, and those Rectangle *Amblygonia* or *Scalena*, euery of which kindes haue their seuerall Rules, but for breuitie sake I will deliuer one generall Rule for all.

The Rule.

From euery Angle to the middle of his subtending side let downe a right line, these 3 right lines will crosse and cut themselues exactly all in one pointe, and that comune Intersection is the Granitie Center.



Example,

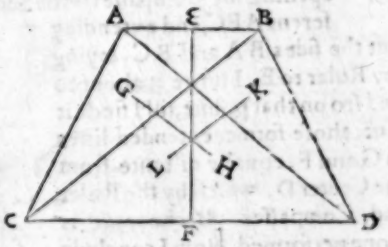
Example.

Behould A B C the Triangle of what kinde soeuer, B E F the Middle points of the sides, whence lines drawne to the opposite Angles crosse all in G, which is alwayes the true Counterpeyze Center of that Triangle, as is most ingeniously demonstrated by Archimedes *in his first Booke of aquall Counterpeyze.*

The 9 probleme,

How to finde the Center of Grauitie of any Mensall Figure or Quadrilater Trapezia, whereof 2 sides are aquedistant or otherwise.

ADmit A B C D the Mensall Figure, proponed E and F the middle pointes of the two Parallele sides, G the Grauitie Center of the Triangle ABC found by the last Probleme H, the Counterpoyze Center of BCD, I, the Intersection of E F and G H, shall euer be the Counterpeyze Center of that Mensula, and G I shall alway carry the same proportion to F H, that the Triangle B C D dooth to A B C interchangeably, which is to bee remembred, because yee shall haue vse thereof heereafter, but especially in my Artillerie Treatize, for discouering the Counterpeyze Center in al kind of cast Ordinance, for placing their Eares artificiallye, &c.



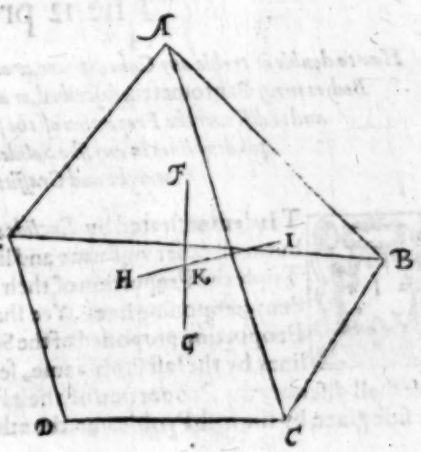
But if no two sides be Parallele, ye may by any other line drawne from A to D, diuide it into two other Triangles, *viz.* A B D and A D C and then drawing a right line from K the Grauitie Center of A B D to L, the Grauitie Center of A D C, where it cutteth G H *viz.* in I, shall be the Counterpeyze Center of that Quadrilater Trapezium whatsoeuer for this rule is vniuersall.

The 10 proleme,

How to finde the Counterpeze Centers of any Quadrangle or other Polygonam right lined Plaine whatsoeuer.

THere is no *Polygonum* but may by right lines bee deuided into Triangles and Quadrangles as hath beene already at large opened in *Planimetra.*

Admitte therefore A B C D E, the *Polygonum*, whose Grauitie Center I desire, drawing therefore the line E B, I resolue that Figure into the Triangle A E B, and the Quadrangle E B C D. Now by the 8 and 9 Problemes, I find F the Counterpeyze Center of the Triangle, and G of the Quadrangle, conioyning them with the line G F. Againe I diuide the same *Pentagonum* by the line A C, into the Triangle A C B, and the Quadrangle A E D C, and conioy-

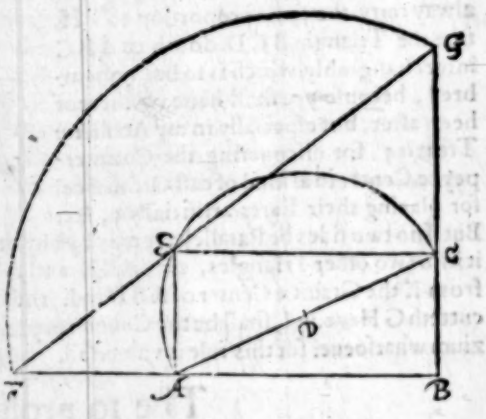


ning likewiſe the Center of the Triangle A B C at I, and the Center of the Quadrangle AE DC at H, with the right line H I. Their commune Interſection in K, ſhall be the Grauity Center of that *Pentagonum*. In this manner as you were firſte taught by the Counterpoyze Centers of Triangles, to finde out the Centers of a Quadrangle, and now by the Grauitie Centers of a Triangle and Quadrangle, to diſcouer the Center of any *Pentagonum*, ſo may you likewiſe by the Centers of a *Pentagonum* and Triangle, ſearche out the Grauitie Center of an Hexagonum, and ſo by one and the ſame Engine, proceede infinitely to all rightlined Plaine Superficies whatſoeuer.

The 11 probleme,

Howe to finde out two meane proportionall lines betweene any two right lines aſſigned.

ADmit A B, and B C, the two lines aſſigned, BC being Perpendicularly placed on A B I drawe AC, and vpon D the middle of that *Hypothenuſa*, as a Center opening my Compaſſe to the Semidiameter DA, I deſcribe the Semicircumferens AEC, and extending out the ſides B A and B C, laying my Rular to E, I turne it about to and fro on that pointe, till I finde it Cut, thoſe former extended lines in G and F, equally diſtante from the Center D, which by the Ruler and Compaſſes *Mechanically* is ſoone performed. Now I conclude A F, and CG ſhall bee the two meane Proportionals deſired.



And this of al other the Inuentions of *Plato*, *Apollonius*, *Sporus*, *Diocles*, *Architas*, *Menechmus*, *Eratoſthenes*, *Diocles Nicomedus*, and many other famous *Geometricians* I like beſt, for ready performance of this concluſion, whoſe ſeueral Methodes I could heere adioyne, but I loue Breuitie.

The 12 probleme,

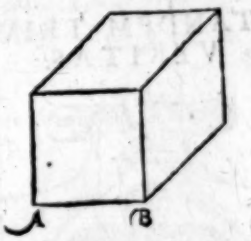
How to double or treble any Cube, Sphere, or other Platonicall regular or tranſformed Bodyes in my Pantometria *deſcribed, or any other Solide of what forme ſoeuer, and to diſcouer the Proportion of the ſides Diameters, or other correſponding lines in any like Solides, the Proportion of whoſe waight and Craſſitude is knowne.*

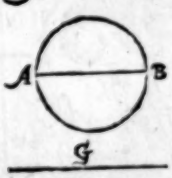
IT is demonſtrated by *Euclide*, that not onely in Spheres and Cubes, but in all other vniforme and like Bodyes, the Solides hould alwayes Triple the Proportion of their Dimetients Sides, or other correſpondent componing lines. Yee ſhall therefore firſt in lines ſet downe the Proportion proponed of the Solide Bodyes, and then between thoſe lines by the laſt Probleame, ſearche out two meane Proportionals, theſe ſhall diſcouer the Proportion of their ſides or Dimetients, and then hauing one ſide giuen by the firſt Probleame, the other is reuealed.

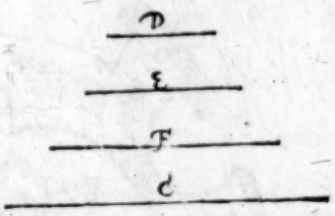
Example

Example.

Admit A B the ſide of the Cube or Diameter of that Sphære, which I deſire to treble, that is to finde out the ſide or Diameter of ſuch an other Cube or Sphære, as ſhall in quantitie or waight if they be *Homogenea* bee treble to the other.







To reſolue this I ſet downe the lines D and C, retaining the preſcribed Proportion *viz.* of 1 to 3, and then by the laſt Probleme I ſearche out E and F, the 2 meane Proportionals between them, and by the firſte Probleme likewiſe the line G, retaining to A B the Proportion of F to E. G therefore I conclude ſhall be exactly the ſide of that Cube or Diameter of that Baull that ſhall be treble to the other. And as you haue doone in this Example of Treble, ſo are you to proceede in any other Proportion, ſetting firſt downe in lines the proponed proportions ſolide, and then proceeding to the 2 lines meane proportionall, the Methode is all one, and therefore ſuperfluous to vſe mo wordes.

THis Rule houldeth not onely in Sphearet, and all other the Platonicall or tranſformed Bodyes, but alſo in all other vniforme or like Solides, of what ſhape or forme ſoeuer they bee, as Tunnes, Tankardes, Columnes, Statues, Pylaſters, Beames, Maſtes, Shippes, and Ordinance, of all ſortes conferred with their like. But the rare vſe of theſe Problemes ſhall appeare hereafter not onely in my Treatize of great Artillerie, but alſo of Architecture Nauticall *for the invention of the Proportion and true quantities of the Keeles, Beames, Maſts, Yardes, and all other Members or Partes of Shippes, for all Quantities and Burdens, and that for all Moulder, Shapes, Proportions, Conditions, and Qualities whatſoeuer.*

FINIS.

VICTRIX TANDEM TRIVMPHABIT
VERITAS.

Faultes eſcaped.

P. *Age* 34, *line* 41, *for* been *reade* betweene, *and for* but ſuch *reade* but for ſutche.
P. 38, L. 2, *for* conſidering *read* concerning.
P. 38. L. 3. *for* if *reade* ſins.
Pa. 66. Li. 1, *for* or meaſure *reade* or otherwiſe meaſure.
Pag. 74. Li. 11, *for* 146 *reade* 140.
P. 76. l. 15 *for* you draw *reade* you not draw.
P. 79. *laſt line*, *for* $\frac{16}{17}$ *reade* $\frac{16}{17}$.
P. 85. L. 32, *for* 1575 $\frac{1}{1}$ *reade* 1437 $\frac{1}{1}$.
P. 91. L. 7, *for* 889 *reade* 880.
Pa. 92. Li 12, *for* 50 *reade* 60.
Pag. 93 Lin. 5, *for* 8000 *reade* 80000, *and likewiſe in the* 14 *line*.
P. 104. L. 3, *for* $\sqrt{3}$ is *reade* $\sqrt{3}$ 3, is.
P. 106. L. 3, *for* 45 + *reade* 49 +
P. 107. L. *the laſt*, *for* $\sqrt{3}$ 3 to 5 *reade* $\sqrt{3}$ 3 to 4
Pa. 109, Li. 6, *for* contayning Circles, *reade* contained Circles, *and in the Margent*, *for* Tetraedron 20, *reade* Tetraedron 10.
Pa. 113, Li. 12, *for* 729 *reade* 720.
Pa. 114. L. 24. *for* 186 *reade* 180.
Pag. 116. Lin. 13, *for* $\frac{1}{10}$ *reade* $\frac{1}{10}$.
Pa. 120, Li. 8, *for* $\frac{1}{7}$ *reade* $\frac{1}{7}$.
Pa. 121. Lin. 2, *for* $\frac{1}{1}$ *reade* $\frac{1}{17}$.
P. 122. L. 28, *for* to times *reade* 20 times.
Pag. 123. L. 35. *for* $\frac{621}{13}$ *reade* $\frac{621}{130}$.
P. 127. L. 12, *for* 16 $\frac{2}{3}$ *reade* 16 $\frac{2}{7}$.
P. 127. L. 17. *for* $\sqrt{3}$ 5 $\frac{1}{2}$ *reade* $\sqrt{3}$ 55 $\frac{1}{2}$.
P. 127. L. 20. *for* $\sqrt{3}$ 888 $\frac{2}{3}$ *reade* 888 $\frac{2}{3}$.
P. 127. L. 22, *for* $\frac{1}{2}$ *reade* $\frac{1}{2}$.
P. 128. L. 27, *for* $\sqrt{3}$ 2 *reade* $\sqrt{3}$ $\frac{2}{1}$.
P. 128. L. 30, *for* + $\sqrt{3}$ $\frac{2}{1}$ *reade* + $\sqrt{3}$ $\frac{2}{1}$.
Pa. 130, L. 8, *for* $\frac{2}{1}$ *reade* $\frac{2}{1}$.
Pa. 130. L. 33. *for* C E in x *reade* C E in x.
Pag. 133. Li. 32, *for* ſuppoſed Octaedrons, *reade* ſuppoſed 1. Octaedrons.
Pag. 133. Li. 42, *for* $\frac{1}{14}$ *reade* $\frac{1}{44}$.
Pag. 134. L. 1. *for* $\sqrt{3}$ $\frac{1}{3}$ *reade* $\sqrt{3}$ $\frac{1}{3}$, *and in the Margent*, *for* Tetraedrons ſide, *reade* Tetraedrons ſide. 1.
P. 135. L. 4. *for* $\sqrt{3}$ $\frac{41}{44}$ *reade* $\frac{41}{44}$.
P. 135. Lin. 36. *for* diuided 36, *reade* diuided by 36.
P. 135. l. 38 *for* ſide ſquare, *read* ſide 5, ſquare
P. 136. L. 13. *for* $\frac{1}{24}$ *reade* $\frac{1}{24}$.
P. 137. L. 20, *for* $\frac{1}{1}$ + *reade* $\frac{1}{1}$ +
P. 138. l. 8 *for* ſide 3 deduct *read* ſide. Deduct
P. 141. L. 18. *for* $\sqrt{3}$ v. 198 *reade* $\sqrt{3}$ v. 168. *and in the Margent*, *for* $\sqrt{3}$ v. 310 *reade* $\sqrt{3}$ v. 30.
P. 141. L. 22. *for* $\sqrt{3}$ 188 *reade* $\sqrt{3}$ 180.
P. 141. L. 24. *for* $\sqrt{3}$ v. 168. *reade* $\sqrt{3}$ v. 108. *and the like in line* 26.
Pa. 141. L. 4. *for* $\sqrt{3}$ $\frac{1}{4}$ *reade* $\frac{1}{4}$.
P. 152. l. 25. *for* concent or *reade* content of.
P. 151. L. 16. *for* $\sqrt{}$ 1220703 $\frac{1}{4}$ *reade* 1220703 $\frac{1}{4}$.
p. 151. L. 17, *for* $\frac{67}{316}$ *reade* $\frac{97}{316}$, *and for* $\frac{2240}{46316}$ *reade* $\frac{2240}{46316}$.
pa. 152. L. 27. *for* $\sqrt{3}$ v. 84 — $\sqrt{3}$ *read* $\sqrt{3}$ v. 784 — $\sqrt{3}$.
P. 152, L. 31. *for* $\sqrt{3}$ v. 2 $\frac{11}{15}$ *reade* $\sqrt{3}$ v. 2 $\frac{11}{15}$.
pa. 153. L. 22. *for* $\sqrt{3}$ $\frac{6613 \cdot 41 \cdot 148 \cdot 440}{3300}$ *reade* $\frac{6613 \cdot 41 \cdot 148 \cdot 448}{3300}$.
pa. 153. L. 26, *for* $\sqrt{3}$ $\frac{3111}{64}$ *reade* $\sqrt{3}$ $\frac{3111}{64}$
pag. 165. L. 16, *for* 48 *reade* 14.
pag. 167. L. 7. *for* + $\sqrt{3}$ $\frac{2}{31}$ *reade* + $\sqrt{3}$ $\frac{2}{30}$.

In page 147, *there wanteth at the ende theſe two Theoremes enſuing.*

The 18 *Theoreme.*

Icoſaedrons Diameter diuided by extreame & meane proportion maketh his greater part the Diameter of his comprehending Dodecaedrons Baſis internall Circle.

The 19 *Theoreme.*

Octaedrons Dimetient exceedeth in power the Dimetient of Icoſaedron by the Diameter of Dodecaedrons Baſis internall Circle.

FINIS.